STUDIES ON EDUCATION, SCIENCE, AND TECHNOLOGY 2025

Editors
Prof. Dr. Abid Ali Khan
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Chapter 1 - Teachers' Perceptions of GenAI Use in Higher Education: Learning Performance, Motivation, Effort Expectancy, Trustworthiness, and Intention to Use

Omid Noroozi , Nafiseh Taghizadeh Kerman , Seyyed Kazem Banihashem

Chapter Highlights

- While Generative Artificial Intelligence (GenAI) tools are increasingly being used by teachers in higher education, we still lack insight into their views on their use and potential applications. This study explores teachers' perceptions of GenAI tools across five key dimensions: learning performance, motivation, effort expectancy, trustworthiness, and intention to use.
- Data were collected through the spread of an online survey among 390 higher education teachers at a Dutch university during the 2023–2024 academic year. The results indicated that teachers generally found GenAI tools useful, easy to use, and moderately trustworthy. Effort expectancy was high, meaning that teachers generally found GenAI tools easy to learn, use, and interact with.
- However, the motivation to use GenAI was moderate, and there was variability in intention. Some teachers expressed a strong desire to continue using GenAI tools, while others were uncertain or less inclined to engage with them in the future.
- These findings suggest that while GenAI tools show promise in supporting teachers' performance, there are challenges related to motivation and long-term intention to use. The study provides insights for institutions aiming to integrate GenAI tools effectively into higher education.

Introduction

Generative Artificial Intelligence (GenAI) is increasingly reshaping higher education, introducing a new paradigm in how knowledge is accessed, produced, and assessed. Tools such as ChatGPT, Gemini, DeepSeek, and other large language models have created novel affordances for educational stakeholders, including students, teachers, and administrators. These systems are capable of generating text, solving problems, and even simulating conversations, thereby offering the potential to support educational goals such as feedback provision, instructional design, and learner support (Farrokhnia et al., 2024; Banihashem et al., 2024). The increasing integration of such technologies into educational systems calls for a deeper understanding of how they are perceived by those directly responsible for teaching and learning. While a growing body of research has started to examine students' perceptions of GenAI in academic contexts (Daher & Hussein, 2024), the views of teachers, who play a foundational role in shaping instructional experiences, remain underexplored.

As the primary designers and facilitators of learning environments, teachers hold significant influence over how technologies are interpreted, adapted, and ultimately used within educational practice (see Masoumi & Noroozi, 2025). Their perceptions matter not only because they impact implementation decisions but also because they reflect the practical, pedagogical, and ethical judgments that shape teaching. Unlike students, who often focus on the convenience and immediacy of GenAI tools for tasks such as summarizing texts or generating ideas, teachers must assess these tools from a multidimensional lens: are they pedagogically sound? Do they align with the intended learning outcomes? Can they be trusted to support rather than hinder student development? (Zawacki-Richter et al., 2019). As such, investigating teachers' perceptions is essential for understanding both the opportunities and limitations of GenAI tools in educational settings (Noroozi et al., 2025; Soleimani et al., 2025).

Teachers' engagement with GenAI technologies is shaped by their expectations, prior experiences, institutional culture, and the specific demands of their disciplines (Zawacki-Richter et al., 2019; Holmes et al., 2019). For example, in fields requiring high degrees of precision, such as science or mathematics, concerns about accuracy and reliability may dominate perceptions of GenAI (Farrokhnia et al., 2024). In contrast, in the humanities and social sciences, issues such as originality, creativity, and ethical implications may be more pronounced (Daher & Hussein, 2024). Moreover, the practical integration of these tools depends on teachers' digital competencies and access to training (UNESCO, 2023; Holmes et al., 2019). If teachers lack confidence in evaluating or guiding AI-generated content, their hesitation may stem not from resistance but from concerns about professional responsibility and potential misuse (Zawacki-Richter et al., 2019; UNESCO, 2023). Therefore, perceptions are not static judgments but dynamic evaluations shaped by disciplinary context, institutional support, and broader pedagogical values.

In addition to disciplinary and institutional factors, teachers' perceptions of GenAI are often guided by broader pedagogical concerns. One recurring theme in the literature is the tension between efficiency and depth. GenAI tools offer the ability to quickly generate explanations, assessments, or lesson plans, which can reduce the time burden on teachers. However, this efficiency may come at a cost: concerns have been raised about the de-skilling of teachers, overreliance on automated suggestions, and the erosion of pedagogical authenticity (Holmes et al., 2019; Zawacki-Richter et al., 2019). Teachers may wonder whether using GenAI for feedback or content creation undermines their professional agency or diminishes the teacher-student relationship. This ambivalence necessitates a careful and context-sensitive exploration of how teachers perceive these tools, not just in terms of usefulness, but also about their core educational values.

Another dimension of concern involves ethical considerations, particularly around academic integrity, bias, and transparency. Teachers are responsible for upholding educational standards and ensuring that assessments reflect genuine student learning. The potential for GenAI tools to generate misleading or plagiarized content raises questions about how their use can be monitored or integrated ethically (Farrokhnia et al., 2024). Furthermore, the quality of GenAI outputs varies significantly depending on prompts, platform, and underlying datasets, which may contain implicit cultural or disciplinary biases. Teachers must therefore evaluate not only what GenAI can do, but also what it should do in their particular teaching context. Such ethical reflection is integral to responsible AI use and cannot be divorced from the broader question of teacher perceptions.

Despite these concerns, GenAI tools also offer real potential to enhance the teaching profession when thoughtfully applied. Studies have noted that teachers can benefit from GenAI in drafting course materials, brainstorming ideas for class activities, creating illustrative examples, and even simulating student responses for preparation (Banihashem et al., 2024). Moreover, in resource-constrained contexts, GenAI may serve as a support tool that helps teachers meet rising expectations without compromising quality. Whether these possibilities are realized, however, depends heavily on how teachers perceive GenAI in relation to their needs, values, and teaching philosophy. A positive perception could lead to creative adoption, while skepticism may lead to cautious rejection or minimal engagement.

To fully capture the complexity of teacher perceptions, it is necessary to consider multiple dimensions. These include beliefs about whether GenAI improves teaching performance (perceived learning performance), the degree to which it inspires or motivates teachers' (motivation), how intuitive and easy it is to use (effort expectancy), trust in the accuracy and reliability of its outputs (trustworthiness), and whether teachers intend to use such tools in the future (intention to use) (Daher & Hussein, 2024). Each of these dimensions highlights key aspects of how teachers relate to GenAI tools in their professional roles. As educational institutions increasingly adopt AI-assisted teaching models, understanding teacher perceptions becomes essential (Zawacki-Richter et al., 2019; Holmes et al., 2019). Without a grounded understanding of their views, any attempt at systemic GenAI integration risks resistance, misalignment with pedagogical goals, or superficial implementation (UNESCO, 2023). Teachers must be involved not only as users of these tools but also as co-designers of their integration. This requires institutional investment in research, training, and policy design that centers the teacher perspective and supports pedagogical innovation (Banihashem et al., 2025; Zawacki-Richter et al., 2019).

This study seeks to contribute to this understanding by addressing the following research question: How do teachers perceive the use of GenAI tools in higher education in terms of learning performance, motivation, effort expectancy, trustworthiness, and intention to use? By investigating this question, we aim to provide evidence-based insights that support responsible, inclusive, and teacher-informed approaches to GenAI integration in higher education.

Method

Context and Participants

This exploratory and survey-based study was conducted at a Dutch university and involved

390 higher education teachers in five life sciences courses during the 2023-2024 academic year. Of these, 302 participants were included in the final analysis, as 88 teachers (23%) did not complete the survey. The demographic information of the participants is listed in Table 1.

Table 1. Demographic Information of the Study Sample

Variable	Category	N	%
Gender	Female	123	41
	Male	157	52
	Prefer not to say	21	7
Field of Study	Plant Sciences	52	17
	Social Sciences	66	22
	Animal Sciences	27	9
	Environmental Sciences	73	24
	Agrotechnology & Food Sciences	54	18
	Other	30	10
Nationality	Dutch	203	67
	Non-Dutch	92	30
Years of Experience	<1 year	6	2
	1-5 years	124	41
	6-10 years	57	19
	11-15 years	51	17
	16-20 years	21	7
	<20 years	30	10

Measurements and Analysis

To assess teachers' perceptions of using GenAI tools, Venkatesh et al. (2012) and the UNESCO report developed a questionnaire based on frameworks. The questionnaire included 16 items evaluating five components: learning performance (e.g., I find GenAI useful for my work), motivation (e.g., using GenAI is fun), effort expectancy (e.g., learning how to use GenAI is easy for me), trustworthiness (e.g., I can trust the information presented to me by GenAI), and intention to use (e.g., I intend to continue using GenAI in the future).

Responses were measured on a five-point Likert scale ranging from "strongly disagree" (1) to "strongly agree" (5). A score of 5 indicates a highly positive perception, while a score of 1 reflects a negative perception, indicating strong disagreement. The scale demonstrated high reliability, with a Cronbach's alpha of 0.95, and descriptive analyses provided mean and standard deviation for each item and component

Results

The findings revealed insights into teachers' perceptions of GenAI tools across five dimensions.

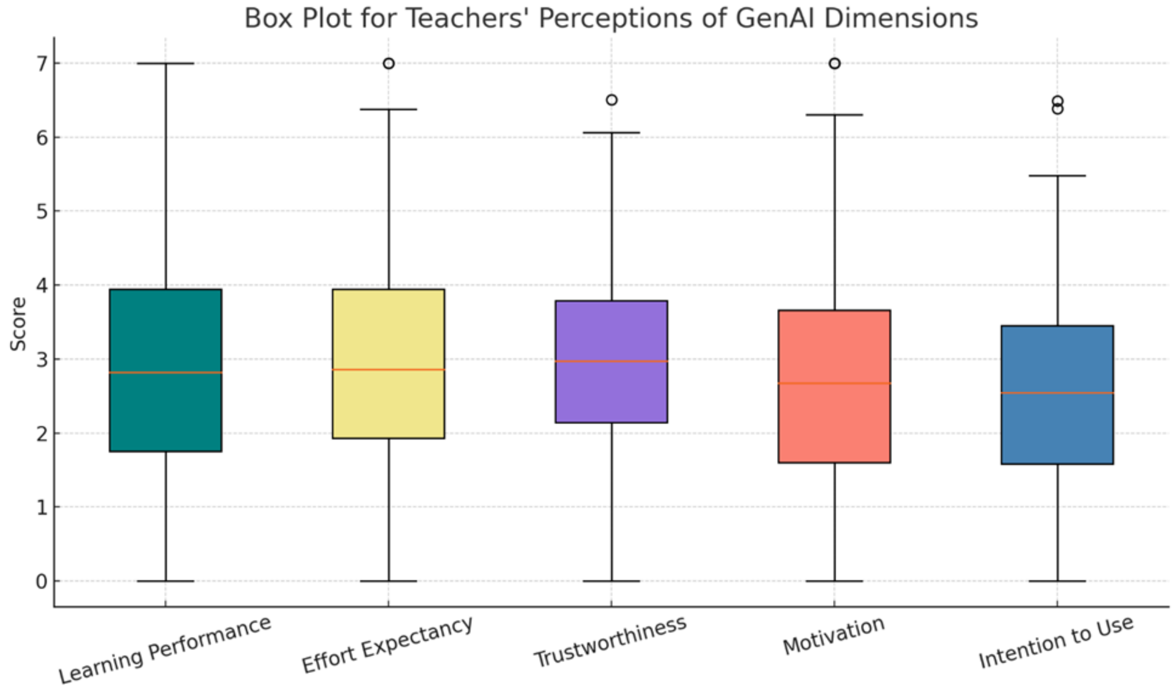

Figure 1. Descriptive Statistics of Teachers' Perceptions of Using GenAI in Higher Education

Learning performance, with an overall mean of 2.70, indicated that teachers generally find GenAI useful for their learning performance and productivity. Among the items, the highest-rated statement was that using GenAI was useful for their work (M = 2.89, SD = 1.69), while the lowest-rated was its impact on productivity (M = 2.50, SD = 1.70). For *motivation*, the results showed a moderate level of enjoyment, with an overall mean of 2.62. Teachers found GenAI moderately fun (M = 2.72, SD = 1.64), though entertainment and enjoyable were rated slightly lower (M = 2.58, SD = 1.56 and M = 2.57, SD = 1.58).

The *effort expectancy* was rated relatively high (M = 3.00, SD = 1.56), suggesting that teachers perceive GenAI as easy to learn and use. Among the items, the statement "I find GenAI easy to use" received the highest score (M = 3.08, SD = 1.68). However, *trustworthiness* in GenAI scored mixed results. While teachers expressed skepticism about its accuracy, as indicated by high agreement with the statement "If I use GenAI I think I would need to check its responses against other sources" (M = 3.83, SD = 1.75), their overall trustworthiness level in the tool was moderate (M = 2.97, SD = 1.27). Finally, *intention to use* showed variability, with an overall mean of 2.50. While some teachers indicated an intent to continue using GenAI (M = 3.08, SD = 1.77), others were less certain about consistent future usage (M = 1.89, SD = 1.38) (see Figure 1).

Discussion

Understanding how teachers perceive GenAI tools is essential for informing their responsible integration into higher education. The findings of this study shed light on the multifaceted nature of teachers' experiences, highlighting both promising opportunities and critical reservations. These insights underscore the necessity of addressing not only the technical affordances of GenAI, but also the professional, pedagogical, and ethical dimensions that shape its adoption.

The relatively moderate perception of GenAI's contribution to learning performance suggests that, while some teachers recognize its practical utility; particularly in automating or accelerating certain tasks, this has not translated into a strong sense of pedagogical enhancement. This cautious stance reflects concerns about depth, relevance, and the ability of such tools to meaningfully support complex educational goals (Banihashem et al., 2024; Holmes et al., 2019). Teachers appear to be weighing the convenience offered by GenAI against the risk of oversimplification or misalignment with their instructional intent.

Equally important are the findings regarding motivation. Although some teachers expressed mild interest in using GenAI, overall motivation levels remained low. Prior research has suggested that without a clear pedagogical purpose or alignment with professional identity, the initial curiosity surrounding new technologies tends to diminish rapidly (Daher & Hussein, 2024; Noroozi et al., 2024). This implies that motivation among teachers may depend less on novelty or entertainment, and more on whether a tool genuinely supports their educational philosophy.

By contrast, effort expectancy received relatively favorable ratings, indicating that teachers found GenAI tools easy to learn and use. While this dimension is often considered a key enabler of adoption, its influence appears to be limited when not accompanied by trust and perceived educational value (UNESCO, 2023). Technical usability, though necessary, is insufficient on its own to drive meaningful or sustained integration.

Concerns about trustworthiness were especially salient. Many participants indicated a need to verify GenAI-generated content before use; an indication of lingering skepticism regarding accuracy, consistency, and bias. Such caution is well-founded, given that GenAI tools may produce plausible but misleading responses, particularly in nuanced academic contexts (Farrokhnia et al., 2024; Holmes et al., 2019). Without a strong sense of reliability, teachers are unlikely to use these tools in pedagogically high-stakes areas such as assessment, feedback, or content curation.

The variability in teachers' intention to use GenAI further emphasizes the importance of context. Differences in disciplinary background, teaching experience, and institutional culture all appear to influence attitudes toward adoption. Some teachers expressed a readiness to explore GenAI further, while others were uncertain or hesitant. This heterogeneity supports calls for more differentiated and responsive implementation strategies; ones that take into account the diverse realities of teaching practice (Banihashem et al., 2025; Zawacki-Richter et al., 2019).

Taken together, these findings point to a central insight: that GenAI adoption in higher education is not simply a matter of technical capacity or institutional mandate. Rather, it hinges on the professional judgment, ethical standards, and pedagogical values of teachers. They are not passive recipients of technological change; they are active agents whose perceptions must be considered foundational in any effort to integrate AI into education meaningfully and responsibly (UNESCO, 2023; Banihashem et al., 2025).

Conclusion

This study provides evidence that teachers' perceptions of GenAI tools are shaped by a complex interplay of ease of use, trust, motivation, and perceived pedagogical relevance. While GenAI was generally seen as user-friendly, teachers' concerns regarding its trustworthiness and educational value limited their willingness to adopt it consistently.

These findings suggest that institutional strategies for GenAI integration must go beyond promoting usability. They should actively engage teachers in dialogue, offer context-sensitive professional development, and prioritize pedagogical alignment and ethical reflection (Zawacki-Richter et al., 2019; UNESCO, 2023). Teachers need to be positioned not merely as users of AI, but as co-designers in shaping its role within higher education (Banihashem et al., 2025).

Future research should examine how these perceptions evolve over time and across disciplinary contexts, particularly as GenAI becomes increasingly embedded in teaching and learning environments. Longitudinal and qualitative approaches could offer deeper insight into how teachers negotiate the risks and rewards of AI integration. Ultimately, the success of GenAI in higher education will depend not just on what the technology can do, but on how well it aligns with the principles, values, and professional agency of the teachers who use it.

References

Banihashem, S. K., Kerman, N. T., Noroozi, O., Moon, J., & Drachsler, H. (2024). Feedback sources in essay writing: Peer-generated or AI-generated feedback? *International Journal of Educational Technology in Higher Education*, *21*(1), Article 23. https://doi.org/10.1186/s41239-024-00455-4

Banihashem, S. K., Noroozi, O., Khosravi, H., Schunn, C. D., & Drachsler, H. (2025). Pedagogical framework for hybrid intelligent feedback. *Innovations in Education and Teaching International*, 1–17. https://doi.org/10.1080/14703297.2025.2499174

Daher, W., & Hussein, A. (2024). Higher education students' perceptions of GenAI tools for learning. *Information*, *15*(7), Article 416. https://doi.org/10.3390/info15070416

Farrokhnia, M., Banihashem, S. K., Noroozi, O., & Wals, A. (2024). A SWOT analysis of ChatGPT: Implications for educational practice and research. *Innovations in Education and Teaching International*, *61*(3), 460–474. https://doi.org/10.1080/14703297.2023.2195846

Holmes, W., Bialik, M., & Fadel, C. (2019). *Artificial intelligence in education: Promises* *and implications for teaching and learning*. Center for Curriculum Redesign. https://discovery.ucl.ac.uk/id/eprint/10139722/

Masoumi, D., & Noroozi, O. (2025). Developing early career teachers' professional digital competence: A systematic literature review. *European Journal of Teacher Education*, *48*(3), 644–666. https://doi.org/10.1080/02619768.2023.2229006

Noroozi, O., Khalil, M., & Banihashem, S. K. (2025). Artificial intelligence in higher education: Impact depends on support, pedagogy, human agency, and purpose. *Innovations in Education and Teaching International*, 1–6. https://doi.org/10.1080/14703297.2025.2539579

Noroozi, O., Soleimani, S., Farrokhnia, M., & Banihashem, S. K. (2024). Generative AI in education: Pedagogical, theoretical, and methodological perspectives. *International Journal of Technology in Education*, *7*(3), 373–385. https://doi.org/10.46328/ijte.845

Soleimani, S., Farrokhnia, M., van Dijk, A., & Noroozi, O. (2025). Educators' perceptions of generative AI: Investigating attitudes, barriers and learning needs in higher education. *Innovations in Education and Teaching International*. Advance online publication. https://doi.org/10.1080/14703297.2025.2530767

UNESCO. (2023). *Guidance for generative AI in education and research*. United Nations Educational, Scientific and Cultural Organization. https://unesdoc.unesco.org/ark:/48223/pf0000386217

Venkatesh, V., Thong, J. Y. L., & Xu, X. (2012). Consumer acceptance and use of information technology: Extending the unified theory of acceptance and use of technology (UTAUT2). *MIS Quarterly*, *36*(1), 157–178. https://doi.org/10.2307/41410412

Zawacki-Richter, O., Marín, V. I., Bond, M., & Gouverneur, F. (2019). Systematic review of research on artificial intelligence applications in higher education – Where are the educators? *International Journal of Educational Technology in Higher Education*, *16*(1), 1–27. https://doi.org/10.1186/s41239-019-0171-0

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Chapter 2 - Formative Assessment in Physics Education: A Systematic Literature Review

Fauziah Rasyid, Heru Kuswanto, Supahar Supahar

Chapter Highlights

- The implementation of Formative Assessment (FA) has been extensively documented in numerous studies as an effective tool for enhancing physics learning. However, various challenges have been identified in implementing effective formative assessment. Therefore, this study conducts a Systematic Literature Review (SLR) to explore the application of FA in physics education and evaluate its effectiveness.
- The SLR procedure follows the PRISMA guidelines, incorporating inclusion and exclusion criteria. A systematic literature search was conducted using the Scopus and ERIC databases. After a comprehensive search and screening process of 178 studies published between 2015 and 2024, we selected 25 articles for in-depth analysis.
- Our findings identify potential tools and strategies that can enhance the effectiveness of physics learning, such as technology-assisted FA and computer-simulation-based FA. Furthermore, our findings reinforce existing literature, demonstrating that FA supports students' learning progress.
- FA impacts not only students' academic performance and attitudes in physics learning but also influences physics teachers' instructional practices and attitudes. This study contributes to researchers and practitioners by providing valuable insights into the effective implementation of FA tools and strategies in physics education.

Introduction

Physics is a subject known for its difficulty, containing abstract and complex concepts (Pals et al., 2023). Therefore, efforts are needed to make physics learning more effective for students. One such effort is the implementation of assessment strategies that support and enhance students' learning processes

The implementation of formative assessment has been confirmed to enhance the effectiveness of physics learning. The application of formative assessment (FA) in physics education serves as an effective strategy to improve students' conceptual understanding of physics (Peter & Uwamahoro, 2023)), enhance the learning process, and boost student performance (Nurjanah et al., 2024). Formative assessment, also known as "assessment as or for learning," holds significant potential to influence student engagement, students' perceptions, and overall learning outcomes (X. Chen et al., 2018). Formative assessment can assist students in improving their performance and academic achievement (Ganajová et al., 2021). Assessing the effectiveness of FA, physics educators should implement FA to enhance physics teaching and learning.

FA can function optimally only if teachers use the data collected through formative assessment to adjust their instruction (Black & Wiliam, 1998). Furthermore, the formative assessment process is only effective if assessment techniques are valid, if the feedback provided is useful for guiding learning, and if the feedback is actively used (Sarwar & Trumpower, 2015). However, studies by Khan et al. (2020) indicate that teachers have not been able to fully optimize FA, as they do not adequately monitor students' physics learning progress and fail to provide effective feedback It is widely known that the practice of formative assessment (FA) is one of the challenges faced by teachers (Atasoy & Kaya, 2022; Gotwals, 2018). Therefore, the practice of physics teachers in applying FA in the learning process remains under scrutiny.

The effective implementation of FA faces numerous challenges. The ideal application of formative assessment in learning presents considerable challenges (Ganajová et al., 2021). These challenges include requiring more time (Bleckmann & Friege, 2023), difficulties in implementing FA in large classes (Beerepoot, 2023) and teachers' lack of understanding in applying FA (Arrafii & Sumarni, 2018; Halim et al., 2024; Zivanayi & Nwaigwe, 2024).

Addressing these challenges requires an investigation into how FA is implemented in the learning physics as guidance for teachers and researchers in physics education.

To provide deeper insights into FA practices in physics learning, a systematic literature review (SLR) study is needed. Systematic literature reviews are essential for identifying research priorities and understanding the initial conceptual framework for future research (Higgins et al., 2019). Systematic reviews help answer questions that individual studies cannot, as well as explain and evaluate how phenomena occur (Page et al., 2021). Moreover, systematic reviews clarify and deepen the information obtained (Ilma et al., 2023). Therefore, a systematic review is used to comprehensively explore FA implementation in physics education.

Previous researchers have conducted SLR on FA such as in the context of higher education. Morris and Rohs (2023) examined FA and feedback practices in higher education categorized into timing and spacing; quizzing and testing; peers and technology. Sembey et al (2024) examined new technologies in higher education assessment and feedback practices. Sembey et al. (2024) identified the application of FA into personalized assessment; assessing self-regulated learning; automated assessment and feedback design and incorporating effectiveness evaluation in the form of tool performance, student performance and student perceptions and attitudes.

Meanwhile, the previous SLR on FA in secondary schools is Halim et al. (2024) They examined the implementation of FA in teaching and learning in secondary schools and categorized FA diversity into peer assessment, teacher assessment and self-assessment. In addition, FA strategies were categorized into teacher planning, assessment methods and instruments and grading. Previous SLR studies examined based on educational levels, but studies based on disciplines such as physics are still limited.

So far, only one SLR study in the discipline of physics was found, namely Nurjanah et al. (2024) who studied FA in high school and categorized FA into technology-based, test-based, reflection and informal feedback, card-based, chart or graph-based and others. In addition, study by Nurjannah et. al (2024) involved the high school level of education. Therefore, to obtain a broader scope, it is necessary to study the application of FA in physics education at various levels of education.

The study of FA in the subject of Physics is still very limited, while the practice of FA should not be generalized. The application of various assessments is highly dependent on the specificity of the teaching field (Bennett, 2011) and it is important to examine teachers' practices within the discipline to fully characterize expertise in FA (Gotwals et al., 2015).Therefore, this SLR aims to examine findings research results related to how FA is implemented in physics classrooms. The study will focus on the FA strategies and tools applied as well as how effective FA is in learning Physics. This study contributes to providing deeper insights into FA practices that can be used as a guide for physics teachers to enhance their teaching practices and improve student achievement in learning physics. Additionally, to complement the understanding of FA in physics education based on the literature, mapping the current state of FA studies in physics learning is conducted. This is to identify research trends and provide insights into areas requiring further development and future research. Therefore, the following research questions are formulated for this systematic review study:

RQ 1: What is the distribution of studies according to publication year, education level, research method, and physics topic?
RQ 2 : What is strategy of implementing FA in physics education?
RQ 3 : What is the effectiveness evaluation of FA studies in physics education?

Method

Literature Search Strategy

This study aims to review the literature on formative assessment in physics education. This systematic literature review follows the PRISMA guidelines with a descriptive content analysis approach (Page et al., 2021). The literature sources were obtained from the Scopus and ERIC databases. A literature search was conducted in both databases using the keywords “formative assessment” AND “physics.”

The search in Scopus was performed within the Article Title, Abstract, and Keywords fields, with a restriction to the Subject Area “Social Sciences,” yielding 107 articles. The literature search in ERIC was limited to peer-reviewed articles, resulting in 71 articles. After identifying the search results from both databases, 34 duplicate articles were found, leading to a total of 144 screened records.

Selection Criteria

After identifying the literature search results from the databases, article selection was carried out based on predefined inclusion and exclusion criteria. These criteria were established to filter articles that align with the study's objectives and scope of formative assessment in physics education. The inclusion and exclusion criteria are presented in Table 1.

Table 1. Inclusion and Exclusion Criteria

Type of criterion	Criteria	Inclusion	Exlusion
Type of publication	Journal articles	X	
	Conference paper	X	
	Reports		X
	Dissertations		X
	Books and book chapters		X
Open access	Open access	X	
	Limited access		X
Publication period	January 2015-December 2024	X	
Language	English	X	
	Other		X
Place of study	Worldwide	X	
Type of study	Empirical research	X	
	Literature reviews		X
	Theoretical reviews		X
Focus on the subject	Physics	X	
	Other		X
Topic	Formative assessment to enhance physics teaching and learning.	X	
	Other		X

These inclusion and exclusion criteria were carefully applied to the identified articles, resulting in 25 selected articles that were included and analyzed in this study. The process of selecting the included articles followed the stages of identification, screening, eligibility, and inclusion, which are detailed in the following flowchart.

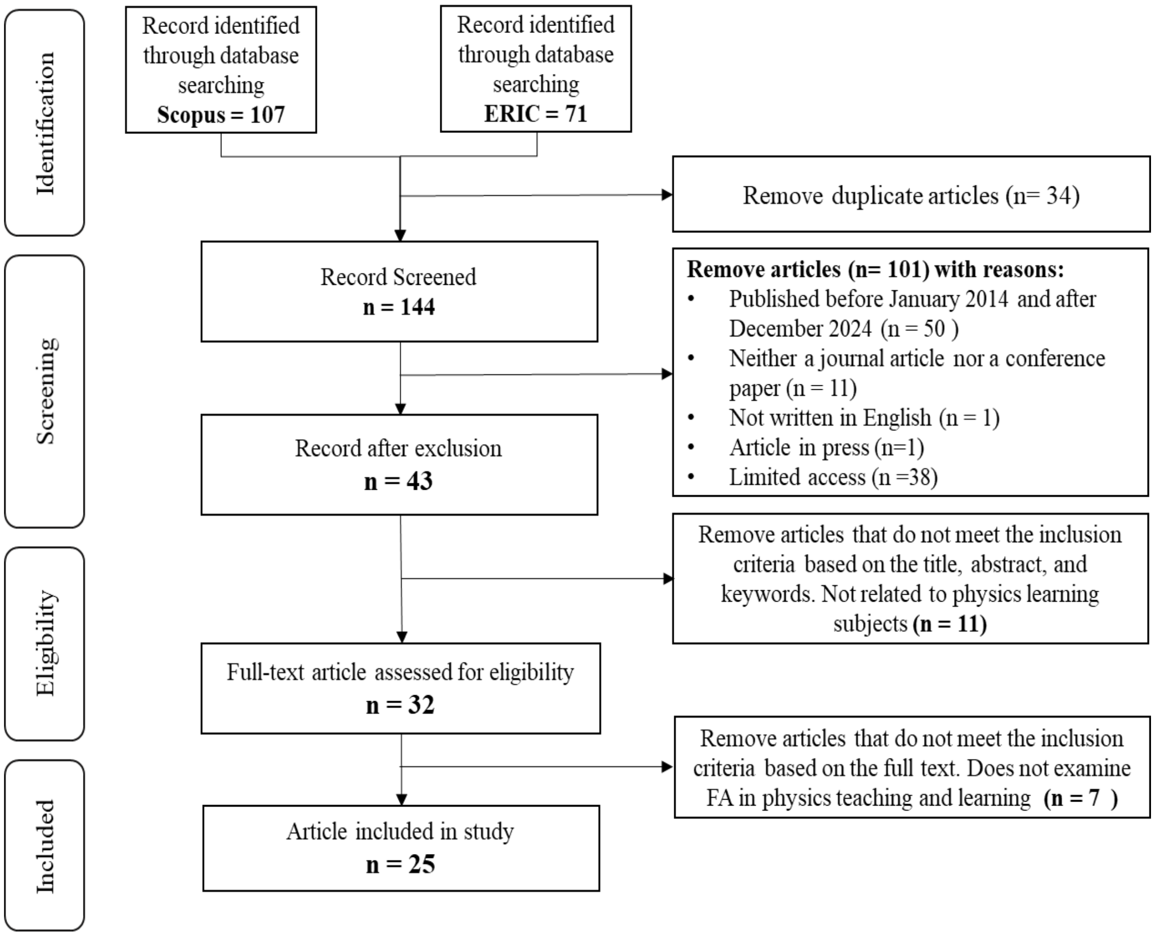

Figure 1. Flowchart of Study Selection Procedure

Data Exraction

The data extracted from the 25 relevant articles included in this systematic review consist of the author's name and publication year, research objectives, research methods, education level, physics topics, formative assessment tools, formative assessment implementation strategies, and key findings (main findings, effectiveness indicators, and research contributions).

After data extraction, the findings in this study were categorized based on FA tools and strategies, as well as the evaluation of FA effectiveness in studies adapted from the approach used by Sembey et al. (2024).

Results

Distribution of Studies

Based on the selection and screening process, a total of 25 studies were identified, all of which are journal articles. Based on the year of publication, the 25 studies examining the implementation of FA in physics education are presented in the diagram in Figure 2.

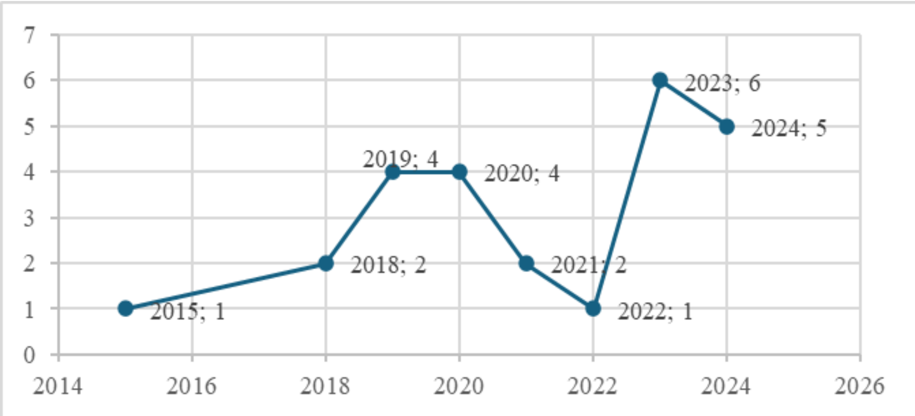

Figure 2. Number of Studies included by Year

Research on FA in physics education has gained attention over the past decade. Since 2015, the number of publications discussing this topic has shown an increasing trend, reaching its peak in 2019. However, after that, there was a decline in the number of publications, which continued until 2022. In 2023, interest in this topic increased significantly again, although there was a slight decline in 2024. The studies examining the implementation of FA in physics education involve 12 countries as contributors to the literature. The distribution of studies by country is illustrated in Figure 3.

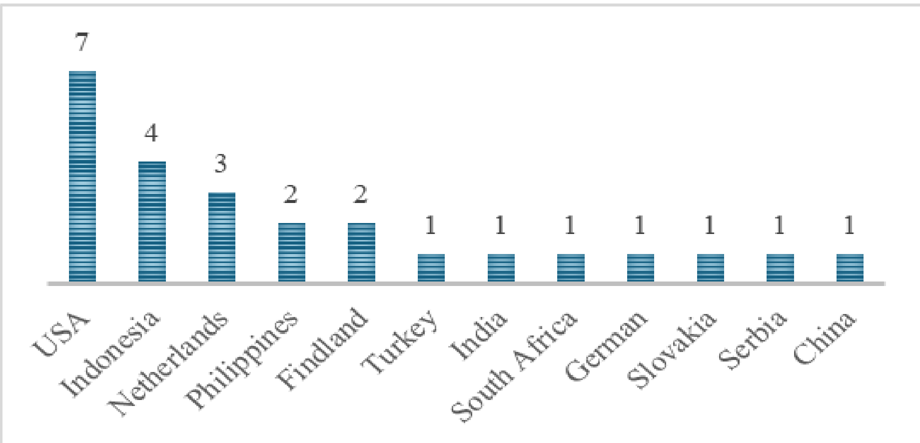

Figure 3.The Number of Studies included by Country

Figure 3 shows that the highest number of studies comes from the USA (n=7), followed by Indonesia (n=4) and the Netherlands (n=3). Meanwhile, the countries with the fewest studies are Turkey, India, South Africa, Germany, Slovakia, Serbia, and China, each with one study (n=1).

Meanwhile, based on the education level, these studies involve research subjects from four educational levels. The distribution of studies based on educational levels is presented in Figure 4.

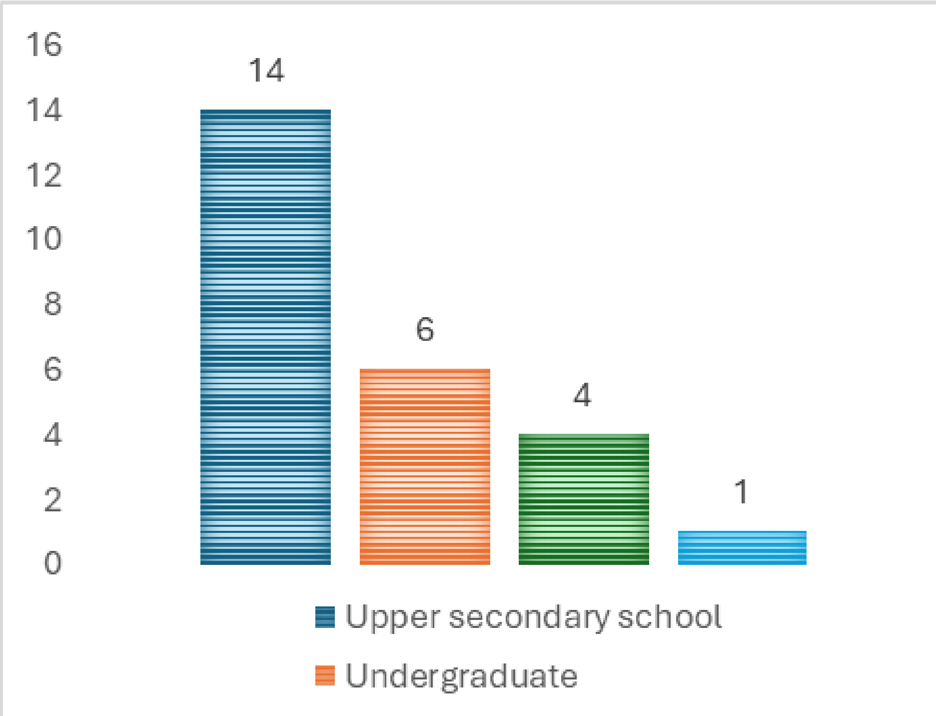

Figure 4. The Number of Studies included by Educational Level

Figure 4 shows the education levels of students involved in studies on FA in physics learning. Figure 4 indicates that teachers and students at the upper secondary school level are the primary beneficiaries of studies on the implementation of FA in physics (n=14). Meanwhile, the least involved education level is pre-university education (n=1).

The research methods used in the studies consist of four types: quantitative, qualitative, research development, and mixed methods. The research methods included in the studies are detailed in Table 2.

Table 2. Research Method Used by Studies Included

Method	n	Description
Quantitative	9	Quasi experiment (Pusawale & Kadam, 2024; Pals et al., 2024; Hidayat & Irdiyansyah, 2023; Ganajová et al., 2021) Experiment (Đorić et al., 2019; Gladding et al., 2015; Molin et al., 2021) Pre experimental (Park, 2019); Non experimental (Chen et al., 2018)
Qualitative	8	Case study (Ketonen et al., 2020; Muslu & Siegel, 2024; Nieminen et al., 2020; M. Park, 2020; Zhai et al., 2018; Zivanayi & Nwaigwe, 2024) Design-based research (Alonzo et al., 2022; Deverel-Rico et al., 2024)
Mixed method	4	Convergent parallel mixed-methods (Buggé, 2023) Concurrent triangulation design (Ole & Gallos, 2023a) Quantitative and qualitative (Kusairi et al., 2019; Ole & Gallos, 2023b)
Research and Development	4	Test development (Pals et al., 2023) Development of machine learning model (Bleckmann & Friege, 2023) Development of game-based feedback (Kusairi et al., 2020) and web-based feedback (Kusairi, 2019)

Table 2 shows the number of studies based on the research methods used. Table 2 indicates that the quantitative method is the most frequently used in studies on FA in physics learning (n=9), followed by the qualitative research method (n=8). The least commonly used research methods are the mixed method and research and development (n=4). The distribution of studies based on the research methods used is presented in the pie chart in Figure 5.

Figure 5 shows that the distribution of studies based on the research methods used indicates that 36% (n=9) of the studies employed the quantitative method, making it the most commonly used research method in studies on FA in physics education. Quantitative studies are used to examine the effectiveness of FA in developing students' abilities and performance in physics (X. Chen et al., 2018; Ganajová et al., 2021; Hidayat & Irdiyansyah, 2023; Kusairi et al., 2019; Pusawale & Kadam, 2024) or in shaping the attitudes and perceptions of physics teachers (Zivanayi & Nwaigwe, 2024).

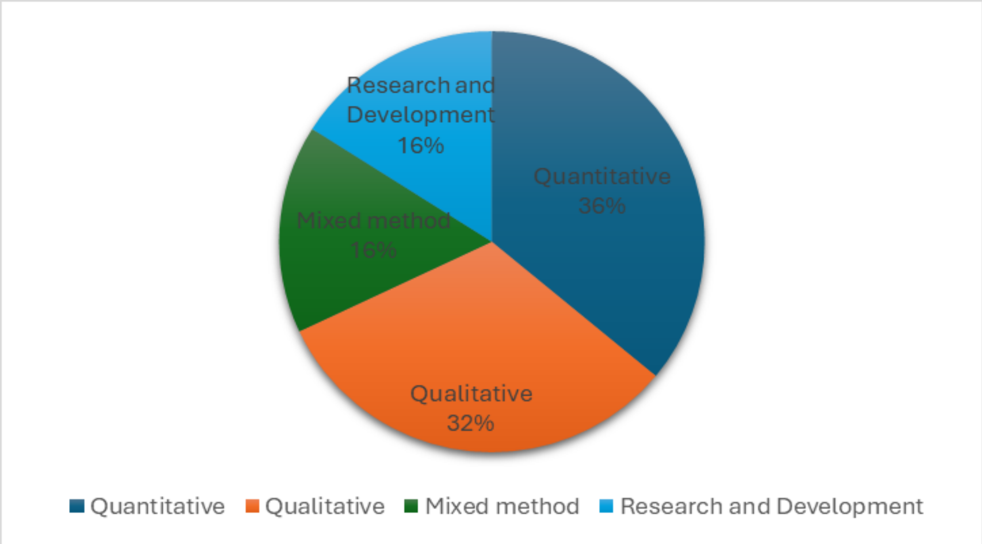

Figure 5. Distribution of Studies by Research Method

In addition to the quantitative method, the qualitative method is also widely used, accounting for 34% (n=8) of the studies, making it the second most frequently used method. Qualitative studies are conducted to explore the implementation of FA in physics teaching practices in greater depth (Alonzo et al., 2022; Deverel-Rico et al., 2024; Nieminen et al., 2020; Pals et al., 2023) and to analyze teacher-student interactions in the application of FA ((Z. Chen et al., 2021; Gladding et al., 2015; Ketonen et al., 2020; M. Park, 2020).

Other research methods used in the included studies are the mixed method and research and development, both of which have an equal proportion of 16% (n=4). Studies using the mixed method, which integrates quantitative and qualitative data, aim to determine teachers' classroom attitudes and practices regarding FA (Ole & Gallos, 2023a) and to assess the effectiveness of FA, student engagement, and learning difficulties related to FA (Buggé, 2023; Kusairi et al., 2019; Ole & Gallos, 2023b). Research and development studies are used to develop formative assessment tools such as formative tests (Pals et al., 2023) and technology-enhanced assessment and feedback systems (Bleckmann & Friege, 2023; Kusairi, 2019; Kusairi et al., 2020).

Implementation Strategies of FA in Physics Education

Based on the 25 analyzed studies, the tools and strategies for implementing FA in physics education, along with the indicators for evaluating effectiveness used in the studies, were identified. The various tools used in formative assessment and feedback applied in physics learning, which can generally be divided into eight areas, as presented in Table 3.

Table 3. Areas of FA Strategies in Physics Education

Area	Description
Technology-enhanced assessment and feedback	Feedback through website (Kusairi, 2019), Feedback game (Kusairi et al., 2020) digital applications (Muslu & Siegel, 2024), web polling (Molin et al., 2021). Computer based immediate feedback "checkable answer feature" (X. Chen et al., 2018) Using machine learning to analyzing concept map (Bleckmann & Friege, 2023) Simulation based FA (Đorić et al., 2019; M. Park, 2019, 2020; Pusawale & Kadam, 2024) Formative E-Assessment (Kusairi et al., 2019) Online homework system (Gladding et al., 2018)
Test	Diagnostic test (Pals et al., 2023); i*somorphic multiple choice* (Kusairi, 2019; Kusairi et al., 2020)
Task and Learning Progressions	LP (Zhai et al., 2018; Alonzo et al., 2022) Task and LP (Deverel-Rico et al., 2024)
Comprehensive FA	Comprehensive FA (Hidayat & Irdiyansyah, 2023); Formative Assessment Classroom Techniques *(*FACTs) (Ganajová et al., 2021).
Formal and Informal FA	LP is used in both informal FA and formal FA practices (Zhai et al., 2018) On- the-fly FA (Nieminen et al., 2020)
Feedback model and strategy	Feedback loop model (Ole & Gallos, 2023a; Ole & Gallos, 2023b) Personalized feedback using stycky notes (Pals, et al., 2024)

Area	Description
Cartoon	Concept Cartoons (Zivanayi & Nwaigwe, 2024)
Peer assessment	Peer assessment of laboratory reports (Ketonen et al., 2020)

Technology-Enhanced Assessment and Feedback

Technology-enhanced assessment and feedback is the most rapidly growing field, examined in nearly half of the studies (n = 12). This area consists of two two sub area: technology-enhanced feedback and technology-enhanced assessment. First, we will discuss technology-assisted feedback, a popular topic studied in six studies. The following is a discussion of each study on technology-enhanced feedback.

Kusairi, (2019) developed a web-based application to provide specific and timely feedback, named Tryout and Webvoing. This application is integrated with isomorphic multiple-choice questions, displaying not only scores but also indicators of physics concept mastery. In line with this, Kusairi et al. (2020) also developed a game-based feedback system, the Physics Formative Feedback Game (PF2G), which integrates isomorphic tests and explanatory videos to enhance students' understanding of kinematics concepts. The trial results of these two feedback products show their potential in supporting physics teaching, but further research is needed to test their broader effectiveness.

Muslu & Siegel (2024) investigated the use of digital applications in providing feedback. This study emphasizes that effectiveness is influenced by teacher implementation, which plays a role in supporting aspects of feedback that applications cannot handle. Molin et al. (2021) studied teacher and peer feedback strategies based on polling technology. The results show that teacher feedback can be beneficial for student learning outcomes, but the greatest benefit comes from peer discussions followed by teacher feedback. Both studies emphasize that technology serves as a complementary tool that aids the feedback process, but teacher practice significantly influences feedback effectiveness.

Chen et al., (2021) examined student interaction with computer-based feedback, the Checkable Answer Feature (CAF), in blended learning-based mechanics instruction. The results show that students who actively and persistently utilize feedback and apply self-regulation strategies achieve better outcomes. These findings highlight the importance of perseverance and student learning strategies. The findings also provide insights for instructors in designing more effective online learning. Bleckmann & Friege (2023) developed a machine learning model to evaluate student performance on concept maps in mechanics. This model shows high agreement with human evaluation, indicating that machine learning has the potential to provide automated and immediate feedback that assists teachers in daily formative practice.

The second subsection, technology-enhanced formative assessment, discusses computer simulation-based FA and online FA. Simulation-based FA is discussed in three studies. The study by Pusawale & Kadam (2024) shows that simulation-based formative assessment is more effective in improving students' conceptual understanding than simply using simulations on quantum and optics topics. Park (2019, 2020) found that simulation-based formative assessment helps students understand kinematics concepts more validly, although misconceptions due to fragmented knowledge still exist, highlighting the role of equations as a conceptual understanding tool in problem-solving.

Đorić et al. (2019) examined the impact of three types of software simulations on Ohm's Law and resistors. The results show that simulations allowing students to build their own electrical circuits have a greater impact on academic performance compared to simulations with pre-built electrical circuits. Additionally, simulations that provide immediate computer-based feedback are more effective in enhancing student understanding than computer simulations with teacher feedback.

Online FA was examined by Kusairi et al. (2019), who investigated the implementation of formative authentic e-assessment to analyze students' conceptual understanding of motion topics. This e-assessment is integrated with modeling instruction, which includes online diagnostic tests, laboratory investigations, worksheets, practical sessions, quizzes, and online feedback. The findings show that e-assessment helps students improve conceptual understanding, although difficulties remain in distinguishing speed and velocity and interpreting graphs.

The study by Gladding et al. (2015) compared two online homework system formats to explore students' interaction with online homework systems. The mastery group, which completed a series of questions with animated solutions until mastery was achieved, showed greater learning improvements compared to the group using direct feedback homework. The mastery approach, which combines formative assessment and multimedia worked examples, resulted in more efficient learning.

Test

Tests as FA tools were examined in three studies (i.e., Kusairi, 2019; Kusairi et al., 2020; Pals, 2023). Tests examined in this study include isomorphic multiple-choice and diagnostic tests. Isomorphic multiple-choice tests were integrated into web-based FA and feedback (Kusairi, 2019) and game-based feedback (Kusairi et al., 2020). The findings show that isomorphic tests help identify students' weaknesses and provide learning indicators on their concept mastery. Meanwhile, the diagnostic test was developed by Pals (2023) as a FA tool to identify high school students' problem-solving abilities in kinematics. This study shows that the diagnostic tool has acceptable inter-rater reliability, allowing teachers to identify students' mastery of kinematics and provide meaningful corrective feedback.

Task and Learning Progression

Three studies report the use of LP in supporting FA in physics education (Zhai et al., 2018; Alonzo et al., 2022; Deverel-Rico et al., 2024). Zhai (2018) used LP in Informal Formative Assessment (IFA) and Formal Formative Assessment (FFA) on the topic of work and energy. Teachers adjusted learning objectives and activities, with most adjustments supporting student learning development, especially in IFA compared to FFA. Although LP aligns more with FFA, teachers mostly used it as a reference for interpreting student responses rather than as a content structure. Direct interaction with LP helped them better understand student comprehension and adjust instructional sequences more effectively.

Alonzo et al. (2022) found that LP helps teachers optimize FA in mechanics instruction by increasing attention to student ideas and reasoning. LP enables teachers to support student idea revisions rather than simply replacing them. Teachers also use LP to encourage student engagement in formative assessment.

Deverel-Rico et al. (2024) examined the use of LP and tasks in interdisciplinary formative assessment related to energy concepts. LP helps express student ideas, but its implementation does not always allow teachers to develop them further. This study emphasizes the importance of high-quality formative assessment and teacher training to effectively foster student-centered discussions.

Comprehensive FA

The comprehensive area of formative assessment (FA) was examined in two studies (i.e., Ganajová et al., 2021); Hidayat & Irdiyansyah, 2023). Hidayat & Irdiyansyah (2023) implemented tests, peer assessment, self-assessment, and prompts for creating concept maps or summaries. Their findings show that comprehensive FA, compared to single assessments, has a substantial effect on learners' educational attainment in introductory physics courses. Ganajová et al., (2021) investigated teaching accompanied by Formative Assessment Classroom Techniques (FACTs), including true or false statements, frayer model, k-w-l chart, card mapping self-assessment card, task cards, metacognition, concept map, exit card, and checklist. Their findings show that teaching with FACTs is more effective than teaching without FACTs. Both studies indicate that the implementation of FA through a combination of various strategies can enhance physics learning.

Formal and Informal FA

Formal and informal FA were examined in two studies (i.e. Niemenen, 2020; Zhai, 2018). Zhai (2018) examined the potential of learning progression (LP) in supporting FFA and IFA in the topic of Work and Energy. The findings indicate that teachers make more adjustments in IFA, suggesting that IFA is more flexible and allows teachers to directly respond to students' needs to foster their learning progress. However, FFA is more aligned with LP. Meanwhile, Niemenen (2020) explored the forms and functions of IFA in the form of on-the-fly assessment in inquiry-based physics learning on the topic of plane mirrors. The study provides insights into how teachers collect information through on-the-fly assessment conversations and how students initiate assessment discussions. The results show that the IFA used by teachers helps students understand the material and continue their inquiry process.

Feedback Model and Strategy

Two studies examined feedback models (Ole & Gallos, 2023a; Ole & Gallos, 2023b). The feedback model studied is the Feedback Loop Model (FLM), which consists of goal, tool, data, and inference. Ole & Gallos (2023a) studied changes in attitude, self-efficacy, and classroom practices of high school teachers toward FLM in kinematics instruction. Their findings show that the implementation of FLM by teachers leads to positive attitude changes and increased confidence in applying FA in their classes.

Ole & Gallos (2023b) examined the effectiveness of FLM in physics learning on high school students' understanding of kinematics and their engagement. Their findings show that FLM influences students' conceptual understanding levels in kinematics and significantly impacts their engagement.

The feedback strategy discussed is personalized feedback after students take a diagnostic test. Pals (2024) studied personalized feedback practices in high school and pre-university kinematics learning. Teachers used diagnostic tests to determine students' understanding levels and provided personalized hints using sticky notes. The findings show that students with initially low scores who received sufficient personalized feedback made significantly more progress in problem-solving than those who did not receive personalized feedback.

Concept Cartoon

Zivanayi & Nwaigwe (2024) investigated the use of cartoons as formal assessment in Newton’s Laws topics. Pre-intervention results show that teachers did not use formal assessments in their classes. Post-intervention results indicate that teachers appreciated Concept Cartoons as a formal assessment tool that enhances conceptual understanding of Newton’s Laws.

Peer Assessment

Ketonen et al. (2020) studied the paths taken by lower secondary school students through peer assessment and the factors influencing their learning in physics, covering fundamental physics investigation, mechanics, and universe dimensions. Their findings show that students' critique of peer work improves the quality of their lab reports. However, this effect is not always positive, as some students fail to utilize peer assessment to improve their reports.

Evaluation Effectiveness of Formative Assessment Studies

The effectiveness indicators of FA studies encompass three aspects: students, teachers, and assessment tools. These indicators provide information on how effective FA is in supporting students' physics learning progress, enhancing teachers' instructional practices, and evaluating the effectiveness of the assessment tools used. The effectiveness indicators examined in the studies are presented in Table 4.

Table 4. The Effectiveness of the Included Studies on Formative Assessment

Area	Description
Student performance and attitude	• Students' conceptual understanding (Kusairi et al., 2019; Ole & Gallos, 2023b; M. Park, 2019; Pusawale & Kadam, 2024). • Students' academic performance (Đorić et al., 2019; Hidayat & Irdiyansyah, 2023); learning gain (Molin et al., 2021) • Student scientific ability (Buggé, 2023) and inquiry skills (Ganajová et al., 2021) • Students' attitude (Buggé, 2023) • Student interaction toward FA (Z. Chen et al., 2021; Gladding et al., 2015; Ketonen et al., 2020; M. Park, 2020)
Teacher practice, perspective and attitude	• Teacher practices on FA (Alonzo et al., 2022; Deverel-Rico et al., 2024; Nieminen et al., 2020; Ole & Gallos, 2023a; Pals et al., 2024; Zhai et al., 2018) • Teachers' perceptions (Zivanayi & Nwaigwe, 2024); Teachers' attitudes and self-efficacy (Ole & Gallos, 2023a)
Performance and affordances of assessment tools	• Performance of diagnostic test (Pals et al., 2023) • The affordance of digital application feedback (Muslu & Siegel, 2024) and machine learning in evaluating concept maps (Bleckmann & Friege, 2023) • The feasibility of game-based feedback (Kusairi et al., 2020) and web-based feedback (Kusairi, 2019)

Table 4 presents studies that investigate Formative Assessment (FA) in physics education by evaluating its effectiveness in three key areas: student performance and attitude; teacher practice, perspective, and attitude; and the performance and affordances of assessment tools.

Student Performance and Attitude

The majority of studies on FA effectiveness focus on students' performance in physics learning, with a particular emphasis on students' conceptual understanding of physics. Several FA strategies have been implemented to enhance students' conceptual understanding, including computer simulation-based FA (Pusawale & Kadam, 2024; Park, 2019), the feedback loop model (Ole & Gallos, 2023b), and game-based feedback combined with isomorphic tests (Kusairi et al., 2019).

Other aspects of student performance studied include inquiry skills and scientific competencies. Bugge (2023) found that FA using the Investigative Science Learning Environment (ISLE) approach, which allows students to revise their written lab reports, helps develop their scientific skills and fosters a positive attitude toward physics experiments. Similarly, Ganajova (2021) reported that physics instruction incorporating Formative Assessment Classroom Techniques (FACTs) effectively enhances students' inquiry skills.

FA plays a crucial role in optimizing student learning, as evidenced by improvements in academic achievement. Đorić (2019) reported that implementing FA through simulations, electronic tests, and feedback significantly enhanced students' academic performance in physics. Consistently, Hidayat & Irdiyansyah (2023) found that a comprehensive FA approach involving formative tests, self-assessment, peer assessment, and prompts for creating summaries or concept maps led to increased academic achievement. Furthermore, Molin (2019) highlighted that teacher feedback strategies combined with peer discussions using polling technology had a positive impact on student learning outcomes. Beyond academic performance, some studies explored student interactions with FA in physics education. Four studies (e.g., Park, 2020; Chen et al., 2018; Gladding et al., 2015; Ketonen et al., 2020) examined how students utilize FA, the factors influencing their engagement with it, and the types of student interactions that have the greatest impact on learning outcomes.

Teacher Practice, Perspective, and Attitude

The second area of FA effectiveness evaluation in physics education focuses on teachers, including teacher practices in FA (Zhai, 2018; Alonzo et al., 2022; Pals et al., 2024; Deverel-Rico et al., 2024; Ole & Gallos, 2023a; Nieminen et al., 2020), teachers' perceptions (Zivanayi & Nwaigwe, 2024), and teachers' attitudes and self-efficacy (Ole & Gallos, 2023a). Studies examining teacher practices in FA aim to identify effective FA strategies that enhance physics learning. In contrast, studies on teachers' perceptions of FA consistently reveal that teachers hold positive perceptions of FA. Zivanayi & Nwaigwe (2024) reported that teachers expressed positive attitudes toward concept cartoons as an FA tool. Similarly, Ole & Gallos (2023a) found that the formative feedback model, specifically the feedback loop model practiced by teachers, led to positive changes in teachers' attitudes toward assessment use and increased their confidence in implementing FA in physics classrooms.

Performance and Affordances of Assessment Tools

The third area of FA effectiveness evaluation in physics education concerns the performance of FA tools, including tests, assessment instruments, and technology-assisted feedback. Pals et al. (2023) developed a diagnostic test tool with high reliability to help teachers identify students' kinematics problem-solving abilities and provide more meaningful feedback. Several studies have evaluated the performance of technology-assisted assessment tools. Muslu & Siegel (2024) investigated the effectiveness of digital applications in providing feedback, finding that their effectiveness depends largely on teachers' proficiency in implementing them. Bleckmann & Friege (2023) explored the performance of a Machine Learning model in assessing students' concept maps as an automated feedback mechanism. Their findings showed a high agreement between Machine Learning assessments and human evaluators. Additionally, studies have examined the feasibility of game-based feedback (Kusairi et al., 2020) and web-based feedback (Kusairi, 2019). Both studies reported promising results, demonstrating that these FA tools can provide specific and timely feedback to students.

Discussion

This systematic literature review analyzes 25 studies related to formative assessment (FA) in physics education and identifies three key findings. First, the distribution of studies based on year, education level, country, research method, and physics topics involved. Second, FA implementation strategies in physics education, categorized into eight areas, with the most frequently used being technology-enhanced assessment and feedback, followed by comprehensive FA; test; task and learning progressions; formal and informal FA; feedback models and strategies; cartoon-based FA; and peer assessment. Third, the effectiveness of FA in the included studies is divided into three areas, with most studies focusing on student performance and attitudes, while also discussing teacher practices, perspectives, and attitudes, as well as the performance and affordances of assessment tools.

Before further interpreting the findings, it is important to identify several limitations of this study. This systematic review has several constraints, particularly concerning the inclusion and exclusion criteria applied. First, the selection of literature limited to English-language publications may exclude valuable findings from research disseminated in non-English sources. Second, restricting the literature to open-access studies in the Scopus and ERIC databases may exclude relevant studies with limited accessibility. Future reviews may consider examining studies in multiple languages, including those with restricted access, and expanding the database coverage, such as incorporating Web of Science and others. This approach could provide a more comprehensive perspective on formative assessment in physics education.

Another limitation is that the included studies cover a broad, fragmented, and limited range of areas. This leads to insufficient discussions on certain FA aspects and weaker conclusions. However, this also presents opportunities for future research. As Bennett (2011) stated, formative assessment remains an evolving field both conceptually and practically.

This systematic literature review also has several strengths. First, it is the only systematic literature review that explores FA in physics education across different educational levels. Previous literature reviews have explored FA in high school physics education (Nurjanah et al., 2024). Other literature reviews synthesized qualitative studies on FA practices in science education (Atasoy & Kaya, 2022). This study aims to provide a better understanding of FA implementation in alignment with the characteristics of the physics discipline.

Second, this study serves as a valuable foundation for teachers, offering insights into evidence-based FA practices. The identified studies provide insights into how FA is implemented in physics classrooms and how it can effectively enhance physics teaching and learning. Based on the 25 studies reviewed, various FA tools and strategies were examined in physics education. The FA tools and strategies discussed highlight their potential in improving the quality of physics instruction and learning. Additionally, the evaluation of FA effectiveness in physics education studies was explored to offer richer insights into FA benefits.

Another strength of this study is that it provides a foundation for researchers in physics education to identify research trends and potential areas for further study. By mapping FA studies in physics education, this review serves as a guide for future researchers in exploring FA in physics education. The following sections discuss the interpretation of findings based on study distribution by year of publication, country, education level, and physics topics examined.

Distribution of Studies

The distribution of FA studies in physics education shows an increasing trend over the past decade. However, a decline in the number of studies was observed in 2021 and 2022, likely due to the impact of the COVID-19 pandemic, during which educational institutions adjusted their teaching and learning approaches. Research on FA in physics education is expected to persist and grow, given that FA is a powerful strategy for improving education quality.

The distribution of studies by country indicates that 12 countries contribute to the literature on FA in physics education. Among these, the USA (n=7) has the highest number of publications on FA in physics education. This finding aligns with Nurjannah et al. (2024), who noted that the USA leads in FA-related publications in high school physics education. Additionally, Zhang et al. (2023) highlighted that US universities are at the forefront of FA research in academic literature.

The distribution of studies by education level shows that secondary school education (n=20) is the most frequently studied, while higher education is the least represented (n=6). This finding is not surprising, as most physics education research involves secondary school students, whereas physics education research at the university level remains limited (Lee & Kim, 2018). The lack of FA studies in higher education physics underscores the need for more research at this level. Morris & Rohs (2023) emphasized that empirical research on FA in higher education must continue to grow in both quantity and depth, particularly when compared to the extensive empirical evidence available at the secondary school level.

The distribution of studies by research method indicates that quantitative methods (n=9) are the most popular in FA studies in physics education (see Table 2 and Figure 5). The most frequently used quantitative research design is quasi-experimental. Quasi-experiments offer higher internal validity than pre-experimental designs but lower validity than true experimental designs (Ary et al., 2009). True experiments involve randomized control groups and assignments. The analyzed studies suggest that the methodologies employed are suitable for addressing research questions, particularly given the complexity of educational settings, which makes it difficult to implement randomized experimental designs (Van der Kleij & Lipnevich, 2021). However, there is still room for improvement in experimental design to enhance research validity.

Qualitative studies are also commonly used in the included studies (n=8). Qualitative methods provide a deeper understanding of how FA is implemented in physics learning. The most dominant qualitative approach is case study research, which explains and describes in detail the phenomena under investigation. This approach is effective for research purposes but is less effective in monitoring student progress. Therefore, further research is recommended using action research and design-based research to enhance FA implementation (Atasoy & Kaya, 2022)

The distribution of studies by physics topic shows that 64% (n=16) focus on mechanics (see Table 2 and Figure 6). This finding is consistent with previous SLR, that the majority of FA studies in physics education involve mechanics topics (Nurjannah et al., 2024). The mechanics topics covered in the included studies include mechanics, kinematics, work and energy, motion, and Newton's laws. Mechanics plays a crucial role in physics education, as studying mechanics helps students develop problem-solving skills (Napis, 2024). Problem-solving is a core competency in physics and a key 21st-century skill for students (Niyomufasha, 2024). However, challenges in understanding mechanics persist, including common misconceptions (Kirkpatrick, 2010), difficulties in kinematics comprehension (Mufit, 2022), and conceptual challenges in kinematics (Kusairi et al., 2019; Kusairi et al., 2020). Therefore, efforts to mitigate misconceptions, such as FA strategies focused on conceptual development, are necessary.

Other physics topics covered include general physics, referring to courses that encompass various topics such as basics of physics inquiry, dimensions of the universe and mechanics (Ketonen et al., 2020); mechanics, thermodynamics, waves, electromagnetism, and modern physics (Molin et al., 2021). Less frequently studied topics include quantum physics, optics, electricity, and magnetism. Although these topics are less explored in the included studies, they remain essential as foundational concepts in physics education (Nikolaus, 2024). Given the abstract and complex nature of physics, these topics continue to present challenges for both students and teachers (Pals et al., 2023; Niyomufasha, 2023; Rasyid et al., 2025).

In terms of disciplinary scope, all studies focus on physics, but two studies involve interdisciplinary approaches, including physics, chemistry, and biology (Deverel Rico, 2024) and physics, biology, chemistry, mathematics, and informatics (Ganajová et al., 2021). These studies highlight the potential for interdisciplinary FA design. Prior research suggests that some FA competencies can be applied across disciplines, but it is essential to examine FA practices in different disciplines to fully characterize FA expertise (Gotwals et al., 2015). Further research is needed to adapt FA to disciplinary characteristics and explore its effective implementation across subjects.

This study provides empirical evidence supporting previous literature that FA enhances student achievement and promotes effective physics teaching and learning. While these findings are expected, they offer insights into FA tools and strategies that benefit physics education. Many of the studies reviewed highlight the potential of technology-assisted FA and simulation-based FA in supporting effective physics instruction. The following section discusses FA implementation tools and strategies in physics education.

Implementation Strategies of FA in Physics Education

FA implementation strategies in physics education, categorized into eight areas, include technology-enhanced FA and feedback; tests; tasks and learning progression; comprehensive FA; formal and informal FA; feedback models and strategies; concept cartoons; and peer assessment. This classification differs slightly from previous findings, which categorized FA into technology-based, test-based, card-based, reflection and informal feedback, graph or chart-based, and others (Nurjanah et al., 2024). Assessment tools based on cards and graphs/charts were not examined in this study; however, these tools are part of formative assessment (FA) classroom techniques (i.e., Ganajová et.al, 2021) and fall within the area of Comprehensive FA. Furthermore, this study introduced new FA categories, namely tasks and learning progression as well as formal and informal FA. In addition, this study is consistent with Nurjanah et al., (2024), which found that technology-based FA remains the most popular area among FA studies in the field of physics.

Technology-enhanced FA offers significant potential. This is because, in its implementation, FA requires teachers to provide timely and appropriate feedback throughout the lesson sequence (Pals et al., 2023). However, this is challenging, especially in large classes. It is impossible for a teacher to provide real-time feedback to all students in the class (Zhu et al., 2020). Therefore, the use of technology to enhance assessment and feedback is a potential solution (Tekin, 2025). Technology-enhanced assessment and feedback can address various challenges, such as time constraints, large class sizes, and geographical limitations (Ciddi, 2025; Gilbert et al., 2011).

In the reviewed studies, technology-enhanced formative assessment and feedback were implemented in various formats, including web-based feedback, game-based feedback, computer simulation and digital applicationn. However, each study explored feedback through different technological approaches, with no repeated use of the same feedback format across studies. As a result, while these findings demonstrate the potential of technology in supporting feedback processes, they do not provide sufficient evidence to determine which specific feedback strategies are most effective for improving student learning in physics education. The same issue is evident in other areas of formative assessment as well, limiting the availability of robust evidence to support stronger conclusions, especially in studies evaluating the effectiveness of FA implementation. In line with this, findings from previous SLRs have also indicated that the absence of a solid theoretical framework, the repetition of studies, insufficient methodological rigor, and the limited and fragmented nature of the research have made it challenging to draw robust conclusions (Morris & Rohs, 2023; Van der Kleij & Lipnevich, 2021).

Among the diverse technological approaches to enhancing formative assessment, computer simulation-based FA emerged as a focus in four of the reviewed studies. These studies demonstrated effectiveness of computer simulation based FA in enhancing students' academic performance in physics. Simulation-based FA is quite popular, as evidenced by four studies that reported positive outcomes on students' performance and academic achievement in physics. Using simulations-based formative assessment, which helps simplify complex and abstract physics concepts, is an effective strategy for improving physics teaching (Pusawale & Kadam, 2024). It is predicted that computer simulation-based formative assessment will continue to expand, making it more interactive, real-time, and capable of breaking down abstract concepts in physics (Nurjanah et al., 2025).

This study reveals that technology-enhanced formative assessment and feedback is a rapidly growing area, as examined in 12 studies, all of which reported positive and promising results in supporting physics learning. This highlights the potential of technology-enhanced formative assessment and feedback to improve physics teaching and learning. Nevertheless, there remains a need for further empirical research to obtain comprehensive evidence regarding technology-enhanced formative assessment and feedback strategies.

In the area of technology-enhanced assessment and feedback, different perspectives emerge from the included studies. These perspectives revolve around whether technology serves as a support tool or has the potential to replace teachers' roles in conducting assessments and feedback. Molin (2021) and Muslu & Siegel (2024) emphasize the use of technology as a complementary tool, stating that FA practices are still influenced by how teachers implement them. This aligns with the findings of Yan et al. (2021), which indicate that the effectiveness of formative assessment depends on how well teachers understand and execute FA activities in the classroom.

On the other hand, some studies suggest that technology not only supports FA practices but may also partially replace teachers' roles in FA implementation. For instance, Đorić et al. (2019) found that simulations with computer-generated feedback had a greater impact on students' academic performance compared to simulations where feedback was provided by teachers. This is reinforced by Bleckmann & Friege (2023), who reported that machine learning technology showed a high level of agreement with human evaluators in assessing students' performance in concept mapping.

Faced with this potential paradigm shift, we argue that technology in education should serve as a tool to support, rather than replace, teachers' roles. Sembey et al. (2024) asserts that integrating technology into FA aims to provide more personalized and timely feedback while also reducing teachers' workload. Therefore, further research is needed to examine approaches that optimize the potential of technology while ensuring that teachers remain the primary actors in FA implementation. These findings have significant implications for physics education, which often requires concept visualization and immediate and context-relevant feedback (O. Park & Gittelman, 1992; Shubha & Meera, 2019; Suyatna et al., 2017). However, technology adaptation must consider how teachers utilize it in teaching practices and how the data generated by technology-based systems can be interpreted and applied to improve teaching quality and support students' academic progress.

Most FA strategies have demonstrated a positive impact on physics teaching and learning (i.e. technology-enhanced assessment and feedback, test, comprhensife FA, feedback loop model, personalized feedback, cartoon concept). However, a study does not report entirely positive results regarding peer assessment and its impact on student learning progress. Ketonen (2020) states that peer critique can help students improve their reports, but not all students equally benefit from the feedback received from their peers. Ketonen et al. (2020) emphasize the need to evaluate students' understanding of peer assessment to maximize the benefits of FA. Students who perceive assessment as a learning tool are more likely to accept and utilize feedback to improve their work, whereas those who view PA as a form of negative evaluation tend to reject it. Thus, further research is needed to explore the factors influencing the effectiveness of PA, including students' perceptions of peer feedback. Van der Kleij & Lipnevich (2021) highlights that no studies have explicitly linked students' perceptions of peer feedback with meaningful learning outcomes. Given the limited number of studies focusing on peer assessment, further research is recommended to explore PA in physics education.

In addition to peer assessment, cartoons as a FA tool in physics are also underrepresented in the reviewed studies, as only one study has examined this topic. Zivanayi & Nwaigwe (2024) investigated teachers' attitudes toward cartoons as an FA tool. In line with these findings, Kandil Ingec (2008) revealed that the use of concept cartoons as an alternative assessment tool is still relatively rare but holds significant potential for developing students' understanding of physics concepts. Therefore, empirical evidence is needed to support the potential of concept cartoons as a FA tool in physics education. Further research is needed to explore student interactions, effectiveness analysis, and teacher practices in implementing concept cartoons as an FA tool in physics classrooms.

The Effectiveness of FA Studies in Physics Education

This study shows that the implementation of FA supports the development of students' academic performance. The most frequently examined aspect of student performance is conceptual understanding (Pusawale & Kadam, 2024; Ole & Gallos, 2023b; Kusairi et al., 2019; Park, 2019). Similarly, previous research indicates that most studies in physics education focus on analyzing students' understanding of physics concepts (Docktor et al., 2015; Docktor & Mestre, 2014; Lee & Kim, 2018). Understanding physics concepts is crucial because physics concepts are often challenging due to their complexity and abstract nature (Pals et al., 2023). FA is intended to support students' conceptual learning in physics (Park, 2020).

Various FA strategies have been applied to enhance students' conceptual understanding, including computer simulation-based FA (Pusawale & Kadam, 2024; Park, 2019), the feedback loop model (Ole & Gallos, 2023b), and game-based feedback integrated with isomorphic tests (Kusairi et al., 2019). All studies investigating FA's role in developing students' conceptual understanding in physics reported positive results, making these tools and strategies promising for teachers in their physics instruction.

Besides evaluating the effectiveness of FA in cognitive and affective aspects, students' interaction with FA is also a crucial factor in its success. Some studies explore how students' engagement and learning strategies influence their response to FA (Park, 2020; Chen et al., 2018; Gladding et al., 2015; Ketonen et al., 2020). This information is valuable for instructors in designing more effective FA strategies to enhance students' learning experiences in physics.

The evaluation of FA's effectiveness also focuses on teaching practices, teachers' attitudes, and perceptions toward FA. Many studies have examined how teachers implement FA, acknowledging that teachers play a key role in FA's success (Yan et al., 2021). Even with technological support, teachers' ability to implement FA significantly determines its effectiveness (Molin et al., 2021; Muslu & Siegel, 2024).

The findings indicate that teachers experience positive changes in their perceptions and confidence in using FA in physics classes. From the teachers' perspective, FA is seen as an effective strategy for improving teaching and learning (Zhang et al., 2023). Previous research also suggests that recognizing FA's benefits increases teachers' intention to implement it (Yan et al., 2021). Given the limited research on teacher-related aspects in FA studies, further empirical studies are needed to support teachers in effectively implementing FA in physics education.

Conclusion

Based on an analysis of 25 studies on formative assessment (FA) in physics education published in Scopus and ERIC between January 2015 and December 2024, research on FA in physics has shown an increasing trend, although there was a decline during the COVID-19 pandemic, with the USA being the most significant contributor. In terms of education level, high school is the most frequently studied level, with mechanics as the dominant topic. Our findings identify several key FA strategies, including technology-enhanced assessment and feedback, comprehensive FA, tests, tasks and learning progressions, feedback models and strategies, informal and formal FA, concept cartoons, and peer assessment. Among these, technology-enhanced assessment and simulation-based FA emerge as the most promising startegies. This study reinforces existing literature on the effectiveness of FA in enhancing physics education. FA has been shown to improve students' conceptual understanding and academic performance, while also influencing teachers' attitudes and perceptions toward assessment. This review provides evidence-based insights into effective FA tools and strategies across various educational levels. It serves as a foundation for physics educators seeking to implement FA effectively and offers research directions for future studies in physics education.

Recommendations

This review provides an evidence-based foundation for FA practices in physics teaching and offers insights for future research. The following recommendations are proposed:

1) Maximizing the potential of technology in improving formative assessment and feedback necessitates further research, focusing on identifying effective technological strategies and understanding the critical role of teachers in implementing technology-supported FA in practice
2) Improve teachers' proficiency in FA implementation, as FA effectiveness heavily

relies on teachers' ability to apply it (Muslu & Siegel, 2024; Alonzo, 2024).

3) It is recommended that future research continues to explore formative assessment within the context of physics education at contextually appropriate educational levels. Particular attention should be given to underexplored areas such as peer assessment and the implementation of concept cartoons as formative assessment strategies.

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References

Note : The studies inluded in the final review are marked with an asterisk (*)

*Alonzo, A. C., Wooten, M. M., & Christensen, J. (2022). Learning progressions as a simplified model: Examining teachers’ reported uses to inform classroom assessment practices. *Science Education*, *106*(4), 852–889. https://doi.org/10.1002/sce.21713

Arrafii, M. A., & Sumarni, B. (2018). Teachers’ Understanding of Formative Assessment. *Lingua Cultura*, *12*(1), 45. https://doi.org/10.21512/lc.v12i1.2113

Ary, D., Jacobs, L. C., Sorensen, C., & Razavieh, A. (2009). *Introduction to Research in Education*. Wadsworth.

Atasoy, V., & Kaya, G. (2022). Formative assessment practices in science education: A meta-synthesis study. In *Studies in Educational Evaluation* (Vol. 75). Elsevier Ltd. https://doi.org/10.1016/j.stueduc.2022.101186

Beerepoot, M. T. P. (2023). Formative and summative automated assessment with multiple-choice question banks. *Journal of Chemical Education*, *100*(8), 2947–2955. https://doi.org/10.1021/acs.jchemed.3c00120

Bennett, R. E. (2011). Formative assessment: A critical review. *Assessment in Education: Principles, Policy and Practice*, *18*(1), 5–25. https://doi.org/10.1080/0969594X.2010.513678

Black, P., & Wiliam, D. (1998). Inside the black box: Raising standards through classroom assessment. In *Phi Delta Kappan* (Issue 1). School of Education, King’s College.

https://doi.org/10.1177/003172171009200119

*Bleckmann, T., & Friege, G. (2023). Concept maps for formative assessment: Creation and implementation of an automatic and intelligent evaluation method. *Knowledge Management and E-Learning*, *15*(3), 433–447. https://doi.org/10.34105/j.kmel.2023.15.025

*Buggé, D. (2023). Improving scientific abilities through lab report revision in a high school investigative science learning environment classroom. *Physical Review Physics Education Research*, *19*(2). https://doi.org/10.1103/PhysRevPhysEducRes.19.020166

*Chen, X., Breslow, L., & DeBoer, J. (2018). Analyzing productive learning behaviors for students using immediate corrective feedback in a blended learning environment. *Computers and Education*, *117*, 59–74. https://doi.org/10.1016/j.compedu.2017.09.013

Chen, Z., Jiao, J., & Hu, K. (2021). Formative assessment as an online instruction intervention: Student engagement, outcomes, and perceptions. *International Journal of Distance Education Technologies*, *19*(1), 50–65. https://doi.org/10.4018/IJDET.20210101.oa1

Ciddi, M. (2025). Analysis of Attitudes of Undergraduate Art Students Toward Painting Workshop Lessons . *International Journal on Social and Education Sciences, 7*(2), 195-204. https://doi.org/10.46328/ijonses.764

*Deverel-Rico, C., Furtak, E. M., Student, S. R., & Burkhardt, A. (2024). How Can Crosscutting Concepts Organize Formative Assessments across Science Classrooms? Results of a Video Study. *Education Sciences*, *14*(10). https://doi.org/10.3390/educsci14101060

Docktor, J. L., & Mestre, J. P. (2014). Synthesis of discipline-based education research in physics. *Physical Review Special Topics - Physics Education Research*, *10*(2). https://doi.org/10.1103/PhysRevSTPER.10.020119

Docktor, J. L., Strand, N. E., Mestre, J. P., & Ross, B. H. (2015). Conceptual problem solving in high school physics. *Physical Review Special Topics - Physics Education Research*, *11*(2). https://doi.org/10.1103/PhysRevSTPER.11.020106

*Đorić, B., Lambić, D., & Jovanović, Ž. (2019). The use of different simulations and different types of feedback and students' academic performance in physics. *Research in Science Education*, *51*(5). https://doi.org/10.1007/s11165-019-9858-4

*Ganajová, M., Sotáková, I., Lukáč, S., Ješková, Z., Jurková, V., & Orosová, R. (2021). Formative assessment as a tool to enhance the development of inquiry skills in science education. *Journal of Baltic Science Education*, *20*(2), 204–222. https://doi.org/10.33225/jbse/21.20.204

Gilbert, L., Whitelock, D., & Gale, V. (2011). *Synthesis Report on Assessment and Feedback with Technology Enhancement*.

*Gladding, G., Gutmann, B., Schroeder, N., & Stelzer, T. (2015). Clinical study of student learning using mastery style versus immediate feedback online activities. *Physical Review Special Topics - Physics Education Research*, *11*(1). https://doi.org/10.1103/PhysRevSTPER.11.010114

Gotwals, A. W. (2018). Where are we now? Learning progressions and formative assessment. *Applied Measurement in Education*, *31*(2), 157–164. https://doi.org/10.1080/08957347.2017.1408626

Gotwals, A. W., Philhower, J., Cisterna, D., & Bennett, S. (2015). Using Video to Examine Formative Assessment Practices as Measures of Expertise for Mathematics and Science Teachers. *International Journal of Science and Mathematics Education*, *13*(2), 405–423. https://doi.org/10.1007/s10763-015-9623-8

Halim, H. A., Hamzah, M. I., & Zulkifli, H. (2024). A systematic review on the formative assessment practice in teaching and learning in secondary school. In *International Journal of Evaluation and Research in Education* (Vol. 13, Issue 2, pp. 1173–1183). Institute of Advanced Engineering and Science. https://doi.org/10.11591/ijere.v13i2.26187

*Hidayat, N., & Irdiyansyah, I. (2023). Optimizing academic achievement through comprehensive integration of formative assessment into teaching. *European Journal of Educational Research*, *12*(1), 71–85. https://doi.org/10.12973/eu-jer.12.1.71

Higgins, J. P. T., López-López, J. A., Becker, B. J., Davies, S. R., Dawson, S., Grimshaw, J. M., McGuinness, L. A., Moore, T. H. M., Rehfuess, E. A., Thomas, J., & Caldwell, D. M. (2019). Synthesising quantitative evidence in systematic reviews of complex health interventions. *BMJ Global Health*, *4*(Suppl 1), e000858. https://doi.org/10.1136/bmjgh-2018-000858

Ilma, A. Z., Wilujeng, I., Widowati, A., Nurtanto, M., & Kholifah, N. (2023). A systematic literature review of stem education in indonesia (2016-2021): Contribution to improving skills in 21st century learning. *Pegem Egitim ve Ogretim Dergisi*, *13*(2), 134–146. https://doi.org/10.47750/pegegog.13.02.17

Kandil Ingec, S. (2008). Use of concept cartoons as an assessment tool in physics education. *US-China Education Review*, *5*(11), 47–54.

*Ketonen, L., Hähkiöniemi, M., Nieminen, P., & Viiri, J. (2020). Pathways Through Peer Assessment: Implementing Peer Assessment in a Lower Secondary Physics Classroom. *International Journal of Science and Mathematics Education*, *18*(8), 1465–1484. https://doi.org/10.1007/s10763-019-10030-3

Khan, M., Zaman, T. U., & Saeed, A. (2020). *Formative Assessment Practices of Physics Teachers in Pakistan*. http://journal.unnes.ac.id/nju/index.php/jpfi

*Kusairi, S. (2019). A web-based formative feedback system development by utilizing isomorphic multiple choice items to support physics teaching and learning. *Journal of Technology and Science Education*, *10*(1), 117–126. https://doi.org/10.3926/jotse.781

*Kusairi, S., Noviandari, L., Parno, & Pratiwi, H. Y. (2019). Analysis of students' understanding of motion in straight line concepts: Modeling Instruction with formative E-Assessment. *International Journal of Instruction*, *12*(4), 353–364. https://doi.org/10.29333/iji.2019.12423a

*Kusairi, S., Puspita, D. A., Suryadi, A., & Suwono, H. (2020). Physics Formative Feedback Game: Utilization of Isomorphic Multiple-choice Items to Help Students Learn Kinematics. *TEM Journal*, *9*(4), 1625–1632. https://doi.org/10.18421/TEM94-39

Lee, B., & Kim, H. (2018). Trends of the Research in Physics Education in Korea. *Journal of the Korean Physical Society*, *72*(12), 1502–1507. https://doi.org/10.3938/jkps.72.1502

*Molin, F., Haelermans, C., Cabus, S., & Groot, W. (2021). Do feedback strategies improve students' learning gain?-Results of a randomized experiment using polling technology in physics classrooms. *Computers and Education*, *175*. https://doi.org/10.1016/j.compedu.2021.104339

Morris, T. H., & Rohs, M. (2023). The potential for digital technology to support self-directed learning in formal education of children: a scoping review. *Interactive Learning Environments*, *31*(4), 1974–1987. https://doi.org/10.1080/10494820.2020.1870501

*Muslu, N., & Siegel, M. A. (2024). Feedback Through Digital Application Affordances and Teacher Practice. *Journal of Science Education and Technology*. https://doi.org/10.1007/s10956-024-10117-9

*Nieminen, P., Hähkiöniemi, M., & Viiri, J. (2020). Forms and functions of on-the-fly formative assessment conversations in physics inquiry lessons. *International Journal of Science Education*, *43*(3), 362–384. https://doi.org/10.1080/09500693.2020.1713417

Nurjanah, S., Sultan, J., Arriza, L., Suardi, I. K., Ramadhani, S., Seran, D. S. F., & Rashid, S.

(2025). Assessment in physics education research: Trends, patterns, and future directions. *Review of Education*, *13*(1). https://doi.org/10.1002/rev3.70043

Nurjanah, S., Suyanto, S., Iqbal, M., Ramadhani, S., Suardi, I. K., & Seran, D. S. F. (2024). A systematic literature review of formative assessment in high school physics learning. *EDUSAINS*, *16*(2), 110–127. https://doi.org/10.15408/es.v13i2.34136

*Ole, F. C. B., & Gallos, M. R. (2023a). Effects of Feedback Loop Model on Teachers' Attitudes, Self-Efficacy and Classroom Practices towards Formative Assessment. *International Journal of Research in Education and Science*, *9*(1), 55–75. https://doi.org/10.46328/ijres.3006

*Ole, F. C. B., & Gallos, M. R. (2023b). Impact of formative assessment based on feedback loop model on high school students' conceptual understanding and engagement with physics. *Journal of Turkish Science Education*, *20*(2), 333–355. https://doi.org/10.36681/tused.2023.019

Page, M. J., McKenzie, J. E., Bossuyt, P. M., Boutron, I., Hoffmann, T. C., Mulrow, C. D., Shamseer, L., Tetzlaff, J. M., Akl, E. A., Brennan, S. E., Chou, R., Glanville, J., Grimshaw, J. M., Hróbjartsson, A., Lalu, M. M., Li, T., Loder, E. W., Mayo-Wilson, E., McDonald, S., … Moher, D. (2021). The PRISMA 2020 statement: An updated guideline for reporting systematic reviews. In *The BMJ* (Vol. 372). BMJ Publishing Group. https://doi.org/10.1136/bmj.n71

*Pals, F. F. B., Tolboom, J. L. J., & Suhre, C. J. M. (2023). Development of a formative assessment instrument to determine students' need for corrective actions in physics: Identifying students' functional level of understanding. *Thinking Skills and Creativity*, *50*. https://doi.org/10.1016/j.tsc.2023.101387

*Pals, F. F. B., Tolboom, J. L. J., & Suhre, C. J. M. (2024). Formative Assessment Strategies by Monitoring Science Students' Problem-Solving Skill Development. *Canadian Journal of Science, Mathematics and Technology Education*. https://doi.org/10.1007/s42330-023-00296-9

*Park, M. (2019). Effects of simulation-based formative assessments on students' conceptions in physics. *Eurasia Journal of Mathematics, Science and Technology Education*, *15*(7). https://doi.org/10.29333/ejmste/103586

*Park, M. (2020). Students' problem-solving strategies in qualitative physics questions in a simulation-based formative assessment. *Disciplinary and Interdisciplinary Science Education Research*, *2*(1). https://doi.org/10.1186/s43031-019-0019-4

Park, O., & Gittelman, S. S. (1992). Selective use of animation and feedback in computer-based instruction. *Educational Technology Research and Development*, *40*(4), 27–38. https://doi.org/10.1007/BF02296897

Peter, G., & Uwamahoro, J. (2023). Effect of formative assessment on students' conceptual understanding of physics concepts among secondary schools in Ngoma district. *East African Journal of Education and Social Sciences (EAJESS)*, *4*(1), 114–119. https://doi.org/10.46606/eajess2022v03i06.0263

*Pusawale, S. N., & Kadam, D. M. (2024). Effect of Simulation based teaching strategy in Engineering Physics course. *Journal of Engineering Education Transformations*, *37*(Special Issue 2), 508–513. https://doi.org/10.16920/jeet/2024/v37is2/24081

Sarwar, G. S., & Trumpower, D. L. (2015). Effects of conceptual, procedural, and declarative reflection on students' structural knowledge in physics. *Educational Technology Research and Development*, *63*(2), 185–201. https://doi.org/10.1007/s11423-015-9368-7

Sembey, R., Hoda, R., & Grundy, J. (2024). Emerging technologies in higher education assessment and feedback practices: A systematic literature review. *Journal of Systems and Software*, *211*. https://doi.org/10.1016/j.jss.2024.111988

Shubha, S., & Meera, B. N. (2019). The Effect of Visual Representation in Developing Conceptual Understanding of Magnetic Force. *Journal of Formative Design in Learning*, *3*(1), 82–87. https://doi.org/10.1007/s41686-019-00031-4

Suyatna, A., Anggraini, D., Agustina, D., & Widyastuti, D. (2017). The role of visual representation in physics learning: dynamic versus static visualization. *Journal of Physics: Conference Series*, *909*(1), 012048. https://doi.org/10.1088/1742-6596/909/1/012048

Tekin, A. (2025). Examining High School Students' Attitudes towards Contemporary and Digital Art and Virtual Exhibitions. *International Journal on Studies in Education, 7*(4), 879-898. https://doi.org/10.46328/ijonse.5561

Van der Kleij, F. M., & Lipnevich, A. A. (2021). Student perceptions of assessment feedback: a critical scoping review and call for research. In *Educational Assessment, Evaluation and Accountability* (Vol. 33, Issue 2, pp. 345–373). Springer Science and Business Media B.V. https://doi.org/10.1007/s11092-020-09331-x

Yan, Z., Li, Z., Panadero, E., Yang, M., Yang, L., & Lao, H. (2021). A systematic review on factors influencing teachers' intentions and implementations regarding formative assessment. *Assessment in Education: Principles, Policy and Practice*, *28*(3), 228–260. https://doi.org/10.1080/0969594X.2021.1884042

*Zhai, X., Li, M., & Guo, Y. (2018). Teachers' use of learning progression-based formative assessment to inform teachers' instructional adjustment: a case study of two physics teachers' instruction. *International Journal of Science Education*, *40*(15), 1832–1856. https://doi.org/10.1080/09500693.2018.1512772

Zhang, Y., Wang, W., Xian, Y., Wang, X., & Huang, J. (2023). The research status of formative assessment in science education. *Journal of Baltic Science Education*, *22*(6), 1103–1119. https://doi.org/10.33225/jbse/23.22.1103

Zhu, M., Liu, O. L., & Lee, H. S. (2020). The effect of automated feedback on revision behavior and learning gains in formative assessment of scientific argument writing. *Computers and Education*, *143*. https://doi.org/10.1016/j.compedu.2019.103668

*Zivanayi, W., & Nwaigwe, C. G. C. (2024). Enhancing Formal Assessment using Concept Cartoons in Physics: A Teacher's Perspective. *Science Education International*, *35*(4), 348–359. https://doi.org/10.33828/sei.v35.i4.6

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Chapter 3 - Fractal Geometry in Secondary Education: Characterization of Mathematical and Didactic-Pedagogical Knowledge Through Continuing Teacher Training

Victoria Artigue, María de los Ángeles Fanaro, Joel Gak

Chapter Highlights

- This study examines the teaching of fractal geometry in secondary education and its integration into the professional development of mathematics teachers in Uruguay. A sequence of activities was designed to develop the mathematical and pedagogical knowledge required to teach fractals, including applications such as multiband fractal antennas based on self-similarity.
- The sequence was implemented in two online professional development courses in 2023 with 83 teachers, with revisions in the second edition emphasizing the formalization of intuitively taught concepts. The activities received positive evaluations and were effective even for teachers with no prior knowledge of fractal geometry.
- Teacher responses were analyzed using the Mathematical Knowledge for Teaching (MKT) framework and its extension MTSK, focusing on levels of formalization and mathematical richness. Teachers showed significant improvement in addressing key concepts such as self-similarity and fractal dimension.
- A Goodman–Kruskal bivariate analysis revealed a strong association between common and curricular knowledge, indicating that teachers successfully connected prior knowledge with official curricula. These findings support the inclusion of fractal geometry in secondary education.

Introduction

This work synthesizes key findings from de doctoral dissertation "*Fractal Geometry in Secondary Education: Characterization of Mathematical and Didact-Pedagogical Knowledge through Continuing Teacher Training", defending in December 2024 at the National University of Central Buenos Aires (UNICEN).*

The primary objective of this study is the teaching of Fractal Geometry (hereafter FG) in secondary education, identifying processes of didactic transposition (Chevallard, 1999). This involves understanding the necessary transformations of mathematical knowledge (as used by mathematicians) to make it teachable and meaningful for high school students. FG is a relatively new mathematical domain, with initial concepts developed less than half a century ago. Various authors recognize its importance for describing and explaining natural phenomena that Euclidean Geometry (hereafter EG) cannot address (Shriki & Nutov, 2016; Faccio, 2013; Redondo & Haro, 2005; Chen et al., 2018; Picoli & Pinto, 2018). Regarding FG instruction, it presents significant challenges as it differs fundamentally from EG and lacks established teaching traditions in mathematics education. While research communities show growing interest in FG pedagogy, it remains underdeveloped in mathematics curricula (Chen et al., 2018; Karakus, 2011, 2013; Karakus & Baki, 2011).

Key obstacles include FG's absence from explicit curriculum requirements - in some countries like China and the US it appears only as optional content. Another barrier is its exclusion from teacher training programs, leaving educators unfamiliar with its epistemology and mathematical foundations (Chen et al., 2018). Karakus (2016) notes most pre-service teachers only understand dimension within EG's coordinate system framework, conceptualizing it through parameters like length, width and height, while remaining unaware of alternative conceptions like fractal dimension. However, countries like Uruguay and Argentina show increasing tendencies to incorporate FG elements in recent curriculum designs. Argentina's Buenos Aires province curriculum mandates fractal teaching in the final secondary year (6th year), presenting it through multiple mathematical lenses including geometry, sequences, transformations, and limits (General Directorate of Culture and Education of Buenos Aires Province, 2010, p. 13). In Uruguay, fractal concepts appear explicitly in the reformed 2006/2010 curriculum for the final secondary year (3rd baccalaureate year) under the "Mathematics and Design" track, focusing on constructing classic fractals (Cantor set, Koch curve, Sierpinski triangle) while connecting to length, area, and coordinate calculations (Secondary Education Council, 2006, p. 9).

As Fusi and Sgreccia (2020) note, these approaches create opportunities to revisit and deepen understanding of various mathematical concepts through FG, including sequences, limits, similarity transformations, and recursive processes. The mathematics education community recognizes FG's importance for both reconceptualizing fundamental notions and its interdisciplinary applications (medicine, biology, engineering, etc.). However, preliminary analysis suggests current FG instruction often overemphasizes visual and aesthetic aspects through limited student engagement - typically reduced to perimeter/area calculations of finite iterations. A common issue involves presenting finite iterations as complete fractals without addressing their limiting nature (Artigue et al., 2021).

A systematic literature review was conducted using Harzing's Publish or Perish software, covering publications from 2000 to 2020. The search targeted articles containing the terms "fractales" and "enseñanza" (or their English equivalents "fractals" and "teaching") in their titles, focusing on Google Scholar and Scopus as primary sources due to their accessibility. The selection process prioritized open-access journals with consistent publication records. From the resulting articles, we developed a bibliographic matrix structured according to Gowin's Vee instrument (Novak & Gowin, 2004), analyzing four key components: focus, conceptual domain, methodology, and main findings.

The data analysis was based on previous work by Pereira and Borges (2017) and Garbin (2007). The first study consists of an exhaustive bibliographic review about the teaching of Non-Euclidean Geometries (hereafter NEG) over the last ten years in Brazil. The second study analyzes the mathematical, didactic, and cognitive foundations of the fractal concept; although the author did not apply these to conduct a bibliographic review but rather to examine university students' perceptions of the notion of fractal, the cognitive perspective used represented a didactic interest when analyzing the surveyed research.

Thus, Pereira and Borges (2017) constructed the following categories regarding research on the teaching of NEG:

- Propose teaching NEG through educational software.
- Analyze NEG in teacher training.

- Examine the potential of fractals for simultaneously exploring different mathematical concepts.
- Propose teaching NEG through manipulatives.
- Analyze teachers' and students' conceptions about NEG.

These categories were adapted for this work, and an additional one was formulated specifically for this analysis, referring to fractals: *the rationale supporting the teaching of fractals*.

On the other hand, Garbin (2007) proposed the following as meanings of the fractal concept:

- Limit object or object resulting from an infinite process.
- Geometric object with fractional dimension.
- Irregular object representing natural phenomena.
- Mathematical set fulfilling certain properties, such as the three previously mentioned.

From the categories developed by Garbin (2007), it was considered that since fractional dimension is a property, it would be included under the category *"Definition of fractal through its properties"*. Additionally, since no author in the bibliographic review uniquely defines a fractal as *"an irregular object representing natural phenomena,"* this was not considered as a category. Likewise, another category was formulated: *"Fractal is not explicitly defined"*. Based on the work of Pereira, Borges, and Garbin, as well as the surveyed research, the analysis categories and subcategories presented in Table 1 were developed. These allow us to answer two questions regarding the construction of the state of the art for this research: *How is the teaching of Fractal Geometry proposed?* and *What characteristics of fractals are highlighted in teaching proposals?*

Table 1. Categories and Subcategories of Analysis for the State of the Art about Teaching and Learning FG. (Source: Prepared by the authors)

Category	Subcategory
C_1: Main interest for the study.	C_{1E}: Teaching FG (emphasis on the use of educational software or manipulatives, or analyzing GF to teach other concepts, with or without presentation of activities).
	C_{1P}: Analysis of teachers' or students' perceptions about FG.

Category	Subcategory
C_2: Reason to teach FG.	C_{2BU}: Beauty of FG due to its presence in nature or usefulness in various disciplines (motivation).
	C_{2ME}: Modeling and explanatory potential.
C_3: Characterization of the fractal geometric object.	C_{3N}: No explicit characterization is presented for fractals.
	C_{3P}: Self-similar mathematical set with fractal dimension (without reference to a limit process).
	C_{3L}: Limit object or object product of an infinite process.

When conducting a temporal analysis, we found growing interest in the topic, with 8 articles published in the last three years, while in the previous 17 years, an average of one publication per year was found. Regarding the publication frequency of this topic by journal, it is noteworthy that the journal with the highest number of publications on FG is the prestigious spanish journal Suma, with a total of 8 articles out of the 28 considered in this study. In order of frequency, next is the journal Unión, where 2 articles were found. The remaining 18 articles are distributed among the following journals: Mathematics Teacher, Números, Paradigma, Lapje, Bolema, Journal of Education and Human Development, Educational Sciences: Theory and Practice, Épsilon, Thema, and American Journal of Education and Information Technology.

Among the research findings, certain focal difficulties were identified when teaching and learning FG in secondary school. On one hand, there are those related to understanding the mathematical object fractal and its properties of self-similarity and fractional dimension (Karakus & Karatas, 2014; Karakus, 2015; Karakus, 2016; Pinto & Desconsi, 2018). On the other hand, there are operational difficulties, including handling operations with rational numbers and analyzing numerical series (Karakus, 2013).

Regarding the research focus (C_1), it was found that more than half of the studies (64%) focus on teaching proposals about FG, either through activities with manipulatives, software, or exercise sequences for students to discover some characteristic of FG. A small portion of those in C_{1E} (18%) implement these proposals, either in the curricular classroom or in extracurricular workshops; the remaining studies only present the proposals. In the remaining 36% of articles, there are 10 studies dedicated to analyzing teacher and student perceptions about FG (category C_{1P}). Thus, 7 studies analyze student productions when teaching FG-related concepts, examining how students distinguish, define, or draw fractals (Karakus, 2013; Karakus & Karatas, 2014; Karakus, 2015; Faccio, 2013). Other cases analyze calculation strategies for perimeters and areas, or construction methods using compass and straightedge or cardboard for different fractal iterations (Picoli & Pinto, 2018; Pinto & Desconsi, 2018; Rezende et al., 2018). The three studies on teacher perceptions respectively examine: their interest in professional development about FG (Chen et al., 2018), their understanding of fractal dimension (Karakus, 2016), and effective use of technology for teaching FG aspects (Yildiz & Baltaci, 2017).

Regarding the rationale for teaching FG (C_2), nearly half of the studies (43%) reference FG's beauty due to its presence in nature or utility across disciplines, which would produce intrinsic motivation (C_{2BU}). Seven of these exemplify FG applications by mentioning plant growth, cancer development, human anatomy, irregular surfaces, lightning patterns, music, painting, fluid mechanics, among others. These references do not go beyond mere mention, meaning no activities are proposed where students could model these situations using FG. The remaining studies (57%, category C_{2ME}) highlight FG's potential for addressing other mathematical concepts or demonstrating its modeling capabilities. For example, connections are shown with geometric concepts like perimeter, area, triangle congruence, and similarity transformations (Figueiras et al., 2000; Moreno, 2002; Rezende, 2018; Pinto & Desconsi, 2018); with calculus through sequence limits and arithmetic/geometric series (Shriki & Nutov, 2016; Karakus, 2011), exponential and logarithmic functions (Karakus, 2011); and with probability through geometric and conditional probability (Lopes et al., 2013).

When analyzing how studies refer to fractals, approximately 35% present fractals somewhat ostensively, showing pictorial representations while merely considering them as "pretty" figures that replicate themselves (C_{3N}); this is the case for activities designed for workshop spaces, extracurricular projects, or recreational activities. The remaining 64% explicitly characterize the fractal object, either alluding to an infinite process (39%, C_{3L}) or focusing on its self-similarity and fractal dimension properties (61%, C_{3P}).

Thus, in the 7 articles of subcategory C_{3L}, the concept of limit is introduced to find lengths, perimeters, and areas of the Cantor set, "Sierpinski triangle," "Koch curve," and "Pythagorean tree" fractals. Only 4 of them design activities targeting this objective. For the Cantor set, questions guide students on calculating its length, first asking them to indicate the number of segments in each iteration, the number of segments removed, and each segment's length. Finally, the fractal's length is requested through questions like: "*What will happen if the process continues to infinity?*" (Sardella et al., 2006), "*What will occur with the number of segments and their total length when iterations become infinite?*" (Redondo & Haro, 2004). For studying other fractals' perimeters and areas, the procedure is similar. Questions include: "*What do you observe happening to the perimeter and total area as iterations increase? What value does it approach?*" (Faccio, 2013), "*Is there any relationship between the number of branches? How could you generalize?*" (Pinto & Desconsi, 2018). In these cases, the treatment of the limit concept is informal, likely due to students' educational level.

Regarding fractal characterization through self-similarity, although this is a property fractals typically possess, nearly half (45%) of articles provide no definition, possibly because they focus conceptually on constructing different iterations, iterative processes, or the fractal dimension concept. In articles that do define self-similarity, the following appear: "*a figure is self-similar if it contains parts resembling the whole*" (9 articles) and "*a figure is self-similar if it's invariant under any scale change*" (5 articles), though two articles present both properties. Mathematically, these definitions are informal, hindering conceptual argumentation since they don't specify what mathematical processes students should perform to justify a figure's self-similarity.

From the C_{1P} category studies analyzing student productions, a drawing demonstrating fractal understanding was considered to require: the seed, iteration rule, and self-similarity property defined similarly to the above definitions (Karakus & Karatas, 2014; Karakus, 2016). It's concerning that teaching proposals don't offer operational methods for determining, through mathematical processes, whether a given geometric figure is self-similar. As discussed earlier, while mathematically complex, this could be adapted for secondary students. Developing approaches to this problem constitutes one of this work contributions. Only 3 of 28 articles mention different self-similarity types. Karakus and Baki (2011) classify fractal self-similarity into three types: self-similarity around a single point, self-similarity in certain parts (approximate), and self-similarity throughout (each part resembles the whole), illustrating this classification in Figure 1.

Figure 1. A Possible Classification of the Self-similarity Concept. [Taken from Karakus (2011).]

Karakus (2016), and Picoli & Pinto (2018), for their part, define strictly self-similar figures as those where any part contains a replica of the whole, or where any portion of the fractal exactly reproduces the form of a larger section. It is noteworthy that in all three articles, this classification is not subsequently applied in the proposed activities, thereby missing the potential advantages it could offer for conceptualizing fractals with strict self-similarity. The objective of analyzing the identified articles was to conduct a literature review on the teaching and learning of FG in secondary education. Through this search and analysis, the need to work on didactic transposition for teaching FG was established.

From the very core of Mathematics, establishing a formal definition of fractals proves resistant; therefore, pursuing this same goal in secondary education appears equally unreasonable. As analyzed, it is not possible to classify everything that "resembles" a fractal under the same label. Nevertheless, in this work we determine that one possible approach to the concept - while remaining faithful to its mathematical roots - is through its constitutive properties: fractal dimension and self-similarity. Specifically, regarding self-similarity, few studies propose mathematical work that would enable students to determine whether a figure possesses self-similarity, moving beyond intuitive notions of identical copies. This motivates developing targeted didactic interventions to address this aspect.

On another note, while most articles emphasize the applicability of FG across various disciplines, none demonstrate the design of interdisciplinary activities to leverage this strength. Such an approach would involve teaching FG through mathematical modeling - an absence likely resulting from the scarce implementation of genuinely mathematical processes with these figures, rarely progressing beyond their figurative and aesthetic aspects. According to this review's findings, it appears essential to design teaching activities that connect FG with:

Limit processes and the approximation to the notion of infinity, and the property of self-similarity as a means to develop mathematical processes such as pattern recognition and generalization.

Research Objectives and Questions

The general objective of this work is to contribute to the teaching and learning of key aspects of FG through the professional development of secondary school mathematics teachers. It seeks to move beyond the typical approach to teaching FG, which often lacks mathematical depth for secondary students, by providing tools for effective instruction. The specific objectives addressed in this research was:

- To design a sequence of activities that enable the teaching of key FG concepts to a group of mathematics teachers.
- To implement these activities with in-service secondary school mathematics teachers through specially designed courses focused on FG instruction.
- To examine teachers' mathematical and pedagogical knowledge as they engage with the designed proposal.
- To develop a set of recommendations for teaching key aspects of FG at the secondary school level.

In that sense, the research questions was:

- What activities can be designed for mathematics teachers to study the mathematical and pedagogical content knowledge of FG for teaching purposes?
- How can key elements of FG be introduced to mathematics teachers for its instruction in secondary schools?
- How can the mathematical knowledge for teaching key FG topics be described and characterized through an analysis of the mathematical-pedagogical potential of a teaching proposal?
- What recommendations and suggestions can be proposed for teaching FG in secondary schools?

In this work, the mathematical-pedagogical potential of the proposal is understood as the capacity to:

- Support secondary teachers in studying key FG concepts such as self-similarity and fractal dimension, as well as the mathematical description of these objects' construction.
- Highlight the benefits of incorporating FG into secondary education by linking classroom work with mathematical rigor.
- Promote technological mediation (e.g., interactive applets, GeoGebra, videos, educational websites) as didactic tools for teaching key FG concepts.
- Generate interest among mathematics teachers in studying FG through extramathematical contexts, such as multiband prefractal antennas—selected specifically for their modeling potential, as detailed in the following chapter.

Operational Definitions for Strict Self-Similarity and Fractal Dimension

The mathematical quality of a teaching process can be analyzed through the descriptor of *mathematical richness* (Font et al., 2015). These authors built upon categories previously proposed by Hill et al. (2008), who considered that such richness can be examined through: detection of mathematical errors, justifications, mathematical representations, solution methods, generalizations and mathematical observations.

This closely relates to the concept of *operativity*, as the meaningfulness of a definition (as an expression) partly depends on what actions it enables, what it allows one to do, and its potential for transformation into other representations. The operativity of a definition thus refers to the possibility of operating within a reference framework according to the task at hand. Consequently, an operative definition requires students to select valid and relevant transformations. This pertains to substituting one mathematical representation for another through content changes to advance reasoning or calculations (Panizza, 2015).

In this work, we propose reconstructing an operative form – with mathematical quality and richness in Hill et al.'s (2008) sense – of the *dimension* concept for strictly self-similar fractals. This extends our prior reconstruction of self-similarity (Artigue et al., 2021), which led to another publication in *Unión* journal (Artigue et al., 2022). There, we analyzed and exemplified various mathematical definitions of dimension, ultimately reconstructing a *strict self-similarity dimension* concept. The paper includes productions from mathematics teachers who participated in courses implementing the instructional approach. Results indicate the proposal demonstrates mathematical richness by formalizing intuitive ideas typically found in textbooks and teaching materials.

Specifically for this work, we established the following self-developed definition of strict self-similarity:

A figure is strictly self-similar if it is obtained as the limit of an iterative process starting from an initial figure called the seed, where the result of each step admits a partition such that each subset composing it is similar to the seed with the same similarity factor, less than 1; furthermore, if we designate:

- p_k *as the number of subsets in the partition corresponding to the k-th step,*
- α_k *as the similarity factor between each subset of the partition and the seed,*

it follows that the quotients $p = \frac{p_{k+1}}{p_k}$ *and* $\alpha = \frac{\alpha_{k+1}}{\alpha_k}$ *are constants.*

To establish an operational definition of fractal dimension, we must first observe that when the afore mentioned quotients are raised to the same integer power - let us call it r - they connect the results of any given step with those corresponding to the step obtained after r iterations from it, thus extending the relationship between the results of two consecutive steps as before, that is:

$$p^r = \frac{p_{k+r}}{p_k} \text{ y } \alpha^r = \frac{\alpha_{k+r}}{\alpha_k}.$$

We observe that the exponent matches the difference in value between the two iterations being considered. A similar analysis of the quotients between the similarity factors (or scaling factors) of the subsets corresponding to the previous step and the seed (α) leads us to conclude that the exponent coincides with the difference in value between the two iterations considered. Therefore, if we wish to analyze the quotients independently of r, since it appears as an exponent, we can take the logarithm of both expressions and now consider their quotient, because:

$$\frac{\log \frac{p_{k+r}}{p_k}}{\log \frac{\alpha_{k+r}}{\alpha_k}} = \frac{\log p^r}{\log \alpha^r} = \frac{r.\log p}{r.\log \alpha} = \frac{\log p}{\log \alpha}$$

The dimension, thus constructed, whose value is the result of this quotient, represents the size of the fractal (it depends on how large p is and how small α is). We will designate the value of this invariant quotient as the strict self-similarity dimension of the figure.

Theoretical Foundations for Analyzing the Knowledge Required to Teach Fractal Geometry

Mathematical Knowledge for Teaching

The theoretical model called *Mathematical Knowledge for Teaching* (MKT) was proposed by Ball et al. (2008) and is widely used in research due to its potential to describe the knowledge required by teachers in the process of teaching mathematics (Ball et al., 2008; Hill et al., 2005; Hill et al., 2008; Carrillo et al., 2013). The model distinguishes between:

- The mathematical knowledge needed by teachers (*Specialized Content Knowledge*).
- The knowledge needed by other users of the discipline, who could be either educated citizens with some mathematical knowledge or specialists in fields requiring advanced mathematics but not its teaching (*Common Knowledge*).

Specialized Content Knowledge refers specifically to teaching tasks rather than the depth of mathematical content knowledge. Therefore, we can speak of complex mathematical knowledge even for elementary school teachers or early-grade educators (Carrillo et al., 2013; Ball, 2000).

Knowledge concerning the structure of mathematics - including relationships between different mathematical topics and connections with other disciplines - is reflected in *Horizon Content Knowledge*. Thus, both Horizon Content Knowledge and Specialized Content Knowledge demonstrate that the difference between elementary and advanced mathematics is not associated with whether the teacher works in primary versus secondary education (Carrillo et al., 2013).

In this way, the MKT model represents an evolution of the subdomains described by Shulman (1986), establishing six subdomains (shown in Figure 2) that are categorized as: *Subject Matter Knowledge* and *Pedagogical Content Knowledge*.

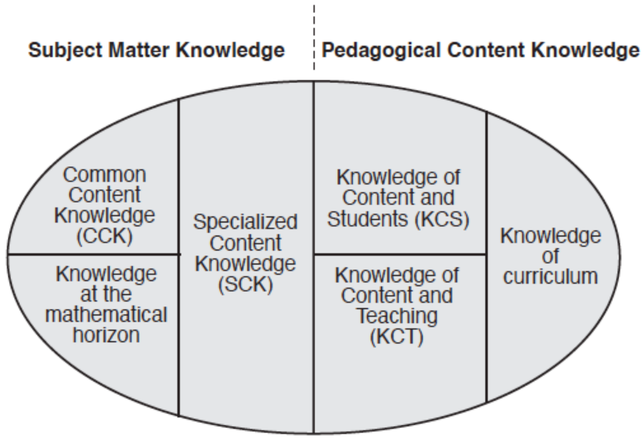


Figure 2. The MKT Model proposed by Ball, Thames, & Phelps (2008). [Source: Ball et al. (2008)]

The specificity of content knowledge addressed by the MKT model has been recognized by the mathematics education research community as its most significant contribution (Flores et al., 2013; Hill et al., 2008). However, this model has been questioned by some authors (Carrillo et al., 2013; Montes et al., 2013) due to the difficulty in finding evidence to clearly delineate some of its subdomains. These authors analyze classroom episodes to demonstrate the overlap between certain MKT subdomains, indicating they are not mutually exclusive.

The alternative proposal by some authors is the Mathematics Teacher's Specialised Knowledge (MTSK) model. It is important to emphasize that all the teacher's knowledge engaged in the previous example is specialized and unique to their profession. Thus, the MTSK model aims to continue the study and analysis of what kind of knowledge a teacher must employ to contribute to student learning, positioning this knowledge as specific to mathematics teachers and contrasting it with that of other professionals who use the discipline (Ball et al., 2008).

In the MTSK framework, three subdomains are considered for the Mathematical Knowledge (MK) domain: profound knowledge of mathematical content itself or Knowledge of Topics (KoT), knowledge of the structure of mathematics or Knowledge of the Structure of Mathematics (KSM), and knowledge about how mathematical procedures are carried out and how knowledge is produced in mathematics or Knowledge of Practices in Mathematics (KPM) (Carrillo et al., 2013). A general schematic of the MTSK model is shown in Figure 3.

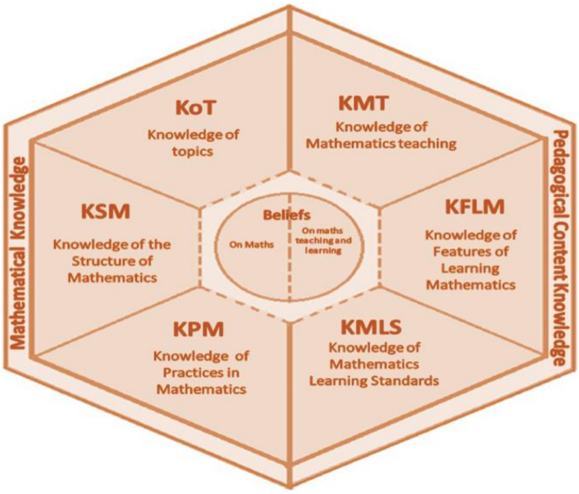


Figure 3. MTSK Subdomains. Source: [Carrillo et al. (2013).]

In line with the knowledge of mathematical content and mathematical practices, it is essential for this work to have a mathematically formulated definition of key aspects of FG, as well as an analysis of its suitability for secondary education. This is because the research problem is based on an exhaustive state-of-the-art review, which revealed the absence of a formal didactic transposition of the concepts of self-similarity and fractal dimension, at least at the secondary education level.

In this context, it is worth noting that a mathematical definition includes at least two essential elements to consider:

1. Mathematical content, which relates to understanding the defined object and its properties.
2. Operativity, which refers to the possible actions the definition enables (Panizza, 2015).

The operativity of an expression is reflected in its capacity to be substituted by other mathematical representations, leading to consideration of how the definition is treated according to the task and its objective. Choosing appropriate representation registers involves understanding which are valid for the activity - whether a mathematical operation, equality, equation, function, graph, table, matrix, etc.

The treatment of a definition therefore depends on:

- The ability to substitute mathematical expressions.
- Their corresponding representations.

These must be both valid and relevant, enabling the subject to advance their reasoning or calculations to achieve the predetermined goal. Within the mathematical framework of this work, the mathematical formulation of a possible definition of strict self-similarity and fractal dimension - predominantly expressed in verbal language - required:

1. Identifying necessary geometric transformations for constructing an iterative figure.
2. Analyzing the characteristic quotients that describe and support its behavior in both consecutive iterations and those varying by a constant.

The calculations suggested in the formulated definition demonstrate what the fractal dimension means for an iteratively constructed figure.

The Didactic Suitability Criteria as a tool for analyzing pedagogical content knowledge

The Didactic Suitability construct was introduced by Godino, Bencomo, Font, and Wilhelmi (2006), defining the following six suitability types:

1. Epistemic Suitability: Refers to the degree of representativeness of the institutional meanings implemented (or planned) relative to a reference meaning.
2. Cognitive Suitability: Expresses the extent to which the intended/implemented meanings fall within students' zone of proximal development, as well as the proximity of achieved personal meanings to the intended/implemented meanings.
3. Interactional Suitability: The degree to which didactic configurations and trajectories allow both the identification of potential semiotic conflicts (detectable a priori) and the resolution of conflicts arising during the instructional process through meaning negotiation.
4. Mediational Suitability: The degree of availability and appropriateness of material and temporal resources required for the teaching-learning process.
5. Emotional Suitability: The degree of student engagement (interest, motivation) in the learning process.
6. Ecological Suitability: The degree of adaptation of the learning process to the school's educational project, curricular guidelines, and social environment conditions (p. 3).

Below is a brief description of each suitability type (Font, 2011; Seckel et al., 2019), some contextualized to mathematics teaching:

1. Epistemic Suitability: The extent to which the mathematics taught is considered "good mathematics." Indicators include contextualized problems, activities with varying difficulty levels, diverse task typologies, and the promotion of relevant mathematical processes such as argumentation and modeling.
2. Cognitive Suitability: Evaluates whether the intended content is reasonably close to students' prior knowledge before instruction and whether the acquired learnings align with the intended outcomes afterward.
3. Interactional Suitability: Assesses whether classroom interactions address students' doubts and difficulties.
4. Mediational Suitability: Measures the appropriateness of material and temporal resources used during instruction, including whether manipulatives or digital tools are employed and whether time is allocated effectively to core content.
5. Emotional Suitability: Evaluates student engagement, interest, and motivation during instruction while preventing math-related rejection or anxiety.
6. Ecological Suitability: Assesses the alignment of instruction with the school's educational project, curricular guidelines, and socio-professional context, including connections to other disciplines.

In this sense, Didactic Suitability (DS) criteria help determine the extent to which mathematics instruction possesses characteristics qualifying it as suitable for achieving its objectives. In this process, it is vital to articulate Epistemic Suitability (institutional mathematical meanings) and Cognitive Suitability (personal, psychological, and individual meanings), as they model the mathematical knowledge to be taught and students' learning outcomes. Other equally necessary suitabilities include Mediational (availability of material and temporal resources) and Ecological (related to institutional, social, and policy contexts).

The DS framework was employed by mathematics teachers in their final course assignment. The task required analyzing a FG teaching proposal from the DS perspective. Not every activity or the entire proposal necessarily reflects all suitability criteria. Thus, depending on the chosen aspect of the course, specific criteria served as analytical tools. The following section presents the teaching proposal designed for this work.

Teaching Proposal on Fractal Geometry for a Teacher Professional Development Course

The teaching proposal presented below includes a series of activities specifically designed for N=83 practicing secondary mathematics teachers in Uruguay. It was constructed considering certain didactic-pedagogical assumptions that support it, for example: there is an introduction to FG through one of its mathematical applications such as multiband antennas; it always includes questions aimed at reflecting on the distinctive characteristics of FG; it continually attempts to relate new concepts to what participating teachers already bring with them; and it tries to generate interest by connecting FG with its beauty and mathematical complexity.

These teachers voluntarily participated as students in three editions of an online course on FG and its teaching. The general objective of the course was to contribute to the continuing education of practicing teachers, and particularly in FG, since most participants expressed having had little or no prior exposure to the topic. The course covered two activities that specifically address the concept of strict self-similarity and the concept of fractal dimension.

Activity 1: Videos on Fractal Antennas

The first course activity aimed to introduce elements of FG through one of its applications: multiband antennas based on fractal iterations, also known as prefractal antennas. This latter term refers specifically to antenna shapes constructed according to specific fractal iterations. The initial objectives were to study this geometry's applicability and spark curiosity/motivation - essentially creating an engaging entry point for participant involvement. Two prefractal antenna videos (links provided in activity instructions below) were selected, which mention self-similarity and fractal dimension concepts in relation to multiband capability and miniaturization.

Participant questions targeted:

1. Prior knowledge/common content knowledge about FG.
2. Newly acquired ideas from the videos.
3. Existing horizon content knowledge and curricular knowledge.

Responses were uploaded to a collaborative online board (visible to all participants with commenting enabled). The instructions were: "*Study the linked videos below and answer:*

1. *What new concepts did you encounter?*
2. *How do these concepts relate to mathematics?*
3. *Could you teach these concepts in your math class? Why?*

   Upload responses to the designated Padlet."

Video1: https://www.youtube.com/watch?v=50KUGdAvMGw

Video 2: https://www.youtube.com/watch?v=La7qSZ7Uxig

Activity 2: Mathigon Reading

The interactive Mathigon textbook (web-based, previously mentioned in the literature review) introduced participants to both linear and nonlinear fractals. Through this resource, they studied foundational concepts of self-similarity and self-similarity dimension. The guiding question analyzed knowledge of key FG topics:

> "*Interact with Mathigon's textbook, complete all embedded activities, then answer: What mathematical processes and entities construct the Sierpinski triangle and Julia sets? Identify similarities/differences. Submit a single PDF (max 1 page).*"

Activity 3: Fractal Antenna Applet

This activity analyzed knowledge regarding perimeter-area relationships in linear fractals. Instructions:

> "*Fractenna Company (https://www.fractenna.com) designs space-efficient miniaturized antennas, beginning with a basic copper antenna (segment AB in Figure 4). The slider generates iterations.*

1. *Describe/explain antenna generation processes using both colloquial and mathematical language.*
2. *Calculate each antenna's length if iteration 0 measures 12cm.*
3. *Determine minimum dimensions for:*

   a) Rectangular embedding material.

   b) Triangular embedding material.
4. *Reflect on material efficiency implicatio*".

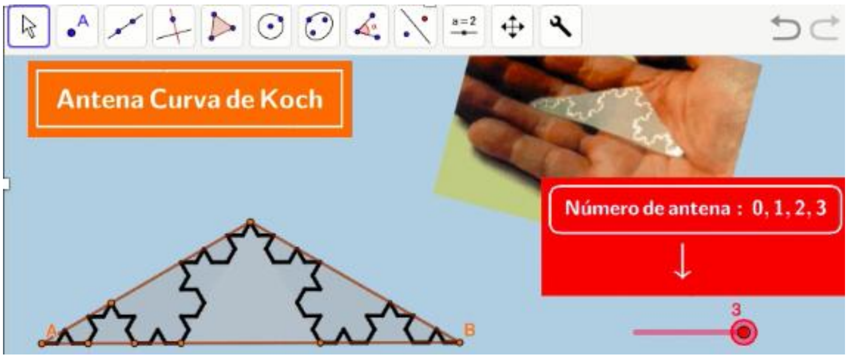


Figure 4. Applet Designed and Developed in GeoGebra by the Author of This Work, to Study Properties of CK (Content Knowledge) in the Context of Fractal Antennas

Activity 4: strict self-similarity and fractal dimension

This activity was designed to assess mathematical knowledge of key FG concepts. The selected educational website, tailored for teachers specializing in fractal instruction, features role-specific interfaces with separate tabs for learners and instructors. The instructor interface includes: 1. a simulated student-teacher dialogue demonstrating self-similarity and dimension instruction, and 2. interactive applets with configurable parameters for scaling factors and iteration quantities of strictly self-similar fractals.

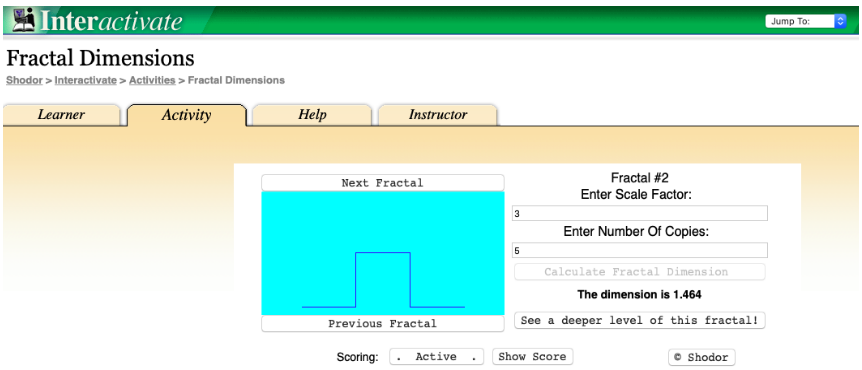


Figure 5. Homepage of the Website Hosting the "Self-Similarity and Fractal Dimension" Activity. [Source: http://www.shodor.org/]

The activity instructions were: *"We invite you to explore the "InterActivate" website to familiarize yourself with content dedicated to fractal dimension concepts. Navigate through these tabs: "Learner," "Activity," "Help," and "Instructor." Then, under the "Activity" tab:*

1. *For each fractal, complete the text fields with the scaling factor and number of*

copies. Note the site only displays each fractal's dimension when correct values are entered.

2. *Select one displayed fractal and justify its strict self-similarity using the course's definition.*
3. *Sort all page fractals by ascending dimension, then formulate conclusions relating dimension to "space-filling" degree. Mathematically justify the dimension calculation for your chosen fractal (from step 2).*

Create a PowerPoint, Genially, or similar presentation (max 3 slides) addressing questions 2-3. Upload to the "self-similarity and dimension task" space".

Activity 6: Multiband Prefractal Antennas

This activity design involved researching antenna studies using FG principles. Engineering/electronics researchers (Gianvittorio et al., 2003) use "prefractal" for specific fractal iterations. Since Moore's (1965) work on miniaturization, antennas have faced fundamental size limitations (Herrera & Inclán, 2004), driving research into ultra-compact designs. Wireless communication advances demand multifrequency operation within small devices (e.g., cellphones, laptops), requiring broad operational bandwidths (Sandoval & Vire, 2008). For instance, GSM compliance requires 900MHz and 1800MHz bands. Frequency-independent antennas need multi-scale structures, where efficient space use maximizes bandwidth. FG-based designs intuitively satisfy both requirements through:

- Multiband capability (via self-similarity)
- Miniaturization (infinite lengths in finite areas)

We present multiband prefractal antennas as a FG application. Their key feature is pattern repetition across scales, enabling simultaneous multi-frequency gains unlike traditional single-band antennas. While fractal antenna theory combines Maxwell's equations with FG principles, the field remains experimental. Each iteration theoretically adds operational bands, but practical implementation requires computational simulations (e.g., CST Microwave Studio) testing materials, substrates, and other telecom parameters (intensity, power, etc.). Multiband prefractal antennas exploit scalability: each iteration generates new perimeters, creating inversely proportional frequencies. A seed antenna operating at frequency f, when scaled by factor k, operates at f/k and multi-scaled copies produce identical electromagnetic behavior across all scale factors' bands.

The activity instructions were:

1. "*Identify seed structures and geometric processes in these fractal-inspired designs that are shown in Figure 6.*
2. *Classify their self-similarity type*
3. *Calculate theoretical operational frequencies and justify*
   Suggested approach: Analyze the fractals' construction processes."

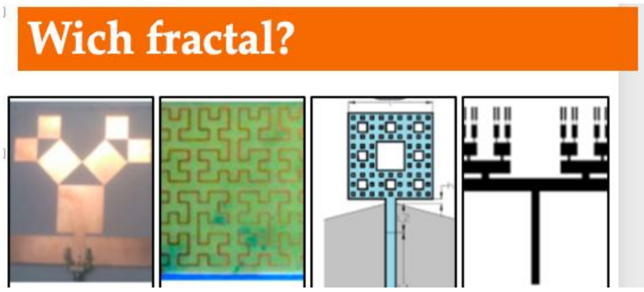


Figure 6. Antennas Inspired by Different Fractals. [Sources: (1) Bakytbekov (2017), (2) Oltra (2009), (3) Aggarwal & Kartikeyan (2010), (4) Sandoval & Vire (2008), (5) Brande (2012).]

Activity 7: Pedagogical Knowledge Based on Didactic Suitability Criteria

The course was designed to incorporate different tools and varied didactic intentions. In addition to addressing basic FG concepts, the Didactic Suitability framework was introduced as a reference for analyzing teaching interventions. The Didactic Suitability criteria mentioned earlier served as the instrument to study and analyze how participating teachers acquired Pedagogical Knowledge of the proposed FG teaching approach.

Final submission must include at least the following elements:

1. A detailed report of the analysis process, explaining what evidence was significant for applying each relevant criterion.
2. A personal reflection on the experience of completing this task, including:
   - Challenges encountered and strategies to overcome them
   - Perceived contributions to professional development

The task may be completed individually, but we encourage group work (up to three members). The submission will consist of a maximum 5-minute video in which the group presents their critical analysis.

Methodology of the Study

The study conducted is qualitative, exploratory, and descriptive in nature, although some quantitative techniques were also employed. The objective is to examine, describe, interpret, analyze, and characterize the mathematical and pedagogical knowledge of teachers who participated in the continuing education course on FG, based on their response protocols for each activity.

This analysis was carried out using the theoretical constructs of Shulman (1986), Ball et al. (2008), and Font (2011). The state-of-the-art review on the teaching and learning of FG was crucial for designing and developing the course activities, as it provided insight into current trends in FG education. After conducting and analyzing this review, the activities were designed with the goal of using FG to mathematize real-world situations—an area identified as underdeveloped in existing research.

The activity design considered FG's potential for modeling real-world contexts, ensuring that the prompts and questions targeted the construction of key FG concepts. Mathematical modeling is understood here as the creation or use of mathematical models to solve applied or contextual problems (Blum & Niss, 1991). An applied problem in mathematics is framed within a real-world situation (the "rest of the world" outside mathematics) and involves questions that connect mathematical concepts to that situation (Blum & Niss, 1991).

Some activities involved the use of applets, specifically designed for this course (e.g., the *"Koch Curve Antenna"* and *"Sierpinski Triangle Antenna"*), created using the open-source software GeoGebra. Other activities, such as *"Self-Similarity and Fractal Dimension,"* relied on existing FG-focused websites, though the prompts were adapted to fit the course context.

The course was hosted on CREA, a platform provided by Uruguay's CEIBAL initiative (https://www.ceibal.edu.uy), as shown in Figure 7. All public secondary school teachers in Uruguay have access to this platform. Each course edition lasted six weeks.

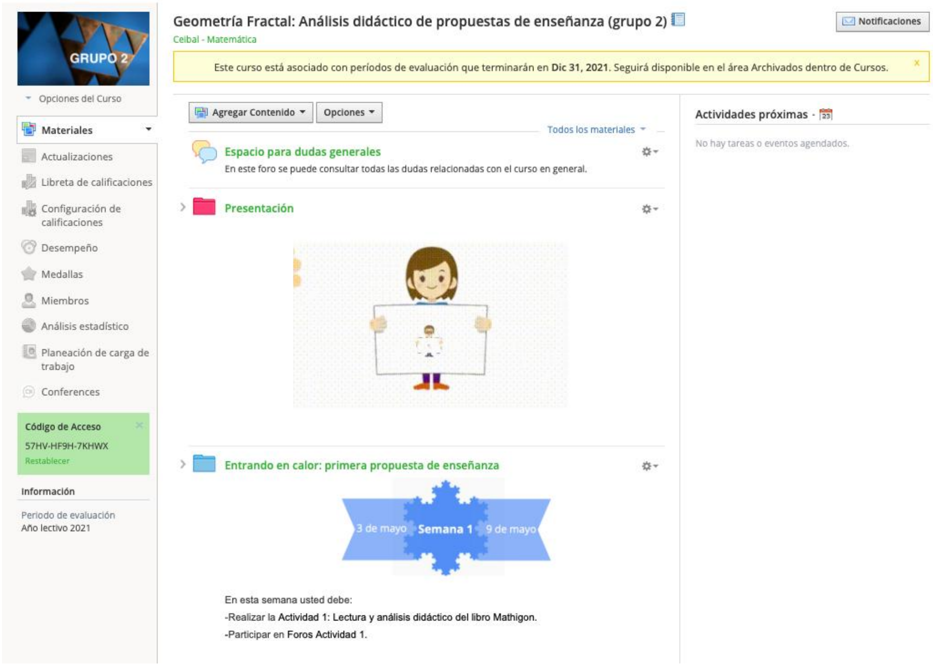


Figure 7. Homepage of the Course *"Fractal Geometry: Didactic Analysis of Teaching Proposals"*. [Source: Personal creation.]

To describe the Mathematical Knowledge and Pedagogical content Knowledge, a categorization of the responses and procedures used in the assigned tasks was carried out based on the conceptual framework, followed by univariate and bivariate analyses of the established categories, which were treated as (qualitative) variables in the study. The data were constructed from the theoretical-conceptual framework and the written and audiovisual productions submitted by course participants in response to the proposed activities. From this analysis, categories and subcategories of Disciplinary Content Knowledge about FG and Pedagogical Content Knowledge of the proposal were developed.

Analysis of Results

This chapter examines the two main types of knowledge according to the adopted theoretical framework: Subject Matter Knowledge and Pedagogical Content Knowledge. For each, categories were drawn from both the Ball et al. (2008) and Carrillo et al. (2013) models, adapting these frameworks to the data obtained in this study. This adaptation was necessary because FG is not included in Uruguay’s mathematics teacher training curriculum, making it impossible to analyze the Specialized Content Knowledge characteristic of the MKT model.

It was assumed that merely participating in a course on FG teaching would not suffice for teachers to develop specialized knowledge of fractal instruction. Furthermore, neither course completion nor the objectives of this study required teachers to design FG activities for their secondary school classes. Instead of Specialized Content Knowledge, the Knowledge of Topics (Carrillo et al., 2013) was considered, given its potential to analyze and understand teachers' disciplinary knowledge.

For this research, the Knowledge of Topics category was further subdivided to focus on:

- The concepts of self-similarity and fractal dimension (and their operationalization).
- The geometric transformations required to construct certain linear fractals.
- The relationship between perimeter and area in key FG cases.

Thus, the analysis began with disciplinary knowledge, resulting in a new analytical framework, as shown in the following Table 2.

Table 2. Categories to be Considered, Derived from the Ball et al. (2008) Model and the Carrillo et al. (2013) Model, for Knowledge of the Discipline. [Source: personal elaboration.]

SMK (Knowledge of the discipline)	CCK (common knowledge of the content), taken from Ball et al. (2008).	Subdomain del MKT
	HCK (knowledge on the mathematical horizon), taken from Ball et al. (2008).	Subdomain del MKT
	KoT (knowledge of mathematical topics or themes), taken from Carrillo and collaborators (2013).	Subdomain del MSKT

Once the categories were selected through an inductive, interactive process of analyzing the protocols and in dialogue with the adopted theoretical frameworks, an ascending ordering of knowledge was created for each category based on the concept of "mathematical quality of instruction" proposed by Hill et al. (2008). This ordering considered: the identification of mathematical content related to FG, the mathematical depth (or formality) with which it was addressed, and the completeness of the task. These aspects were identified from the participants' productions

In this sense, the weakest level in mathematical quality is Level 1, where responses focus on general aspects of the mathematical object under study or describe it in intuitive terms. The next level, Level 2, is stronger as the response incorporates some specific consideration of mathematical content, though without delving deeper. Finally, Level 3 represents the strongest mathematical quality, manifested in productions that offer a more complete treatment of the object, addressing mathematical processes that define it more formally.

Regarding common knowledge, the first level refers to knowledge of mathematical content that is part of official secondary school curricula and is necessary to operationally solve activities designed to work mathematically with FG. For example, similarity transformations are fundamental for constructing some linear fractals, the logarithmic function is essential for the concept of fractal dimension, and the concept of limit is necessary to understand self-similarity. The second level refers to mentioning some characteristics that define certain fractals, and the third level involves describing at least one of these characteristics, even if informally. It was assumed that, since course participation was voluntary, participants had some interest in studying FG and its teaching, and therefore we examined what mathematics they knew about fractals - knowledge they likely had not acquired during their teacher training. The Knowledge on the Mathematical Horizon was analyzed mathematically in relation to the epistemological origins of FG, its connection with EG, and also its differences from EG, taking into account its potential for modeling natural structures or phenomena.

As subdomains of Knowledge of Topics, it was considered that the lowest levels of knowledge regarding strict self-similarity (topic 1) and fractal dimension (topic 2) refer to productions that are closer to intuitive ideas about these properties, as opposed to higher levels that reflect an intention to mathematically formalize these ideas. For geometric transformations needed to construct some linear fractals (topic 3), the highest level had quality indicators related to describing all elements of the transformations and how they are mathematically composed. Finally, for the subdomain of perimeter-area relationships of some fractals (topic 4), the highest level was assigned to productions that explicitly present expressions or functions that generalize (and model) perimeter and area for a given iteration.

Univariate Statistical Analysis

A descriptive analysis of the categories is displayed, with a mosaic plot presented for each one. In all cases, the blue section of the graph represents the percentage of productions at the lowest level of specific knowledge about FG (coded as 1), the orange section corresponds to level 2, and the green section represents the highest level of specific knowledge achieved (coded as 3). Each level is exemplified with one or more productions made by participating teachers. The N=83 participants were coded with the letter P followed by an assigned teacher number (from 1 to 83).

Analysis of Mathematical Knowledge about Fractal Geometry

To analyze common knowledge about FG, it was considered that among participating teachers, 67% hold a secondary mathematics teaching degree, while the remaining percentage consists of teachers who, without a teaching degree, are authorized by Uruguay's General Directorate of Secondary Education to teach mathematics in secondary schools. Therefore, participating teachers are mathematically educated at least at a level slightly more advanced than secondary school, and despite FG being virtually absent from both secondary curricula and teacher training programs, they felt sufficient motivation and interest to voluntarily enroll in the course.

Evaluating common knowledge regarding FG was the objective of the first course activity, which involved studying two videos about fractal antennas to introduce this geometry through an extramathematical context and explore teachers' prior knowledge about fractals. Participants answered: What concepts were you exposed to for the first time? What mathematical content and processes are necessary to learn about fractals? The audiovisual materials analyzed did not delve deeply into fractals, and their particular properties were only mentioned for practical purposes of antenna operation; therefore, responses to this activity referred to knowledge acquired either during secondary school, teacher training years, or through personal motivation.

The levels developed for this category are:

1. Mention of basic mathematical content needed to understand FG characteristics and up to three defining properties
2. Statement of several (more than three) fractal characteristics (without arguments or justification)
3. Description of at least one characteristic property of FG, albeit informally (without mathematical rigor)

Results shown in Figure 8 indicate that levels 2 and 3 account for nearly two-thirds (71%) of teachers, interpreted positively as evidence that teachers possess mathematical knowledge related to FG. Slightly less than one-third of participants (29%) only recognized superficial aspects of fractals. It can therefore be established that participants have substantial common knowledge about fractals, which is a favorable indicator when considering the incorporation of this geometry into official secondary school curricula and teacher training programs.

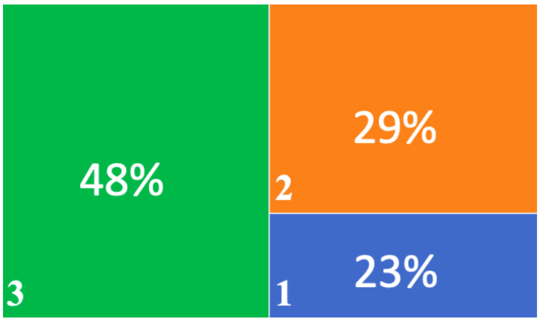


Figure 8. Results of the Shared Knowledge of FG among the Mathematics Teachers who Participated in the Course. [Source: personal elaboration.]

Horizon Content Knowledge about Fractal Geometry

The horizon knowledge about FG was analyzed considering intrinsic aspects of this geometry in comparison to EG. For this work, it is accepted that EG was studied by participants throughout their secondary education or during their initial years of teacher training. Therefore, the levels describing the horizon knowledge subdomain refer to aspects that both connect and differentiate these two geometries. There is no intention to analyze the relationship of mathematical contents with others present in curricula at different educational levels, since FG still has little presence in official Mathematics programs.

The mathematical quality levels to describe the horizon knowledge shown by participating teachers are as follows:

1. Expression of intuitive ideas that FG has applications in extramathematical contexts, without reference to its epistemological origins or its differences from EG.
2. Expression referring to FG being clearly different from EG, with description of at least one difference.
3. Expression of the idea that FG is clearly different from EG, considering it as a continuation or deepening of EG.

As shown in Figure 4.7, most (80%) of the analyzed protocols are concentrated at level 1. This result indicates that teachers have weak knowledge about the reasons why FG emerged, although they do vaguely mention the potential of this geometry for mathematically modeling natural phenomena. In the analyzed productions, there was no mention of why FG can be considered different from EG. The remaining 20% of teacher productions correspond to levels 2 and 3.

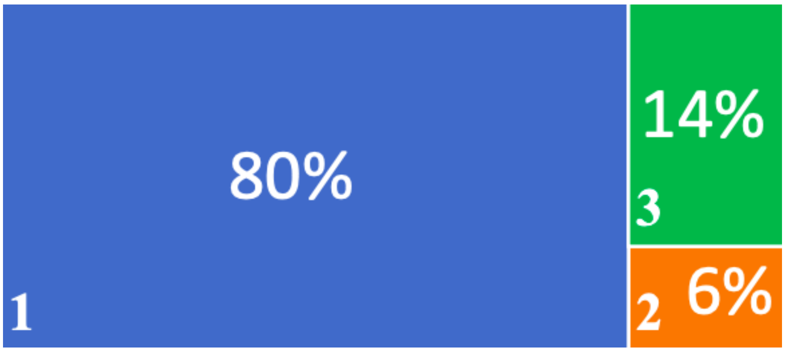


Figure 9. Knowledge Results on the Horizon regarding FG. [Source: personal elaboration.]

Topic 1: Definition of Strict Self-Similarity

This section describes teachers' knowledge regarding the concept of self-similarity. Specifically, it analyzes teachers' interaction with a mathematically strict definition of self-similarity designed by the course tutors This involved examining what mathematical actions and practices were necessary to argue that a figure is strictly self-similar.

The description of the levels is as follows:

1. Use of only the notion of strict self-similarity, with colloquial descriptions based on intuitive ideas.
2. Mention with incomplete or specific justification that the ratios indicated in the formal definition of strict self-similarity are constant.
3. Mention with detailed and general justification of why the ratios indicated in the formal definition of strict self-similarity are constant.

For the participants of the second edition (N=38), the results indicate that slightly more than half of the analyzed protocols (55%) were placed at Level 1, meaning that the formal definition of strict self-similarity was treated informally, colloquially, or with intuitive ideas.

About one-third (37%) mentioned the presence of invariant ratios specific to each fractal but failed to argue why. Only 8% managed to work with the definition under all the conditions it establishes—that is, identifying the ratios that are constant in a strictly self-similar fractal and justifying it mathematically.

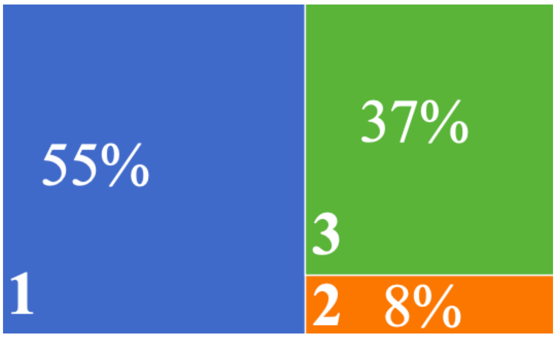


Figure 10. Distribution of Results for Topic 1: Definition of Strict Self-Similarity corresponding to the 2nd Edition of the Course. [Source: personal elaboration.]

Topic 2: Fractal Dimension (DIM)

The analysis for Topic 2 (fractal dimension) was not conducted for the first edition, as it was not included in any activity of the teaching proposal. In the second edition, this topic was introduced to participants as a corollary of the strict self-similarity property, with both concepts being addressed in the same activity. In this sense, the construction of the levels was as follows:

1. Only the presence of an ordering of fractals based on their fractal dimension value or the use of the idea of "plane-filling" to interpret fractal dimension.
2. Interaction with the definition of fractal dimension in an incomplete manner or without justification for why it is calculated using a ratio of logarithms.
3. Proper use of the definition and correct calculation of the fractal dimension value, along with a mathematical interpretation of the logarithm ratio and its connection to the idea of "*how much the fractal fills space*."

Out of a total of N=38 participants in the second edition, nearly three-quarters (76%) of the protocols corresponded to Level 1. This result is concerning from a research perspective, as the concept of fractal dimension was primarily acquired intuitively, equating it with ideas such as "*how much the fractal fills a flat surface*." This highlights the complexity of the concept, though the remaining responses suggest that it is not an unapproachable topic. Only 5% of the productions were at Level 2, while almost one-fifth (19%) of the protocols corresponded to Level 3, meaning that the teachers were able to calculate the dimension of the chosen fractal using the formal definition.

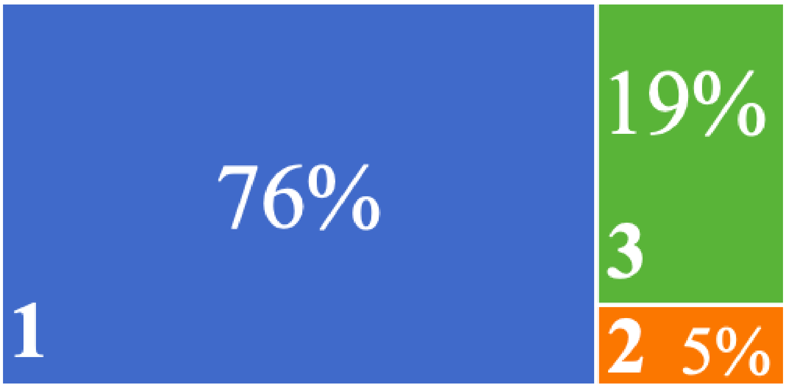


Figure 11. Distribution of Results for Topic 2: Fractal Dimension corresponding to Edition 2 of the Course. [Source: personal elaboration.]

Fractal dimension showed higher percentages at Level 1 and lower percentages at Level 3. This finding suggests that understanding fractal dimension proved more challenging compared to strict self-similarity, which could be attributed to its nature as a more abstract mathematical property often described through equations and more sophisticated calculations. In contrast, strict self-similarity has an advantage in terms of comprehension, as it allows for simple analogies between this concept and real-world objects such as a fern leaf or a snowflake. These visual and tangible comparisons provide better intuition about self-similarity, making it easier to grasp.

From the protocols, it can be interpreted that the teachers ordered the fractals according to their dimension value and drew conclusions about "space-filling" based on this ordering and the visual representation provided by the applet on the website. It is worth noting that the applet displays images corresponding to higher iterations once the fields (scaling factor and number of copies) are correctly filled in. Figure 12 correspond to the protocol of teachers P45 who were selected as representative of this level. Even though the images shown correspond to the first iteration of the fractal, the teacher's conclusion appears to incorporate the intuitive idea that a higher dimension implies greater "space-filling" phrases such as *"greater filling capacity"* or *"more complete coverage of the plane"*suggest this reasoning.

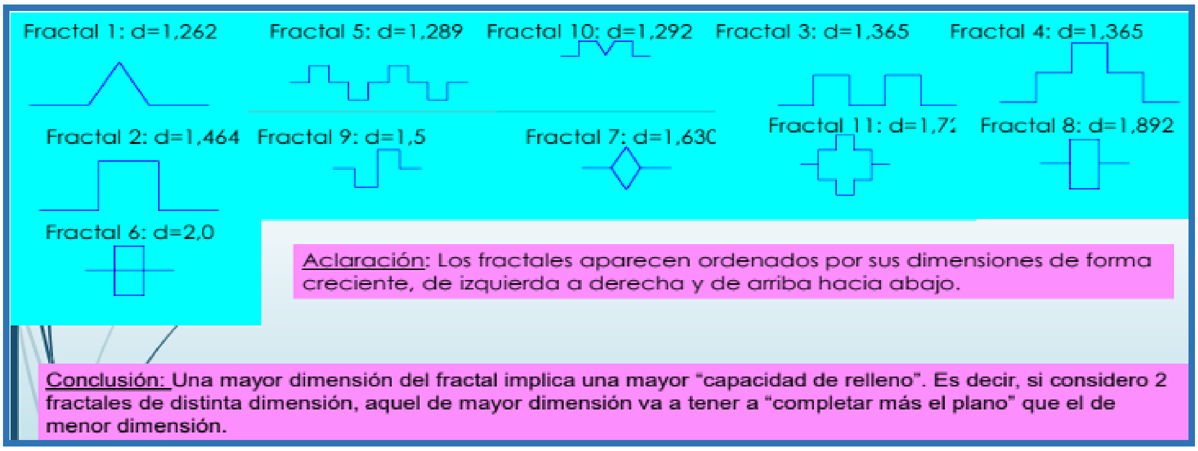


Figure 12. Production of P45 regarding Topic 2: Fractal Dimension corresponding to the 2nd Edition of the Course. Example of Level 1.

This result (even though it corresponds to Level 1) is encouraging, as it demonstrates that despite the more abstract nature of fractal dimension, the activity succeeded in motivating participants to approach the concept from a visual and intuitive perspective. The activity involved comparisons between several strictly self-similar fractals, which was a valuable aspect of its design, as it stimulated an appreciation of similarities and differences in fractal dimension in relation to "space-filling." This helped foster a richer understanding of the concept. This finding underscores the importance of designing activities that combine recursive, geometric, and algebraic elements to address more abstract topics like those specific to FG.

As representative examples of Level 3 performance, the work of teachers P81 and P76 is presented in Figures 13 and 14 respectively. These participants demonstrated mathematically rigorous observations regarding the logarithmic ratio involved in the formal definition of fractal dimension.

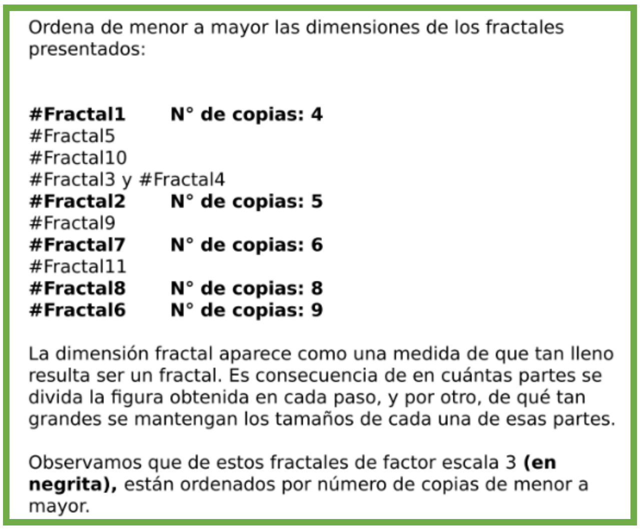
Specifically, their analysis involved:

1. Systematic organization of fractals
2. Comparative analysis of fractals sharing identical scaling factors (particularly emphasizing those with a factor of 3 through bold formatting)
3. Derivation of the principle that fractal dimension increases proportionally with the number of congruent copies

These methodological observations reflect:

- Enhanced mathematical sophistication in analysis
- Attainment of formalized mathematical understanding
- Mastery of Topic 2's conceptual framework

The work exemplifies the highest level of conceptual engagement with fractal dimensionality, particularly in demonstrating the inverse relationship between scaling factors and dimensional values through precise logarithmic computation.


Ordena de menor a mayor las dimensiones de los fractales presentados:

#Fractal1 **N° de copias: 4**
#Fractal5
#Fractal10
#Fractal3 y #Fractal4
#Fractal2 **N° de copias: 5**
#Fractal9
#Fractal7 **N° de copias: 6**
#Fractal11
#Fractal8 **N° de copias: 8**
#Fractal6 **N° de copias: 9**

La dimensión fractal aparece como una medida de que tan lleno resulta ser un fractal. Es consecuencia de en cuántas partes se divida la figura obtenida en cada paso, y por otro, de qué tan grandes se mantengan los tamaños de cada una de esas partes.

Observamos que de estos fractales de factor escala 3 **(en negrita),** están ordenados por número de copias de menor a mayor.

Figure 13. Production of P81 regarding Topic 2: Fractal Dimension corresponding to the 2nd Edition of the Course. Level 3 Example.

In the case of P76 (shown in Figure 14), the data were organized in tabular form, with fractals ordered by their dimension value in the first column. The last column of the table included the dimension value (though labeled as "density"). The teacher then analyzed two specific cases:

- Fractals with identical scaling factors (highlighted in red), specifically those with a scaling factor of 3.
- Fractals with the same number of partition subsets (highlighted in green).

This systematic distinction led to two key conclusions:

1. When the scaling factor is identical, the fractal with *more partition subsets per iteration* has a *higher dimension*.
2. When the number of partition subsets matches, the fractal with the *smaller scaling factor* has a *higher dimension*.

Nº DE FRACTAL	DENSIDAD	CÁLCULO DE LA DENSIDAD
1	1,261	$\frac{\log 4}{\log 3}$
5	1,289	$\frac{\log 17}{\log 9}$
10	1,292	$\frac{\log 8}{\log 5}$
3 - 4	1,365	$\frac{\log 9}{\log 5}$
2	1,464	$\frac{\log 5}{\log 3}$
9	1,500	$\frac{\log 8}{\log 4}$
7	1,630	$\frac{\log 6}{\log 3}$
11	1,722	$\frac{\log 16}{\log 5}$
8	1,892	$\frac{\log 8}{\log 3}$
6	2,000	$\frac{\log 9}{\log 3}$

Cuanto mayor es la densidad de un fractal, podemos decir que el mismo será mas "lleno" .

La dimensión fractal D, la podemos definir así:

$$D=\frac{\log p}{\log \alpha}$$

Donde p es la cantidad de subconjuntos de la partición correspondiente a un determinado paso de la iteración, y α es el factor de semejanza de cada subconjunto de la misma partición con la semilla.

Como la densidad es un cociente, éste dependerá de que tan grande sea el numerador y que tan chico sea el denominador.

Si consideramos a p como la cantidad de subconjuntos que hay en la partición, y α el factor de reducción, podemos pensar que cuanto menor es α, mayor es la longitud de los segmentos (subconjuntos de la partición).

Observando los resultados obtenidos:

- si comparamos fractales con **igual α**, es mas lleno el fractal que tenga mayor p, o sea mayor cantidad de subconjuntos en la partición. (segmentos)
- Si comparamos fractales con **igual p**, es más lleno el fractal que tenga menor α, o sea el que tenga segmentos de mayor longitud.

Figure 14. Production of P76 for Topic 2: Fractal Dimension corresponding to the 2nd Edition of the Course. Level 3 Example.

Topic 3: Description of Geometric Transformations for Fractal Construction

The proficiency levels for *geometric transformations in fractal construction* (Topic 3) were defined as follows:

1. Incomplete or mathematically informal description of the required geometric transformations.
2. Adequate description of the geometric transformations needed to construct the fractal, but without specifying their compositions, making the procedure unclear.
3. Complete description of the geometric transformations (including all elements) required to build the fractal, with explicit composition rules.

The level distribution is shown in Figure 15 and includes all participants from both editions (N=83). Notably:

- Over half (61%) of responses were classified as Level 1, a surprising outcome given that these concepts are typically covered in secondary school and stem from *EG*.
- Very few (5%) reached Level 2.
- Approximately one-third (34%) achieved Level 3, demonstrating mastery in describing and composing transformations.

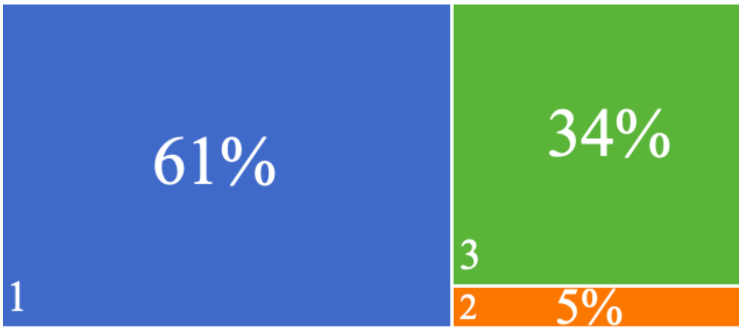


Figure 15. Distribution of Results for Topic 3: Description of Geometric Transformations for Constructing a Fractal, corresponding to the Two Editions of the Course. [Source: Personal Elaboration.]

The Level 3 example shown in Figure 16 corresponds to the work produced by teacher P28. In this case the teacher properly and completely described all required geometric transformations. Additionally, they graphically represented the transformations applied to the seed shape, using the "+" symbol to indicate function composition. While this notation isn't mathematically standard, it demonstrates the teacher's conceptual understanding of transformation composition.

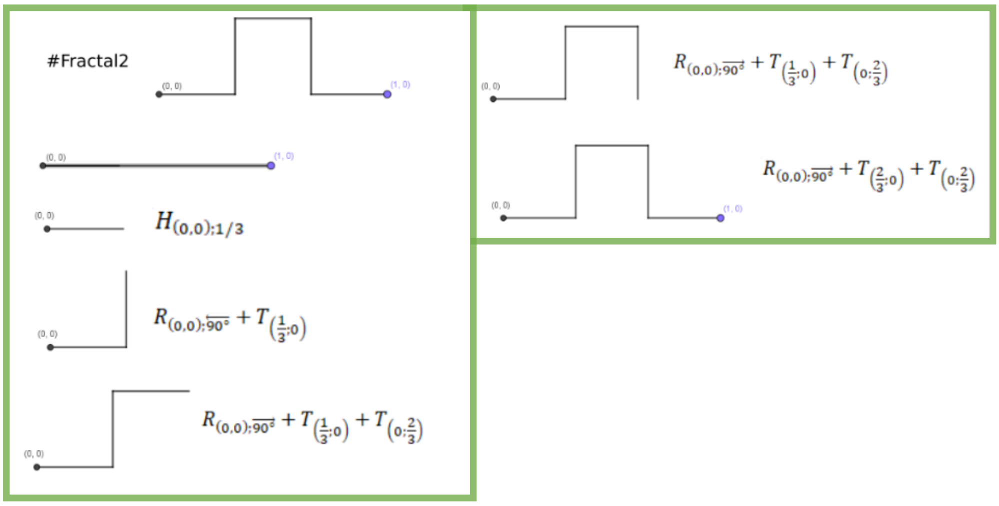


Figure 16. P28 Production for Topic 3: Description of Geometric Transformations (DTG) for Constructing a Fractal, corresponding to Both Editions of the Course. Level 3 Example.

The work clearly shows the mathematical union of the seed's images through similarity transformations, as evidenced by how they progressively "added" segments in the drawing to build the first iteration's result. This approach reveals:

- A practical understanding of how transformations combine geometrically.
- The ability to visualize and implement iterative fractal construction.
- Genuine comprehension of the underlying mathematical concepts.

Topic 4: Relationship between Perimeter and Area

The levels for the topic of the relationship between perimeter and area of certain fractals (Topic 4) are as follows:

1. No consideration of the relationship between perimeter and area, or only a very rudimentary one (incomplete or with errors). No generalization is presented.
2. Mention that the studied fractals have an infinite perimeter enclosed within a finite area, or generalization of some expression.
3. Generalization of results for any iteration (with possible minor errors), along with mathematical arguments relating the fractal's perimeter and area.

The level distribution for Topic 4 is shown in Figure 17 and corresponds to a total of N=83 (all participants across both editions). In this case, half of the protocols reached Level 3 in understanding Topic 4. The percentage of teachers concentrated in Levels 2 and 3 is very high (93%), with only 7% assigned to Level 1. This was the topic in which teachers demonstrated the highest level of knowledge, likely due to the prior inclusion of perimeter and area concepts (applied to Euclidean figures) in the secondary school curriculum. The challenge of the teaching proposal lay in applying these concepts to iteratively constructed figures (such as the linear fractals studied), a task that was successfully accomplished with mathematical rigor.

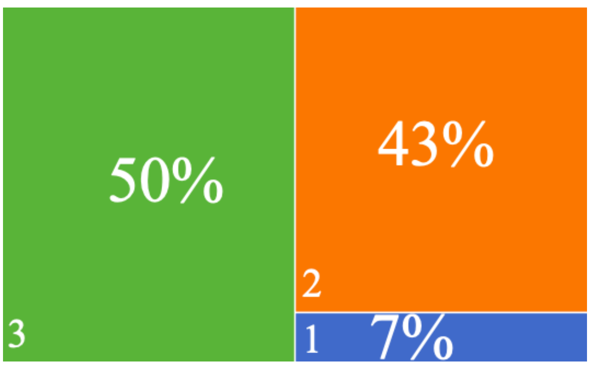


Figure 17. Distribution of Results for Topic 4: Relationship between the Perimeter and Area of a Fractal, corresponding to the Two Editions of the Course. [Source: personal elaboration.]

As an example of Level 1, the work of P56 is presented in Figure 18, from which it can be inferred that there was some reflection regarding the area of the antenna based on the Sierpinski triangle fractal, though there was no consideration whatsoever about the perimeter of the different antennas. It would have been preferable to express the area using decimal numbers (rather than ratios involving irrational numbers) to better observe the trend of the area approaching zero.

¿Cuál es el perímetro de cada antena, considerando que la antena número 0 es un triángulo equilátero de lado 1 unidad? ¿Y el área de cada una de ellas?

Antena	perímetro	Área
0	3	$\frac{\sqrt{3}}{4}$
1	$\frac{9}{2}$	$\frac{3\sqrt{3}}{16}$
2	$\frac{27}{4}$	$\frac{9.\sqrt{3}}{64}$
3	$\frac{81}{8}$	$\frac{27\sqrt{3}}{256}$

¿Qué reflexiones le merece los resultados anteriores respecto al aprovechamiento del material?

A medida que se realizan iteraciones va disminuyendo el área de la antena logrando así un mayor aprovechamiento del material.

Figure 18. P56 Production for Topic 4: Relationship between the Perimeter and Area of a Fractal, corresponding to Both Editions of the Course. Level 1 Example.

Analysis of Pedagogical Content Knowledge on Fractal Geometry

This section examines pedagogical content knowledge within the MKT model, focusing on the following subdomains: knowledge of content and curriculum, knowledge of content and students, and knowledge of content and teaching. Each of these knowledge domains was described through specific didactic suitability criteria, with the construct of didactic suitability being used to organize the evaluation of teaching and learning processes occurring during the implementation of the proposed activities. In this regard, teachers were expected to conduct a didactic analysis of the proposal after reviewing indicators of didactic suitability - that is, to critically assess the mathematical and didactic decisions made when designing the activity sequence.

An ascending ordering was constructed based on the evaluated Didactic Suitability. Thus, level 1 was assigned to the evaluation made by teachers of the proposal when pointing out weaknesses and limitations for teaching or learning FG. Level 2 was constructed from the productions indicating that the proposal is adequate in both cognitive and didactic aspects for secondary school, but there are clear aspects to improve. Level 3 is the strongest regarding the mathematical and didactic potential of the proposal, showing higher quality in terms of the possibilities it offers for teaching and learning fractals.

Knowledge of Curricular Content about Fractal Geometry (KCU)

The levels for this category were as follows:

1. Mention of the possibility of teaching FG without specifying in which courses or educational level it could be implemented.
2. Indication that FG can be taught in some basic cycle course or in some secondary school course.
3. Statement that FG can be taught throughout secondary school by organizing its contents, arguing based on its interdisciplinary approach and applicability.

Fewer than half (41%) of the participants expressed interest in teaching FG but did not elaborate on how this could be done; therefore, their responses were assigned to Level 1. There was only a mention of their concern for teaching it, without reflecting on the transversal contents of this geometry in relation to, for example, the official Uruguayan curricula at different educational levels. A similar percentage (43%) of participants understood that FG can be taught in any secondary school course, with their responses corresponding to Level 3. Less than a fifth (16%) of the total were assigned to Level 2, mentioning the possibility of teaching FG in at least one of the two cycles of secondary school (basic cycle or high school). The distribution of results is shown in Figure 19.

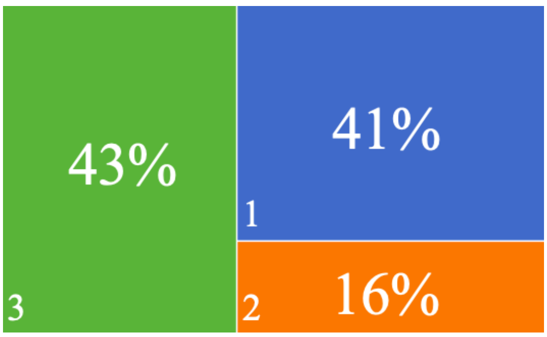


Figure 19. Distribution of Results for the Category of Knowledge of Curriculum Content regarding FG. [Source: personal elaboration.]

Bivariate Statistical Analysis

A bivariate analysis was conducted to study, describe, and interpret potential relationships between categories treated as ordinal qualitative variables, corresponding to subject matter knowledge, in this case of FG, and pedagogical content knowledge. The analysis examined variable pairs to determine independence or association, based on teachers' responses to the instructional activities. When associations between variables were detected, the strength of these relationships was examined for each course edition. The Chi-square test was employed, with the following hypotheses:

- Null hypothesis (H_0): Variables are independent.
- Alternative hypothesis (H_1): Variables are not independent (i.e., an association exists).

If the p-value was $\leq$ the significance level (α), H_0 was rejected, indicating a statistically significant association. If $p > \alpha$, H_0 was retained due to insufficient evidence for association. Using the Chi-square measure, variable pairs with $p < 0.001$ were identified.

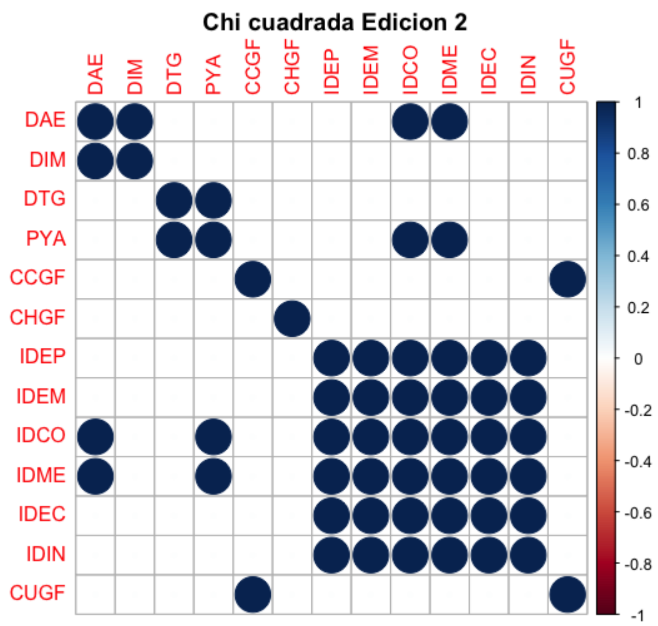


Figure 20. Graph Showing the Chi-square Test for the Variables Constructed in the Second Edition of the Course. [Source: Personal elaboration using the free software Rstudio.]

Using the Chi-square association measure, we analyzed which pairwise variable relationships from the contingency tables for each variable pair showed a p-value less than 0.001. Figure

20 presents the graph with variables meeting this p-value threshold, indicating they are significantly associated with each other. The categories in the graph maintain the original acronyms in Spanish, with the following translations: DAE (Self-Similarity Dimension), DIM (Fractal Dimension), DTG (Description of Geometric Transformations), PYA (Perimeter-Area Relationship), CCGF (Common Knowledge of FG), CHGF (Knowledge of the Horizon of FG), IDEP (Epistemic Suitability), IDEM (Emotional Suitability), IDCO (Cognitive Suitability), IDME (Mediational Suitability), IDEC (Ecological Suitability), IDIN (Interactional Suitability) and CUGF (Curricular Knowledge of FG).

Figure 21 shows the graph related to the Goodman-Kruskal Gamma statistic test, while Table 3 presents the results with greater precision.

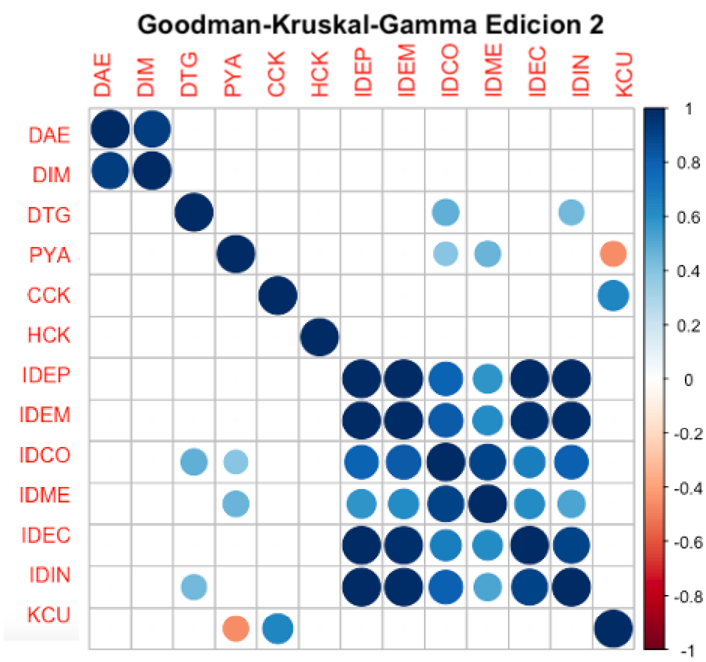


Figure 21. Graph of the Goodman-Kruskal-Gamma Test Statistic for the Variables Constructed in the Second Edition of the Course. KCU is the Same as CCGF. [Source: personal elaboration using Rstudio.]

The significant associations identified through the Chi-square test remained; however, some additional variables showed association when analyzed with the Goodman-Kruskal Gamma statistic. Nevertheless, these were not considered since they showed no dependency according to the initial Chi-square test. Table 3 summarizes both the identified associations and their strength of association.

Table 3. Variables Significantly Associated with the Goodman-Kruskal-Gamma Test, corresponding to the Second Edition of the Course. [Source: personal elaboration.]

Variables	Associated variables	Value of the Goodman-Kruskal-Gamma statistic	Type of association
DAE	DIM	0.925	Very strong
CCK	CUGF	0.64	Strong
IDEP	IDEM, IDEC, IDIN	1.00	Perfect
	IDCO	0.788	Very strong
IDCO	IDEM	0.818	Very strong
	IDME	0.907	Very strong
	IDEC	0.673	Strong
	IDIN	0.794	Very strong

The Very Strong Association Between the Variables Strict Self-Similarity Definition and Fractal Dimension

Since the categories were constructed to analyze the level of knowledge attained about FG, the values obtained in the statistical tests used provide a good indicator of the close relationship between the concepts of strict self-similarity and fractal dimension. Nevertheless, this strong association is also striking. Among the N=38 participants considered in this case (corresponding to the second edition of the course), it was observed that:

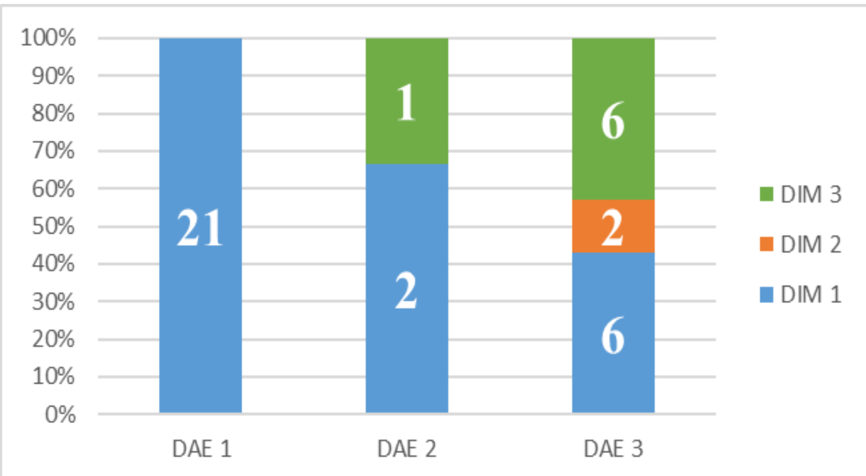


Figure 22. Bivariate Frequency Distribution for the Variables Definition of Strict Self-similarity and Fractal Dimension (DIM).

- All 21 participants who struggled to mathematically justify the concept of strict self-similarity (categorized as DAE 1) also encountered difficulties in calculating the fractal dimension (DIM 1), as seen in Figure 22.
- At the other extreme, of the 14 participants who adequately completed the task on the concept of strict self-similarity (DAE 3), nearly half (6) did not achieve level 1 in calculating the fractal dimension (DIM 1).

It was observed that the strong association predominantly occurred at knowledge Level 1, which was interpreted as indicating that the difficulty in justifying strict self-similarity is related to the formal calculation of fractal dimension.

A potential explanation for this difficulty could be that calculating fractal dimension requires:

1. Identifying the number of subsets in each partition of every iteration
2. Determining the scaling factors of the geometric transformations (which are necessary to justify self-similarity)

Without performing this analysis - which involves justifying that the figure is strictly self-similar - it is impossible to calculate the figure's fractal dimension.

The Strong Association between Common Knowledge and Curricular Content Knowledge about Fractal Geometry

In the analysis of results from the second edition of the course, and unlike the first edition, a strong association was found between the variables common knowledge about FG and curricular content knowledge about FG. According to the theoretical framework adopted in this work, the common knowledge category is a subcategory of subject matter knowledge, and curricular content knowledge is a subcategory of pedagogical content knowledge, therefore, this association reveals a relationship between what teachers know about FG and how it could be integrated into the secondary school curriculum to make its teaching feasible.

This result is evident at knowledge level 3, since of the participants whose protocols were assigned to this level in common knowledge, three quarters of them (75%) also corresponded to level 3 in curricular content knowledge, as shown in Figure 23. At knowledge level 1, of the total protocols at this level in curricular content knowledge, slightly more than half (57%) were also assigned level 1 in common knowledge, indicating that lacking basic knowledge of FG would be related to the place of FG in secondary school Mathematics programs.

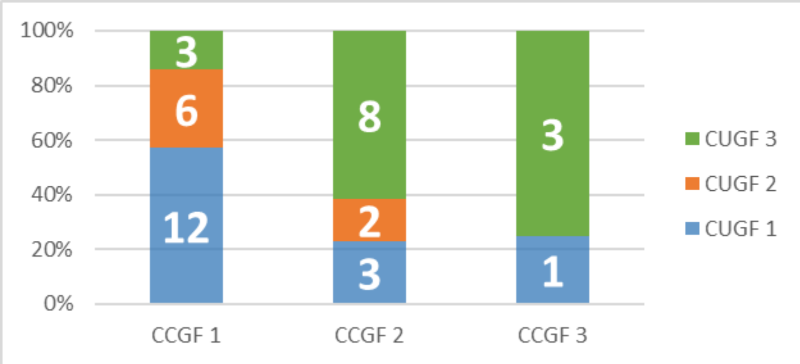


Figure 23. Bivariate Frequency Distribution for the Variables Common Knowledge about FG (CCGF) and Curricular Knowledge about FG (CUGF).

Conclusions

The work focused on the design, implementation, and analysis of a set of activities for learning about Fractal Geometry (FG), as well as studying its feasibility for training in-service mathematics teachers. The research was guided by four questions, the answers to which were developed across different stages and are synthesized in this chapter. An initial exploration of the problem of teaching FG in schools revealed that key FG topics were often taught with minimal mathematical formality, almost intuitively—as if this were the only way to approach fractals, as if they were not inherently mathematical knowledge that could also be addressed from other disciplinary areas like art. Alternatively, fractals were sometimes considered so complex that they were deemed nearly unteachable in secondary school.

This highlighted the need to develop a teaching proposal that would allow teachers to engage with FG concepts, moving beyond intuitive ideas and grounded in the following considerations about FG:

- It is a relatively new field within mathematics and mathematics education research.
- It transcends Euclidean Geometry (GE) in some aspects, aligning more closely with modeling natural phenomena (clouds, mountains, coastlines, etc.).
- It is the focus of extensive popular science material that draws attention due to its aesthetic appeal and broad applicability in the real world.
- It can motivate teachers seeking to connect mathematics with real-world contexts, as it easily links to multiple knowledge domains (biology, medicine, technology, art, etc.).
- It is supported by mathematical areas such as Analysis, Geometry, and Topology.

The first research question was: *What activities can be designed for mathematics teachers to study the mathematical and pedagogical content knowledge of Fractal Geometry for teaching purposes?* Answering this question required an exhaustive review of research on FG teaching and learning, as well as a redesign of activities identified in this review. The sequence focused on key FG topics, building on concepts more familiar to mathematics teachers—such as iteration, perimeter, area, limits, logarithms, and similarity transformations—while also incorporating an application of FG: multiband prefractal antennas.

The activities helped teachers familiarize themselves with fractals and the mathematical processes (operational thinking) needed to work with concepts like self-similarity and fractal dimension. The proposal's design included interaction with applets developed by the author, video studies of prefractal antennas, and interactive books for secondary students, all while maintaining conceptual coherence.

A notable feature of the teaching proposal was that one activity was redesigned for the second course edition. The topic of fractal dimension was not addressed in the first edition, but after evaluating this initial implementation, the course team analyzed the didactic transposition processes required to teach fractals. This underscored the importance of including fractal dimension, leading to its operational treatment in the second edition. However, despite designing an activity for the second edition that addressed strict self-similarity and fractal dimension through an operational definition, the results indicated a need for further redesign, as mathematical justification proved challenging.

Univariate analysis suggested that teachers recognized the connection between self-similarity and fractal dimension but struggled to justify it mathematically. This often led them to focus on calculating the fractal dimension rather than analyzing and justifying the mathematical process behind it. Bivariate analysis confirmed a strong association between these concepts, though low knowledge levels remained prevalent. Thus, modifications to the proposal are suggested, particularly in refining guiding questions, as detailed below. Self-similarity is a key property of fractals, achieved through an infinite iterative process. A fractal is constructed by repeating a process infinitely, with each iteration generating smaller subsets similar to the initial seed or figure, which is then transformed again by the generator. The limit of this process is composed of smaller subsets that are exact copies (in the case of strict self-similarity) of the fractal figure. These subsets repeat at different scales, meaning self-similarity defines the fractal's structure. The teaching proposal aimed to move beyond the misconception that a fractal is synonymous with a single iteration of this process—a common issue in fractal teaching literature.

Fractal dimension measures how "fractal" a figure is or "how much space it fills." Unlike Euclidean dimension, it can be a non-integer value, quantifying how a fractal's complexity increases at smaller scales. A higher fractal dimension indicates a more intricate and detailed figure.

Guided by these informal mathematical ideas, the activities were designed to study:

1. The ratio of congruent subsets (p): Refers to how many congruent subsets exist in two consecutive iterations of an iterative figure, differing by a constant. This calculates how many times the figure repeats at a smaller scale.
2. The ratio of scaling factors (α): Refers to the relationship between scaling factors associated with subsets and the fractal's seed, in consecutive iterations differing by a constant. These factors relate to how the figure shrinks in each iteration.

To improve the teaching proposal, the following enhancements are suggested: To further mathematically justify the relationship between strict self-similarity and fractal dimension, initial recognition questions could be added to identify the number of subsets and scaling factors (i.e., identifying p and α for each fractal, whether from InterActivate or other sources). The expanded prompt might include:

- Identify similar subsets in each iterative figure.
- Study and indicate how many times the same subset repeats at a smaller scale (i.e., in each iteration) Generalize this result. To advance the exploration of ratios, the following items could be included:
- Calculate p for the first three iterations of one figure. What pattern do you observe? Can you generalize and prove this result?
- Calculate α for the first three iterations. How does it relate to the figure's scaling?
- Justify why these ratios are important for describing the figure's self-similarity.
- Why can't we simply measure the length, area, or volume of a fractal figure as we do in EG?
- What does it mean for fractal dimension to not necessarily be an integer?
- How do p and α affect the resulting fractal dimension?

Studying an FG application, such as prefractal antennas, contributed to fractal teaching in the following ways:

1. While reviewing engineering, electronics, and telecommunications theses and articles, the author encountered the term *prefractal*—surprisingly absent from texts aimed at popularizing or teaching key fractal concepts. A prefractal refers to a specific iteration of a fractal. Another finding was the interchangeable use of *fractal* to describe a single iteration of its construction. It is crucial to clarify that a fractal is the limit figure resulting from an infinite iterative process, and understanding the distinction between prefractal and fractal is essential for developing fundamental mathematical processes like generalization or induction.
2. Linking fractals to a concrete application provided a relevant extramathematical context and motivated students (though a small percentage expressed disinterest) to explore how self-similarity and dimension enable antennas to be multiband and compact.

The second research question was: *How can mathematics teachers be introduced to key elements of FG for teaching in secondary school?* To achieve this, a virtual course was designed and implemented in 2021 (during the pandemic), featuring the activities described earlier. The analysis variables were developed after implementing the teaching proposal. These variables are supported by a theoretical framework combining mathematical and pedagogical aspects of mathematics teaching: the *Mathematical Knowledge for Teaching* model, enriched by contributions from the *Mathematics Teachers' Specialized Knowledge* model.

This theoretical-conceptual framework enabled an analytical reading of the solutions provided by teachers in the FG professional development course, allowing inferences about their mathematical and pedagogical knowledge of FG. These inferences were based on interpreting and identifying key mathematical themes, fundamental mathematical processes, and an analysis of the proposal through didactic suitability criteria, which emerged in the context of the course's final activity (presented as videos by participants).
Regarding the results on associations between didactic suitability criteria, several points stand out:

- The *Didactic Suitability* construct refers to disciplinary knowledge and pedagogical knowledge about teaching a subject, which teachers should acquire to develop or evaluate a teaching proposal. These criteria include selecting and organizing disciplinary topics, understanding teaching strategies and resources, recognizing student characteristics, and assessing learning, among others.

- When designing the FG teaching proposal, the possibility of combining or associating these criteria was considered, with the aim of analyzing these associations through teachers' critical judgments as they engaged with the proposal. This required them to analyze the proposal based on their mathematical knowledge of FG as well as their pedagogical knowledge for teaching it. Thus, it was essential to hear their assessments of the proposal's mathematical and pedagogical coherence, valuing both the formal treatment of certain topics and the application of FG to a technological context (fractal antennas), while also appreciating the beauty of fractals through interactions with the Mathigon book

The third research question was: *How can the mathematical knowledge for teaching key FG topics be described and characterized through an analysis of the mathematical-pedagogical potential of a teaching proposal?* This question was answered by justifying the adoption of the MKT framework, which allowed the study and analysis of two key aspects of this work: what mathematical knowledge teachers need to teach fractals, and how they can teach them in secondary school. These questions stem from the absence of FG content in official curricula, both in secondary schools and teacher training programs. In practice, students who encounter a teacher interested in fractals are often the only ones exposed to this living, evolving branch of mathematics.

The fourth research question was: *What aspects should be considered to improve the proposed teaching approach?* Answers to this question were partly addressed in the response to the first question, but key findings from the analysis include:

- No statistical association was found between any FG topics and the Didactic Suitability criteria. This may be because the teaching proposal did not primarily aim for teachers to study *how* to teach fractals but focused instead on studying the topics and analyzing the activities they engaged with.
- There was no activity requiring teachers to design their own fractal-based lesson, which should be a focus of future research.

In closing, it is important to acknowledge the initial obstacle (taken as a challenge in this reserach): the need to mathematically study formal FG concepts as a first step in redesigning a proposal for teachers. A second challenge was designing a proposal that overcame the difficulty of didactically transposing these concepts, given their near absence in teacher training. While significant progress was made, further research is needed on the proposal's feasibility when implemented with secondary students. This raises new questions, such as how to collaborate closely with in-service teachers to refine the sequence with their practical expertise, or how secondary students conceptualize FG based on this proposal. Both questions open new avenues for continuing this research.

These challenges reinforce the value of teaching fractals in secondary school—not only to address mathematical procedures like modeling, pattern recognition, real-number calculations, and limits (promoting advanced mathematical thinking) but also to leverage their applications for understanding the world and appreciating the beauty of their constructions.

Note

The set of activities designed was recognized in 2023 as one of the 10 transformative and innovative experiences in the Project Transformation of the Learning-Teaching-Evaluation Process (TPAE), promoted by the Interuniversity Development Center (CINDA). CINDA is a collaboration network made up of 38 higher education institutions in Latin America. It began its activities in 1971 with the aim of promoting links between universities and generating, systematizing and disseminating knowledge that contributes to higher education policies.

References

Acevedo, J. (2009). Conocimiento didáctico del contenido para la enseñanza de la naturaleza de la ciencia (I): el marco teórico. *Eureka*, 6(1), 21-46.

Aggarwal, A. & Kartikeyan, M. (2010). Pythagoras tree: a fractal patch antenna for multi-frequency and ultra-wide band-width operations. *Progress in electromagnetics research*, 16, 25-35.

Alsina, C. (2007). Teaching applications and modelling at tertiary level. *Modelling and Applications in Mathematics Education*, The 14th ICMI Study, 10(44), 469–474.

doi:10.1007/97803872982212

Aramburú, F. (2021). Dimensiones no enteras y curvas con áreas. Laberintos & Infinitos, 54, http://laberintos.itam.mx/dimensiones-no-enteras-y-curvas-con-area/

Artigue, V., Fanaro, M., y Lacués, E. (2021). Estado del arte sobre la enseñanza y el aprendizaje de la Geometría Fractal en la escuela secundaria. *Pensamiento Matemático*, 11(2), 75-92.

Artigue. V., Fanaro, M., y Lacués, E. (2022). Análisis del concepto de dimensión fractal: una posible reconstrucción para su enseñanza. *Unión*, 18(65).

Bakytbekov, A. (2017). *Fully printed 3D cube cantor fractal rectenna for ambient RF energy harvesting applicaction.* (In partial fulfillment of the requirements for the degree of Master of Science). Kingdom of Saudi Arabia.

Barnsley, M. & Demko, S. (1985). *Iterated Function Systems and the Global Construction of Fractals*. Proc. Roy. Soc. London Ser., (399), 243-275.

Barnsley, M. (1988). *Fractals Everywhere*. Academic Press, Inc.

Binimelis, M. (2012). *Una nueva manera de ver el mundo. La Geometría Fractal. Navarra*, España. RBA editores.

Blum, W. & Niss, M. (1991). Applied mathematical problem solving, modelling, applications, and links to other subjects - state, trends and issues in. Mathematics instruction. Educational studies in mathematics, Editor Dorfler etc, 22, 37- 68. Kluwer Academic Publishers, Dordrecht.

Bolívar, A. (2005). Conocimiento didáctico del contenido y didácticas específicas. Profesorado. *Revista de currículum y formación del profesorado*, 9(2).

Brande, D. (2012). *Diseño de nuevas antenas ultra wide band con filtros notch integrados*. (Proyecto fin de carrera de grado). Universidad Autónoma de Madrid. Recuperado de http://arantxa.ii.uam.es/~jms/pfcsteleco/lecturas/20121219DanielBrandeHernandez.pdf

Canibaño, M., Sastre, P. y Gandini, M. (2011). Ideas para enseñar dimensión fractal en la enseñanza secundaria. *Unión*, (28), 191-196.

Carrillo, J., Climent, N., Contreras, L.C., & Muñóz - Catalán, MC. (2013). *Determining specialised knowledge for mathematics teaching*. En B. Ubuz, C. Haser & M.A. Mariotti (Eds.). Actas del CERME 8 (pp. 2985-2994). Middle East Technical University, Ankara, Turquía: ERME.

Cenich, G., Araujo, S. y Santos, G. (2020). Conocimiento tecnológico pedagógico del contenido en la enseñanza de matemática en el ciclo superior de la escuela secundaria.

Perfiles Educativos, 167, 53-67. Recuperado de http://perfileseducativos.unam.mx/iisue_pe/index.php/perfiles/article/view/59276

Contreras, L.C., Climent, N., Escudero-Avila, D., Flores - Medrano, E., y Montes, M.A. (Eds.). (2014). *Un marco teórico para el conocimiento especializado del profesor de matemáticas.* Huelva, España: Universidad de Huelva Publicaciones.

Carrillo, J., Montes, M., Contreras, L. y Climent, N. (2017). El conocimiento del profesor desde una perspectiva basada en su especialización: MTSK. Annales de Didactique et de Sciences Cognitives. 22. https://doi.org/10.4000/adsc.756

Castiblanco Hernández, S.E. y Montana Páez, S.Y. (2018). Geometría y dimensión: representación y caracterización de objetos 2d, 3d y 4d. (Tesis de Licenciatura). (Tesis de Licenciatura en Matemáticas). Universidad Pedagógica Nacional de Bogotá. Recuperado de: http://repository.pedagogica.edu.co/bitstream/handle/20.500.12209/11153/TE-22698.pdf?sequence=1&isAllowed=y

Castro, A. (2018). *Medida, dimensión y fractales*. (Tesis final de grado en Matemática). La Laguna.

Chen, S., Herron, S., Ding, J. & Mohn, R. (2018). Assessing United States and Chinese secondary mathematics teachers` interest in fractal geometry. *Journal of mathematics education*, 11(2), 17-34.

Chevallard, Y. (1999). *La Transposición Didáctica.* Del saber sabio al Saber enseñado. Buenos Aires: AIQUE

Comas, J. y Herrera, M. (2010). Cálculo de la dimensión fractal del contorno de una ciudad como trabajo de investigación en secundaria. *Suma*, 65, 23-32.

Consejo de educación secundaria. Programa del curso de Matemática I para 3er año de bachillerato, opción Matemática y diseño, plan reformulación 2006. Recuperado de https://www.ces.edu.uy/files/Planes y programas/Ref 2006 Bach/6to matematica diseño/mat1matdise6.pdf

Dirección General de Cultura y Educación De La Provincia De Buenos Aires. Diseño Curricular para la Educación Secundaria Ciclo Superior, 2010. Obtenido de http://servicios.abc.gov.ar/lainstitucion/organismos/consejogeneral/disenioscurriculares/secundaria/sexto/materias%20comunes/matematica_6.pdf

Drakopoulos, V. & Panagiotis-Vlasios, S. (2020). Teaching Recursion to Junior-High School Students by Using Fractals: A Complete Lesson Plan in Python. *American Journal of Education and Information Technology*, 4(2), 50-55.

Dummit, E. (2015). Dynamics, Chaos and Fractals (Part 4): Fractals, Course Notes. https://web.northeastern.edu/dummit/handouts.html

Edgar, G. (2008). *Measure, topology, and fractal geometry*. USA. Springer, second edition

Faccio, T. (2013). Aprendizajens Matemáticas a partir da construccion de fractais. *Relato de Experiencia*, 5-48.

Falconer, K. (2003). *Fractal Geometry. Mathematical Foundations and Applications*. UK: Wiley, Segunda Edición.

Figueiras, M., Molero, M., Salvador, A. y Zuasti, N. (2000). Una propuesta metodológica para la enseñanza de la Geometría a través de los fractals. *Suma*, (35), 45-55.

Flores, E., Escudero, D. y Carrillo, J. (2013). A theoretical review of specialized content knowledge. Proceedings of the CERME 8 (3055-3064). Turquía: ERME.

Font, V. (2011). Competencias profesionales en la formación inicial de profesores de matemáticas de secundaria. Unión, 26, 9-25.

Font, V., Adán, M. y Ferreres, S. (2015). *Valoración de la idoneidad de las Matemáticas enseñadas.* En P. Scott & A. Ruíz (Eds.). Educación Matemática en las Américas, 2. Formación Inicial para Secundaria (338-345). República Dominicana: Comité Interamericano de Educación Matemática.

Font, V., Pino-Fan, L., y Breda, A. (2020). Una evolución de la mirada sobre la complejidad de los objetos matemáticos. *Paradigma*, 41,107-129.

Frame, M., Mandelbrot, B. & Neger, N. (2023). *Fractal Geometry*. Yale University. Recuperado de: https://users.math.yale.edu/public_html/People/frame/Fractals/

Fusi, F. y Sgreccia, N. (2020). ¿Por qué enseñar la noción de fractal en el último año de la escuela secundaria? Opiniones de especialistas en Geometría. *Épsilon,* 105, 31-50.

Garbin, S. y Mireles, M. (2009). Un estudio sobre la noción de dimensión en la enseñanza-aprendizaje de las matemáticas. *Enseñanza de las ciencias*, 27(2), 223-240.

Garbin, S. (2007). La problemática fractal: un punto de vista cognitivo con interés didáctico. *Paradigma*, 28(2).

García, E. (2018). Teoría de la dimensión topológica. (Tesis de fin de grado. Universidad de Cantabria. https://repositorio.unican.es/xmlui/bitstream/handle/10902/15262/Garcia%20Fernandez%20Esther.pdf?sequence=1&isAllowed=y

Gianvittorio, J., Romeu, J., Blanch, S., & Rahmat-Samii, Y. (2003). Self-Similar Prefractal Frequency Selective Surfaces for Multiband and Dual-Polarized Applications. *IEE transactions on antennas and propagation*, 51(11), 3088-3096.

Godino, J., Bencomo, D., Font, V. y Wilhelmi, M. (2006). Análisis y valoración de la idoneidad didáctica de procesos de estudio de las matemáticas. *Paradigma*, 27(2), 221-252.

Godino, J., Contreras, A. y Font, V. (2006). Análisis de procesos de instrucción basado en el enfoque ontológico-semiótico de la cognición matemática. *Recherches en Didactiques des Mathematiques*, 26(1), 39-88.

Godino, J., Giacomone, B., Batanero, C. y Font, V. (2017). Enfoque Ontosemiótico de los Conocimientos y Competencias del Profesor de Matemáticas. *Bolema*, 31(57), 90-113.

Goonatilake, R. y Casas, R. (2020). Fractals, Self-Similarity, and Beyond. *Mathematics teaching research journal*, 21(1).

Herrera, M. e Inclán, J. (2004). *Estudio y metodología de diseño de antenas utilizando geometría fractal* (antenas fractales). Proyecto previo a la obtención del título de ingeniero en electrónica mención telecomunicaciones. Quito.

Hill, H. C., Rowan, B. & Ball, D. L. (2005). Effects of Teachers Mathematical Knowledge for Teaching on Student Achievement. *American Educational Research Journal*, 42(2), 371-406.

Hill, H., Blunk, M., Charalambos, C., Lewis, J., Phelps, G., Sleep, L. & Ball, D. (2008). Mathematical Knowledge for teaching and the Mathematical Quality of Instruction: An Exploratory Study. *Cognition and Instruction*, 24, 430-511.

Hill, H., Ball, D. & Schilling, S. (2008). Unpacking pedagogical content knowledge: Conceptualizing and measuring teachers' topic-specific knowledge of students. *Journal for Research in Mathematics Education*, 39, 372-400.

Karakus, F. & Baki, A. (2011). Assessing Grade 8 Elementary School Mathematics Curriculum and Textbooks within de Scope of Fractal Geometry. *Elementary Education Online*, 10(3), 1081-1092.

Karakus, F. (2013). A Cross-Age Study of Student´s Understanding of fractals. (2013). *Bolema*, 27(47), 829-846.

Karakus, F. & Karatas, I. (2014). Secondary School Students' Misconceptions about Fractals. *Journal of Education and Human Development*, 3(3), 241-250.

Karakus, F. (2015). Investigation into how 8th Grade Students Define Fractals. Educational Sciences: *Theory & Practice*.,15(3), 825-836.

Karakus, F. (2016). Pre-Service Teacher´s Concept Images on Fractal Dimension. *International Journal for Mathematics Teaching and Learning*, 17(2), 1-12.

Kohavi, Y. y Davdovich, H. (2006). Topological dimensions, Hausdorff dimension & fractals. Recuperado de: https://u.math.biu.ac.il/~megereli/final_topology.pdf

Komorek, M.; Duit, R.; Bucker, N.; Naujack, B. (2001). Learning process studies in the field of fractals. In: Behrendt, H.; Dahncke, H. R.; Groaber, W.; Komorek, M.; Kross, A. P. (Ed.). Research in Science Education –Past, Present, and Future. Netherlands: Kluwer Academic Publishers, 95-100.

Lajoie, J. (2006). *La Géométrie Fractale* (Tesis de Maestría en Matemática e Informática aplicada). Québec. Recuperado de https://depot-e.uqtr.ca/id/eprint/1565/1/000135084.pdf

Langille, M. (1996). Studying students' sense making of fractal geometry. (Tesis de maestría no publicada). Simon Fraser University, Canada.

Lopes, J., Salvador, J. y Fernandes, I. (2013). O ensino de probabilidade geométrica por meio de fractais e da resolucao de problemas. *Revista Eletrônica de Educação*, 7(3), 47-62.

Mathigon.org. (2022). Recuperado de: https://es.mathigon.org.

Mandelbrot, B. (1975). *La naturaleza fractal de la naturaleza*. Tusquets Editores. España. Primera edición.

Montes, M., Contreras, L. y Carrillo, J. (2013). Conocimiento del profesor de Matemáticas: enfoques del MKT y del MTSK. En A. Berciano, G. Gutiérrez, A. Estepa y N. Climent (Eds.), *Investigación en Educación Matemática* XVII (403-410). Bilbao: SEIEM.

Montes, M., Aguilar, A., Carmona, E., y Carrillo, J. (2014). Un marco teórico para el Conocimiento Especializado del Profesor de Matemática. Recuperado de: https://www.researchgate.net/publication/267392675_Un_marco_teorico_para_el_Conocimiento_especializado_del_Profesor_de_Matematicas

Moore, G. (1965). Cramming more components onto integrated circuits. *Electronics*, 38(8).

Moreno, P. (2017). Un sierpinski en la fachada. *Épsilon*, (96), 45-60.

Moreno, J. (2002). Experiencia didáctica en Matemáticas: construir y estudiar fractales. *Suma*, (40), 91-104.

Moreno, J. (2002). El juego del caos en la calculadora gráfica. *Suma*. (41), 69-79.

Moreno, J. (2003). Triángulos y tetraedros fractales. *Suma*, (44), 13-24.

Novak, J. y Gowin, B. (2004). *Aprendiendo a aprender*. Barcelona, Martínez Roca.

Oltra, A. (2009). Diseño y construcción de una antena fractal multibanda para telefonía móvil cellular (Tesis final de grado). Catalunya. Recuperado de https://www.researchgate.net/publication/42252031_Diseno_y_construccion_de_una_ antena_fractal_multibanda_para_telefonia_movil_celular

Panizza, M. (2015). Las transformaciones semióticas en los procesos de definición de objetos matemáticos (Tesis de doctorado). Universidad Nacional de Córdoba.

Pascual, F. (1991). El modelo matemático del espacio-tiempo. *Contextos*, 9(17), 231-255.

Peitgen, H., Jürgens, H. & Saupe, D. (1991). *Fractals for the Classroom. Part one. Introduction to Fractals and Chaos*. NCTM Advisory Board. Second Edition. Springer – Verlag. New York, EEUU.

Peitgen, H., Jürgens, H. & Saupe, D. (1991). *Fractals for the Classroom: Strategic Activities Volume One*. NCTM Advisory Board. Second Edition. Springer – Verlag. New York, EEUU.

Peitgen, H., Jürgens, H. & Saupe, D. (1992). *Fractals for the Classroom. Part two. Complex System and Mandelbrot Set.* NCTM Advisory Board. Second Edition. Springer – Verlag. New York, EEUU.

Peitgen, H., Jürgens, H. & Saupe, D. (1992). *Fractals for the Classroom: Strategic Activities Volume Two*. NCTM Advisory Board. Second Edition. Springer – Verlag. New York, EEUU.

Peitgen, H., Jürgens, H. & Saupe, D. (2004). *Chaos and Fractals*. New Frontiers or Science. Second Edition. Springer, New York, EEUU.

Pereira, T. y Borges, F. (2017). A geometria dos fractais no ensino de Matemática: uma revisao bibliográfica categorizada das pesquisas brasileiras dos últimos dez anos. *Acta Scientiae*, (4).

Picoli, A. y Pinto, J. (2018). Explorando a Geometria Fractal no Ensino Médio por meio de una oficina pedagógica. *Thema*, 15(4), 1594-1561.

Pinto, J. y Desconsi, A. (2018). Teorema de Pitágoras e o fractal árvore pitagórica: um experimento no ensino fundamental. *Brazilian Journal of Education, Technology and Society*, 11(3), 444-457.

Rea L. & Parker M. (1992). *Designing & Conducting Survey Research*, third edition. Jossey-Bass A Wiley Brand.

Redondo, A y Haro, M. (2004). Actividades de geometría fractal en el aula I. *Suma*, (47), 19-28.

Redondo, A. y Haro, M. (2005). Actividades de geometría fractal en el aula II. *Suma*, (48), 15-21.

Rezende, V., Moran, M., Mártires, T. y Carvalho, F. (2018). O Fractal Árvore Pitagórica e Diferentes Representações: uma Investigação com Alunos do Ensino Médio. *Jornal*

Internacional de Estudos em Educação Matemática, 11(2), 160-171.

Rubiano, G. (2009). Iteración y fractales (con Mathematica). Vicerrectoría Académica, Colombia, primera edición.

Sabogal, S. y Arenas, G. (2011). Una introducción a la geometría fractal. Universidad Industrial de Santander, Bucaramanga.

Sandoval, F. y Vire, S. (2008). Diseño e implementación de antenas fractales para UHF. (Tesis final de grado Ingeniería en Telecomunicacoines). Universidad Técnica Particular de Loja. Recuperado de https://www.researchgate.net/publication/327582599_Diseno_e_implementacion_de_antenas_fractales_para_UHF

Sardella, O., Zapico, I. y Berio, A. (2006). Fractales: una nueva mirada en la enseñanza de la geometría. *Números*, (65), 14-20.

Schoenfeld, A. (2010). *How we think. A Theory of Goal-Oriented Decision Making and its Educational Applications.* Routledge Taylor & Francis Group.

Schwab, J.J. (1978). *Science, curriculum and liberal education*. Chicago: University of Chicago Press.

Seckel, M., Breda, A. y Font. (2019). Los criterios de idoneidad didáctica en la formación de profesores. *Acta Latinoamericana de Matemática Educativa*, 32(2).

Shriki, A. & Nutov, L. (2016). Fractals in the Mathematics Classroom: The Case of Infinite Geometric Series. *Learning and teaching mathematics*, (20), 38-42.

Shulman, L. (1986). Those who understand: Knowledge growth in teaching. *Educational* Researcher, 15(2), 4-14.

Spinadel, V. (2002). Geometría Fractal y Geometría Euclideana. *Educación y Pedagogía*, 15(35), 85-91.

Stylianides, G., Stylianides, A. & Weber, K. (2017*). Research on the teaching and learning of proof: Taking stock and moving forward.* Compendium for research in mathematics education (pp.237-266). Publisher: National Council of Teachers of Mathematics. Editors: J. Cai.

Vitabar, F. (2010). Imágenes fractales con GeoGebra. *Unión*, (24), 161-175.

Yildiz, A. & Baltaci, S. (2017). Reflections from the Lesson Study for the Development of Techno – Pedagogical Competencies in Teaching Fractal Geometry. *European Journal of educational research*, 6(1), 41-50.

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Chapter 4 - The Image of Women in Consumer Culture in the Context of Art

Derya Özdemir Kibici

Chapter Highlights

- This study aims to critically examine how consumer culture constructs, transforms, and commodifies the image of women through art. It questions how the female body and identity are represented within the framework of capitalist market logic, spanning traditional artworks to contemporary digital art and advertising.
- This chapter of the book is motivated by the thesis that art is not only a means of reflection but also a powerful ideological tool that reinforces gender norms and consumption habits.
- Analyses of movements and practices such as Pop Art, feminist art, and social media art will examine the process by which the image of women is reduced to an object to be "desired," "owned," and "consumed."
- Without the critical intervention of artists, this process serves to deepen gender inequality.

Introduction

The female image is more than just a representation in art history; it is a social, cultural, and ideological indicator. The formation of the female figure, particularly in painting, constitutes a significant field of visual discourse reflecting the aesthetic understandings, gender roles, and dominant ideologies of the periods. The female body has been idealized as a symbol of beauty and fertility, while also being subject to the male gaze (Berger, 1972; Mulvey, 1975). In this context, painting serves as a historical document regarding women's social position and individuality.

The female image has been not only an aesthetic theme in art history, but also a focal point of social, ideological, and cultural representations (Gouma-Peterson & Mathews, 1987; Kibici, 2021). In painting, in particular, the female figure has served as a tool that makes visible the value judgments, moral norms, and gender roles of each period. Throughout much of art history, the representation of women has been shaped by aesthetic criteria determined by the male gaze (male gaze); The female body has often been constructed as an object of desire and ideal beauty (Berger, 1972; Mulvey, 1975).

During the Renaissance, the female image was treated as a sacred or idealized identity through religious and mythological representations (Chadwick, 2012). From the 19th century onwards, the participation of women artists in the production process, along with modern art, led to a radical transformation in the ways this image is represented (Nochlin, 2015). From the 20th century onwards, the feminist art movement opposed the objectification of the female body, enabling women to engage in the process of subjectification and express their own experiences in art (Pollock, 2015).

However, with the modern period, the participation of women artists in artistic production processes called into question this one-sided understanding of representation. The elevation of the female image from mere "subject" to the status of "subject" became apparent with feminist art movements (Nochlin, 2015; Pollock, 2015). In Turkey, this transformation occurred parallel to the institutionalization of art education and the establishment of women artists in academia from the early years of the Republic (Antmen, 2010; Gürçağlar, 2015).

Today, the female image is addressed not only as an aesthetic theme but also within the framework of identity, gender, body politics, and cultural representation (Karaoğlu & Kara, 2014). This study aims to examine the historical transformation of the female image in painting and its ideological and aesthetic dimensions. Furthermore, by discussing how artists' treatment of the female figure relates to periodic and cultural contexts, it aims to evaluate the reflections of the female image in contemporary art practices. This study aims to examine the historical transformation of the female image in painting from the perspective of consumer culture, both globally and locally (Turkish). The representation of the female figure within the framework of consumer culture is analyzed within the context of trends, gender politics, and aesthetic approaches in different periods.

The Image of Women in Art

Studies on the representation of women in art have generally progressed along the lines of gender theory, feminist art criticism, and visual culture analysis. John Berger's Ways of Seeing (1972), by highlighting the concept of the "male gaze" in painting, became a pioneering source for discussing how women have been objectified in art history. According to Berger, "men look, women are watched," a situation that has largely led to the passive position of women in art history.

The first critiques of the female image in art became evident with the rise of feminist art theory in the 1970s. In Ways of Seeing, Berger (1972) argued that the female body has historically been constructed as an "object of spectacle," while Mulvey (1975) analyzed the sexist gaze in visual culture through the concept of the male gaze. Feminist art historians such as Nochlin (2015) and Pollock (2015) have criticized the historical marginalization of women artists and argued for the need to redefine their representation. Laura Mulvey's (1975) concept of the male gaze, developed in her article "Visual Pleasure and Narrative Cinema," has provided an explanatory framework not only for cinema but also for the visual arts in general. Mulvey argues that the female body is rendered an object of male desire and that this representation is a reflection of patriarchal cultural structures.

In her work "Why Have There Been No Great Women Artists?", Linda Nochlin (2015) questions the reasons for the invisibility of women artists in art history. Nochlin emphasizes that women have historically been denied access to artistic production, and that, as a result, their representation has been shaped within a male-dominated language. Griselda Pollock

(2015), on the other hand, developed the conceptual framework of feminist art history, arguing that the work of women artists should be interpreted at both individual and societal levels. According to Pollock, the image of women carries a multilayered meaning not only in the works of male artists but also in works in which women artists express their own experiences.

Recent research demonstrates that the image of women has undergone a transformation in contemporary art. For example, Butler's (1990) theory of gender performativity has brought a new critical perspective to the bodily representations of contemporary women artists. In digital art, photography, and performance art, the female image is no longer merely a representation but also a tool of resistance and redefinition (Jones, 2012; Betterton, 2019). The modernization process of art history in Turkey has brought about a unique transformation in the representation of women. During the late Ottoman period, Mihri Müşfik Hanım both adopted a Western academic style in her paintings and took pioneering steps toward subjectifying female identity (Antmen, 2010). During the Republican era, women artists—names like Hale Asaf, Fahrünnisa Zeid, and Aliye Berger—reinterpreted female identity as part of the modernization ideology (Erzen, 1999).

With the rise of feminist art in Turkey after 1980, the female image became not only an aesthetic element but also a critical discourse tool. Artists such as Nil Yalter, Şükran Moral, Canan Şenol, and İnci Eviner expressed the female experience in political and performative ways, using themes of the female body, identity, social oppression, and migration in their works (Sözen, 2008; Gürçağlar, 2015).

Recent studies demonstrate that the work of women artists in Turkey has been reevaluated from both a feminist art theory and a gender perspective (Çalıkoğlu, 2011; Dastarlı & Cin, 2023). Antmen (2010) discusses how women's art in Turkey creates "spaces of resistance" within a male-dominated system, while Aydın (2021) notes that the female image is intertwined with themes of identity, memory, and resistance in contemporary Turkish painting. This literature reveals that the position of the female image in art history is not merely an aesthetic issue, but also a reflection of cultural identity, gender, consumer culture, and power relations. In painting, the female figure is no longer an image confined to the viewer's gaze; it has become a representation of a plural identity that constructs its own narrative. In this context, the literature reveals the evolution of the female image's function in art history from a passive object of representation to an active subject. Women artists' use of their own bodies and experiences as artistic material has redefined the political and aesthetic boundaries of art. In this context, it explores how consumer culture manipulates the image of women through art and how this image is instrumentalized in transforming women from subjects to objects of consumption. This continuum extends from the female body depicted as a "landscape" in traditional painting to the "desire-inducing" female figure in advertising to the "self-commodifying" representations of women on social media.

The Image of Women in Artistic Consumer Culture

With the rise of industrial capitalism and globalization, consumption has become a central phenomenon in modern societies. Consumer culture is not merely a set of economic activities but also a system of meanings through which individuals construct their identities, desires, and social positions (Baudrillard, 1998) so it is important to measure the attitudes and skills of the people towards the art (Ceran, 2022, 2025; Ciddi, 2025, Çakır et al., 2019; Kibici, 2025; Ozkan & Erdem, 2025; Tekin, 2025). Within this system, visual representations, and particularly art, play a vital role in transmitting cultural codes. The image of women has been a fundamental theme in art throughout history and has been constantly reshaped in response to the needs of consumer culture (Wolf, 1991).

Historical Background: Women and Audience Perspectives in Traditional Art in Terms of Consumption Culture

The roots of the image of women in consumer culture extend deep into Western art history. As John Berger (1972) emphasized, women in traditional art were often depicted as beings "to be viewed." In Berger's famous distinction of "Gaze," female figures in traditional nude paintings were positioned to direct the desire of the viewing (male) spectator. This is a dual process that transforms women into "an object for the men within the artwork" and simultaneously "an object for the men who are the spectators of the painting" (p. 47). For example, during the Renaissance and Baroque periods, depictions of nude women adorned with mythological or religious themes (Botticelli's Birth of Venus or Titian's Venus of Urbino) essentially served to satisfy the aesthetic and sexual desires of wealthy (male) patrons. In this context, the artwork itself became a consumer object, while the female image was commodified as the most valuable part of this object.

Consumer culture, as one of the most powerful ideological tools of the modern capitalist system, profoundly influences individuals' perceptions of identity, belonging, and value (Featherstone, 2007). This culture plays a decisive role, particularly in the production and reconstruction of the female image. Throughout history, art has functioned not only as a reflection of social values but also as a field that critiques or reshapes these values (Berger, 1972). Today, the transformation of the female image into an object of consumption through artworks, fashion, advertising, and digital media has gone beyond aesthetic representation and become a socio-cultural discourse (Baudrillard, 1998).

The representation of women in consumer culture is not limited to themes of beauty, elegance, or attractiveness; it is also associated with the use of the "female body" as a symbol of desire, status, and capital (Gill, 2007). This situation also manifests itself as a critical counterpoint in contemporary art practices; Feminist artists, in particular, expose the consumerist view on the female body (Nochlin, 2015).

The Modern and Postmodern Break: The Criticism and Ambiguous Position of Pop Art

With the rise of the consumer society in the mid-20th century, art could not remain indifferent to this phenomenon. The Pop Art movement brought the imagery of consumer culture directly to the center of art (Sturken & Cartwright, 2009). Andy Warhol's Campbell's Soup Cons and Marilyn Monroe silkscreens questioned mass production and the cult of celebrity while simultaneously celebrating them. Here, the female image (Marilyn Monroe) transcended being an artistic icon into a marketable brand, a consumable image.

Warhol's works clearly demonstrate the place of women in consumer culture: women are an image that can be packaged, reproduced, and sold like a product. However, this critique of Pop Art is often ambivalent; while exposing the absurdity of the system, they are also drawn to its allure and aesthetics. This demonstrates the complex and often contradictory relationship art has with consumer culture (Honnef & Warhol, 2000).

Consumer culture defines a social structure in which individuals are shaped by desire and identity-driven behaviors rather than needs. Baudrillard (1998) emphasizes that consumption is not merely the purchase of material goods; it is the consumption of symbolic meanings, identities, and status indicators. In this context, art plays a dual role as both a site of production and critique in consumer culture.

In the postmodern era, art has been commodified under the influence of marketization; artworks have become "investment objects" or commodities carrying "brand value" (Adorno & Horkheimer, 2002). In this context, the image of women has often been reduced to an aesthetic surface, an object for spectacle (Mulvey, 1975). With the advent of modern art, female artists began to question these forms of representation. Artists such as Cindy Sherman, Barbara Kruger, and Shirin Neshat reversed the cultural codes of female imagery and critically reproduced the patterns of beauty and identity imposed by consumer culture (Pollock, 2015).

Figure 1. Gül (2009) Crisis

The painting (Figure 1), which is a photograph from artist Deniz Gül's work 'Video Crisis' 2009, which features paper bags of branded products such as DKN, Gucci, Zara, Laura Ashley, etc., can be seen as a work reflecting women's sensitivity to brands and their shopping frenzy in the consumer culture (Özdemir, 2014).

Women's Image as an Advertising Tool

As communication tools such as cinema, TV, video, posters, and photography gain increasing importance in daily life, the frequent use of iconic forms such as photographs, graphics, figures, films, engravings, paintings, patterns, and geometric shapes is observed in the imaginative messages they convey and produce. Imaginative messages, varying in expression and form, are also used alongside text and words. In these messages, the pictorial ratio is generally used equivalently to that of written messages (Bilgin, 2006).

Similarly, in Turkey, media and artistic representations of consumer culture shape female identity. In contemporary Turkish art, artists such as Nil Yalter, İnci Eviner, and Canan Şenol have produced works that question the social, political, and consumer dimensions of the female body (Yılmaz, 2021). The foundation of advertising lies in one or more meanings within the intended message. This becomes particularly important when the intended message relies solely on visuals. A common challenge for advertising designers is the difficulty of capturing visual meaning in a visual message (Atabek & Atabek, 2007).

Figure 2. Photo from a Clothing Brand (Jill Sander) Advertising Shoot

At this stage, advertising designers begin the process of creating an image that reinforces visual meaning. In this context, imagery becomes a crucial element, and the impact of the female image is paramount. It would be more accurate to interpret this as attracting the same sex rather than men. The way women are presented to other women should have characteristics that foster emulation and increase demand and competition (Özdemir, 2014).

The meanings of images in advertisements and their impact on viewers are crucial, and the effectiveness of the advertisement depends on this. In advertisements, the perceived meaning, perceived by viewers/readers, is crucial, as opposed to the intended meaning, which reflects the text's creation logic (Atabek & Atabek, 2007).

Critical Intervention in Feminist Art

The feminist art movement, which gained strength in the 1970s, introduced a radical critique of the roles assigned to women by both traditional art history and consumer culture. Artists directly targeted the commodification of the female body, forcing viewers to reflect on its workings (McRobbie, 2008).

Figure 3. 'Virginia Slims Woman'

Barbara Kruger is one of the most iconic figures in this context. Her works, which overlay striking white text on a red background over black-and-white found footage images (for example, "I shop, therefore I am" or "Your body is a battleground"), directly utilize the language of advertising to question the connection between consumerism, identity, and the control of women's bodies. Kruger's works expose how advertising tells women to "buy their identities" and how this process objectifies women. Similarly, Cindy Sherman's Untitled Film

Stills series parodies the stereotypes Hollywood and the media have created about women (hero, victim, femme fatale, etc.), demonstrating the artificiality of these images and how they limit female subjectivity (Baldiny, 2017). Sherman emphasizes that the female image is not "natural" but rather a culturally and socially constructed performance (Sprague-Jones & Sprague, 2011).

The images created in advertisements, which utilize a wide range of artistic possibilities, psychological research, and marketing strategies, play a significant role. Advertisements are shaped by dominant gender images, demonstrating and reinforcing these gendered images. They portray women and men as role models (Kellner, 2001). The desired social environment is artificially created through general perceptions and conditioning instilled in people. The image of the woman in the "Virginia Slims Woman" ads gradually shifted from the smiling, friendly, and suitable potential partner of the early ads to the intimidating, attractive, sexy, and masculine image (see Figure 3). The image of a powerful, confident, and independent woman, with sunglasses and a leather jacket, is presented (Kellner, 2001; Özdemir, 2014).

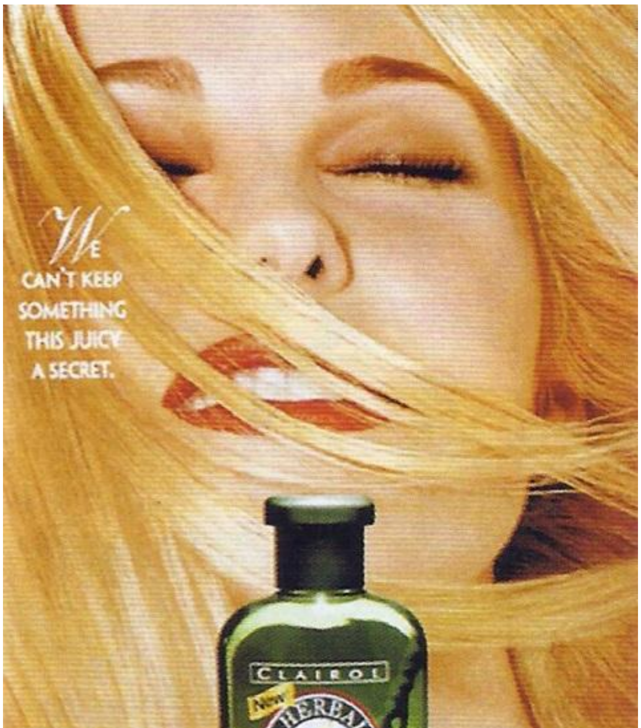

Figure 4. Herbal Shampoo Advertisement

As Susan Sontag explained in 1999: “According to the widely accepted definition, being feminine means being attractive or doing one’s best to be very attractive; it means attracting.” Physical beauty and attractiveness are constantly emphasized and are shown as among the best representations of femininity, thereby emphasizing a planned effort to look good (Atabek & Atabek, 2007). This perception has not only been presented as a narrative to women in different ways, but has also been constantly referred to women through various visuals. The common result of all these narratives is that being beautiful has now become a duty and a condition for women. And it has become so commonplace and normalized that anything done for beauty has been permissible. This created image has become the main element of consumer culture. The images of women that are created and reflected in lenses are created through yet another woman as a model in order to impress other women, to focus on certain areas, and to make certain objects a necessity. Advertisements using the image of women; It is used extensively not only to influence women but also to increase the effectiveness of other advertisements that appeal to men and the general public (Atabek & Atabek, 2007; Özdemir, 2014).

Contemporary Digital Age: Social Media and Self-Commodification

In contemporary art, the image of women is a sign of consumption, intertwined with digital media and popular culture. Idealized bodies, aesthetic surgery norms, and fashion representations on social media transform female identity into a performative sphere of consumption (Gill & Elias, 2014). Within artistic practices, this transformation is seen as both a critique and a process of reproduction. Digital art, NFTs, and AI-powered visual productions, in particular, reproduce the female image through repetitive symbols, while simultaneously carrying the potential to critique the system by manipulating these symbols (Bishop, 2012).

Today, the relationship between consumer culture and women's image has taken on a new and more complex dimension with social media platforms. Platforms like Instagram and TikTok have become "lifestyle" showcases where users create and publish their own images. While women appear to have gained the power to create their own representations against the one-sided representation of traditional media, the economy of "likes," "followers," and "interaction" has brought about a new form of commodification (Shao, 2023).

"Influencer" culture normalizes the use of women's bodies and lifestyles as marketing tools for brands. This process is a practice of "self-commodification" (Marwick, 2013), where women consciously make their own image marketable. While this can be interpreted as a feminist act of "controlling one's own image," it also requires a critical perspective, as it internalizes and reproduces consumer culture's domination of women's bodies and identities at the individual level. Digital artists critically interrogate this new reality through deepfake technology, data visualization, and digital collage techniques.

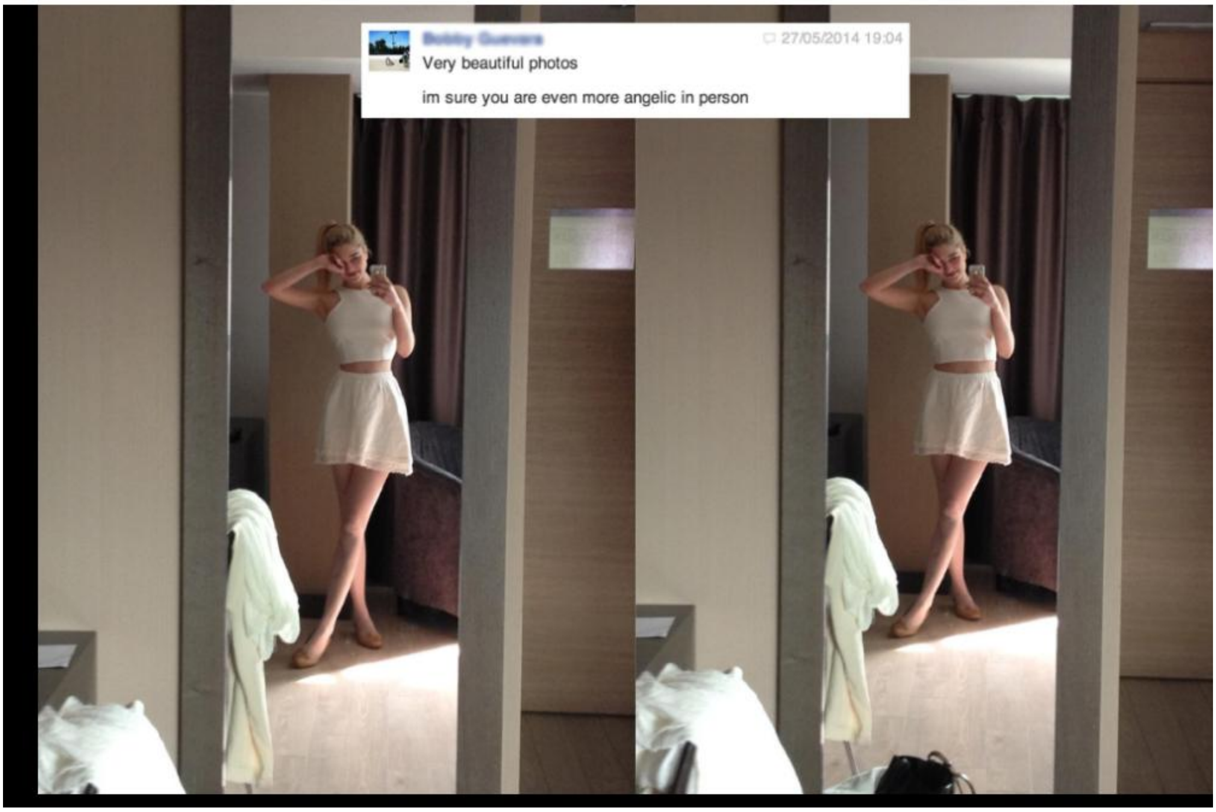

Figure 5. Amalia Ulman - "Excellences & Perfections" (2014-2015)

The work in Figure 5 is a performance art project by Ulman (2014-2015). In 2014, the artist created a fictional persona on Instagram that mimicked the roles of the stereotypical "lifestyle influencer"—the naive country girl, the beauty addict, the sick and recovering "cool" girl. The selfies she shared seemed to document a completely scripted life filled with plastic surgery, luxury brands, wellness, and emotional crises. The project critically explores how female identity is performed and commodified in the economy of "likes" and "followers." Ulman's work is a perfect example of how the act of "controlling one's own image" simultaneously becomes a practice of "self-commodification." It demonstrates how the "authentic" life presented on social media is, in fact, fictional and marketable.

Conclusion

Consumer culture, using art as a tool, constantly redefines the image of women and subjects this image to market logic. A historical examination reveals that the representation of women in art has evolved from an object that provides pleasure to the viewer, to a marketable brand, and ultimately to a self-marketing "digital worker."

However, art not only mediates this commodification process but also serves as one of the most powerful channels for resisting and critiquing it. Barbara Kruger, Cindy Sherman, and their contemporary followers, using the language and power of art, have exposed the dominance of consumer culture over the image of women and invited viewers to question this visual regime. Without these critical interventions, the image of women reproduced through art will continue to function as a reinforcement of gender inequality and a perpetual confinement of women to the status of "consumer object." Therefore, it is crucial to not ignore the dialectical reality that art can be both a part of the problem and a tool for the solution.

References

Adorno, T. W., & Horkheimer, M. (2002). *Dialectic of enlightenment: Philosophical fragments*. Stanford University Press.

Antmen, A. (2010). *Sanatçının cinsiyeti: Türkiye'de kadın sanatçılar ve feminist sanat*. İletişim Yayınları.

Atabek, G. Ş., & Atabek, Ü. (2007). *Medya metinlerini çözümlemek: İçerik, göstergebilim ve söylem çözümleme yöntemleri*. Siyasal Kitabevi.

Aydın, G. (2021). Çağdaş Türk resminde kadın imgesi: Kimlik ve direniş. *Sanat ve Tasarım Dergisi*, *11*(1), 55–72.

Baldini, V. (2017). *Articulate: Feminist text-art from the 1990s-today: Four case studies* [Master's thesis, Fashion Institute of Technology, State University of New York]. FIT Digital Commons.

Baudrillard, J. (1998). *The consumer society: Myths and structures*. SAGE Publications.

Berger, J. (1972). *Ways of seeing*. Penguin Books.

Betterton, R. (2019). *An intimate distance: Women, artists and the body*. Routledge.

Bilgin, N. (2006). *Sosyal bilimlerde içerik analizi, teknikler ve örnek çalışmalar* (2. baskı).

Siyasal Kitabevi.

Bishop, C. (2012). *Artificial hells: Participatory art and the politics of spectatorship*. Verso.

Butler, J. (1990). *Gender trouble: Feminism and the subversion of identity*. Routledge.

Ceran, S. (2022). The relationships between secondary school students' visual creativities in art and their attitudes towards graphic design. *International Journal of Research in Education and Science*, *8*(3), 576–595. https://doi.org/10.46328/ijres.2962

Ceran, S. (2025). Examining high school students' artificial intelligence literacy in visual arts subjects and metaverse-based digital art attitudes. *International Journal of Technology in Education*, *8*(4), 1245–1265. https://doi.org/10.46328/ijte.5360

Chadwick, W. (2012). *Women, art, and society* (5. baskı). Thames & Hudson.

Ciddi, M. (2025). Analysis of attitudes of undergraduate art students toward painting workshop lessons. *International Journal on Social and Education Sciences*, *7*(2), 195–204. https://doi.org/10.46328/ijonses.764

Çakır, E., Öztürk, M., & Ünal, M. (2019). Interpainting as a creating method in digital illustration: Reinterpretations from movie scenes. *Bilim, Eğitim, Sanat ve Teknoloji Dergisi (BEST Dergi)*, *3*(2), 78–88. https://doi.org/10.46328/seat.v3i2.32

Çalıkoğlu, B. (2011). *The transformation of women artists representation: A feminist analysis of the historical narrative and Istanbul Modern's new works new horizons exhibition* [Doctoral dissertation, Yıldız Teknik Üniversitesi].

Dastarlı, E., & Cin, F. M. (2023). *Feminist art in resistance: Aesthetics, methods and politics of art in Turkey*. Palgrave Macmillan.

Erzen, J. N. (1999). Cumhuriyet dönemi Türk resminde kadın imgesi. *Toplumsal Tarih*, *68*, 46–52.

Featherstone, M. (2007). *Consumer culture and postmodernism* (2. baskı). Sage.

Gouma-Peterson, T., & Mathews, P. (1987). The feminist critique of art history. *The Art Bulletin*, *69*(3), 326–357.

Gill, R. (2007). *Gender and the media*. Polity Press.

Gill, R., & Elias, A. S. (2014). "Awaken your incredible": Love your body discourses and postfeminist contradictions. *Feminism & Psychology*, *24*(3), 314–331. https://doi.org/10.1177/0959353514539657

Gürçağlar, M. (2015). Feminist sanatın Türkiye'deki yansımaları. *İstanbul Üniversitesi Sanat Tarihi Dergisi*, *24*(2), 145–162.

Jones, A. (2012). *Seeing differently: A history and theory of identification and the visual arts*. Routledge.

Karaoğlu, A., & Kara, S. (2014). Estetik bir obje olarak takı. *İdil Sanat ve Dil Dergisi*, *13*, 69–79.

Kellner, D. (2001). Popüler kültür ve postmodern kimliklerin inşası (G. Seçkin, Çev.). *Doğu Batı Üç Aylık Düşünce Dergisi*, *4*(15), 194–195.

Kibici, V. B. (2021). Analysis of music teachers' job satisfaction and COVID-19 anxiety levels. *International Journal on Social and Education Sciences*, *3*(4), 752–767.

Kibici, V. (2025). Examining the digital literacy and artificial intelligence attitudes of university students studying arts. *International Journal of Education in Mathematics, Science and Technology*, *13*(5), 1283–1297. https://doi.org/10.46328/ijemst.5839

Marwick, A. E. (2013). *Status update: Celebrity, publicity, and branding in the social media age*. Yale University Press.

McRobbie, A. (2008). Young women and consumer culture: An intervention. *Cultural Studies*, *22*(5), 531–550.

Mulvey, L. (1975). Visual pleasure and narrative cinema. *Screen*, *16*(3), 6–18.

Nochlin, L. (2015). *Women, art, and power and other essays*. Routledge.

Ozkan, Z., & Erdem, T. (2025). Examination of expert opinions on art skills education for individuals with mild intellectual disabilities. *International Journal on Studies in Education*, *7*(4), 1048–1068. https://doi.org/10.46328/ijonse.5766

Özdemir, D. (2014). *Bir anlatı ve anlayış olarak kadının modernizm sonrası sanatsal imajı* [Yüksek lisans tezi, Atatürk Üniversitesi]. Atatürk Üniversitesi Sosyal Bilimler Enstitüsü Veri Tabanı.

Pollock, G. (2015). *Vision and difference: Feminism, femininity and the histories of art*. Routledge.

Shao, C. (2023). The influence of consumer culture on feminist consciousness in social media field. *SHS Web of Conferences*, *179*, Article 03010.

Sözen, M. (2008). Türkiye'de çağdaş sanat ve toplumsal cinsiyet temsilleri. *Sanat Kuramı Dergisi*, *2*(3), 27–40.

Sprague-Jones, J., & Sprague, J. (2011). The standpoint of art/criticism: Cindy Sherman as feminist artist? *Sociological Inquiry*, *81*(4), 404–430.

Sturken, M., & Cartwright, L. (2009). *Practices of looking: An introduction to visual culture* (2. baskı). Oxford University Press.

Tekin, A. (2025). Examining high school students' attitudes towards contemporary and digital art and virtual exhibitions. *International Journal on Studies in Education*, *7*(4), 879–898. https://doi.org/10.46328/ijonse.5561

Ulman, A. (2014). *Excellences & perfections* [Instagram performance]. Rhizome and Arcadia Missa Gallery. https://archive.newmuseum.org/exhibitions/2487

Warhol, A. (2000). First pop star of the art world. K. Honnef & A. Warhol (Ed.), *Warhol* içinde. Taschen.

Wolf, N. (1991). *The beauty myth: How images of beauty are used against women*. William Morrow and Company.

Yılmaz, S. (2021). Türkiye'de çağdaş sanatta kadın temsilleri: Feminist bir okuma. *İstanbul Sanat Dergisi*, *12*(3), 45–63.

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Chapter 5 - Mastering High School Math: The Power of Algebraic Thinking in High School

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Chapter Highlights

- This study investigates how high school students engage with algebraic thinking through classroom problem-solving and collaborative tasks, such as observing algebra classes and analyzing assignments and exams.
- By observing students in Grades 9 and 10, collecting samples of their algebraic work (classwork, homework, mathematical projects, quizzes, and tests), and analyzing patterns of reasoning, the research identifies both productive strategies and common misconceptions among high school students.
- Findings reveal that while students often rely on procedural approaches, opportunities for generalization and symbolic reasoning significantly enhance their conceptual understanding. Procedural strategies are rooted in conceptual knowledge; the ability to read and explain their process reflects students' conceptual understanding in algebra.
- Exercising both procedural and conceptual knowledge leads to the development of algebraic thinking. Finally, the study concludes with recommendations for teaching practices that foster deep algebraic thinking and strengthen problem-solving skills by integrating both procedural and conceptual approaches.
- Consequently, algebraic thinking enables high school students to master mathematical strategies and succeed in STEM subjects, including art.

Introduction

The article examines the impact of algebraic thinking on mastering mathematics in high school through qualitative methods. The first section of the research focuses on the importance of algebra in secondary education, the role of algebraic thinking in mathematical development, the problem statement, the purpose of the research, and the research questions.Subsequent sections of the study center on three primary research questions.

The second section presents a literature review covering definitions and dimensions of algebraic thinking, procedural versus conceptual mathematical knowledge, key studies on algebra learning in secondary schools, and gaps or limitations in existing research.

The third section outlines the research methodology, including the qualitative classroom-based study design, observation, document analysis, participants, data collection, and analysis methods.

The fourth section presents the findings, summarizing major themes observed (e.g., symbolic reasoning, pattern recognition, equation solving), examples of student work demonstrating varying levels of algebraic understanding, notable misconceptions or problem-solving behaviors, and insights into how students transition from arithmetic to algebraic thinking.

The fifth section is the discussion, which compares the findings with existing literature—highlighting confirmations, contradictions, or extensions of prior studies. It also provides insights into how teaching methods influence algebraic thinking, classroom dynamics, student engagement, and implications for curriculum development and teacher training. Finally, the sixth section presents the conclusion.

Importance of Algebra in Secondary Education

Algebra is a foundational subject in high school mathematics that connects a broad range of mathematical topics. This paper highlights several key components of algebra that are essential to mathematical understanding. Algebra serves as a powerful tool for developing students' critical thinking skills, enhancing problem - solving and reasoning abilities, enabling generalization, and fostering an understanding of relationships between patterns. It also involves the application of properties such as the distributive, associative, and commutative laws.

Kortering et al. (2005) state that algebra classes are critical to success in postsecondary education and well-paying careers. Therefore, its role as a "gatekeeper" subject is well-justified. As a result, high school algebra plays a crucial role in promoting students' academic growth.

Students' critical thinking is elevated through algebra by solving mathematical problems using a variety of single and combined strategies, generalizing from specific examples, applying inductive and deductive reasoning, and recognizing recurring patterns. Exposure to problems ranging from simple to complex sharpens their strategic thinking and problem-solving abilities.

Generalizing patterns in word problems presents a significant challenge for high school students. For example, Utami et al. (2023) found that students struggle more with generalizing patterns in word problems than in object configurations or geometric shapes. Generalizing patterns in algebra is generally more difficult for students than doing so in geometry. Understanding relationships between various patterns significantly contributes to the development of students' critical thinking.

Algebra in secondary education equips students with essential skills such as translating word problems into mathematical statements, solving equations and inequalities for unknowns, and recognizing the underlying structure of mathematical expressions. Through this process, students develop logical and critical thinking. Nurrahmawati et al. (2021) emphasized that translating between representations is a vital aspect of mathematics learning. Moreover, algebra extends arithmetic by introducing unknown variables used to solve equations or inequalities. Above all, recognizing key structural elements in mathematical expressions provides students with valuable insights for solving problems and constructing algebraic proofs.

Role of Algebraic Thinking in Mathematical Development

Algebraic strategies for solving various problems influence students' ability to make connections with strategies in other areas of mathematics. For instance, Algebra 1 and Algebra 2 are closely connected to high school precalculus. These courses cover similar topics that are explored in greater depth in precalculus. Precalculus is designed to serve as a bridge between the concepts of algebra and calculus, using previously learned skills to transition into more complex calculus topics later in the year (Weintraut, 2021).

Calculus, in turn, is a prerequisite for many advanced courses across different fields, especially in STEM (Science, Technology, Engineering, and Mathematics). Additionally, calculus serves as the foundation for higher-level mathematics such as real analysis, differential equations, complex analysis, numerical analysis, topology, and mathematical modeling.

Problem Statement and Research Purpose

Algebraic thinking empowers high school students to master key components of mathematics. It is a foundational element of mathematical development, involving the recognition of patterns, analysis of relationships, generalization of mathematical rules, and the use of symbols and variables to represent abstract ideas. Exploring mathematical concepts in algebra classes deepens our overall understanding of mathematics. As we deepen our appreciation of math's importance, we begin to recognize its vast potential to unlock the secrets of the universe and elevate humanity to new heights (Texthelp, n.d.). Algebraic thinking is a powerful tool for mastering high school math, and mathematics plays a vital role in everyday life.

The purpose of this paper is to show how algebraic thinking paves the way for algebra and expands the reach of mathematics, science, and art by providing deeper access to their fields. We already know that algebra is a fundamental part of mathematics. Simply put, algebra is the formal use of symbols and equations to solve problems.

In contrast, algebraic thinking is the ability to recognize patterns, understand relationships, and use logic to solve problems—even without using formal algebra. There are multiple definitions of mathematics; a simple one connected to algebraic thinking, cited by Quinn (2022), states, “Mathematics is about patterns and relations. — Nuh (Noah) Aydin.” Algebraic thinking enhances the power of algebra’s influence on other mathematical subjects because it emphasizes the exploration of patterns and relationships within algebra.

The research paper focuses on algebraic thinking in mathematical problem solving, algebraic reasoning, classroom practices that promote algebraic understanding, and the key characteristics of algebraic thinking. These key characteristics include generalization, the use of symbols and representations, functional thinking, understanding structure, problem solving and reasoning, working with unknowns, and representing and analyzing relationships. The paper explores the significance of algebraic thinking in mastering Algebra 1 and Algebra 2, which are fundamental components of high school mathematics.

Research Questions

- How do teachers and students demonstrate algebraic thinking in problem-solving tasks?
- What common misconceptions arise in students' algebraic reasoning?
- What classroom practices promote deeper algebraic understanding?

Literature Review

The second section of the paper addresses fundamental components of algebraic thinking, including the definition and dimensions of algebraic thinking, procedural versus conceptual knowledge, key characteristics of algebraic thinking, and the gaps or limitations identified in the study. In general, a definition explains what something means. To define something is to limit or bind it completely (Anthro_Poetry, 2022), thereby channeling the dimensions of algebraic thinking to a certain degree. Next, procedural and conceptual approaches form the core of algebraic thinking; therefore, the paper presents a brief discussion that highlights their differences. In addition, it outlines key characteristics of algebraic thinking that contribute to the overall scope of the section. Since the scope of the paper focuses on a relatively narrow area of the topic, it aims to shed light on the broader impact of algebraic thinking in mastering high school mathematics.

Definition and Dimensions of Algebraic Thinking

The dimension of algebraic thinking has a wide scope, such that a single definition cannot encompass all its components. Algebraic thinking is the ability to generalize, represent, justify, and reason with abstract mathematical structures and relationships (Digital Promise Global, n.d.). Broadly speaking, operations and algebraic thinking involve generalizing arithmetic and representing patterns. Algebraic thinking plays a crucial role in deepening students' understanding of arithmetic and supports them in making meaningful connections across various areas of early mathematics. Kriegler (n.d.b) describes algebraic thinking in several ways, based on the work of multiple researchers.

> *Battista and Brown (1998):* For students to meaningfully utilize algebra, it is essential that instruction focus on sense making, not symbol manipulation. Throughout their mathematical careers, students should have opportunities to reflect on and talk about general procedures performed on numbers and quantities. ...Thinking about numerical procedures starts in elementary grades and continues...until students can eventually express and reflect on the procedures using algebraic symbolism.

> *Greenes and Findell (1998):* The big ideas of algebraic thinking involve] representation, proportional reasoning, balance, meaning of variables, patterns and functions, inductive reasoning, and deductive reasoning.

> *Herbert and Brown (1997):* Algebraic thinking is using mathematical symbols and tools to analyze different situations by (1) extracting information from the situation...(2) representing that information mathematically in words, diagrams, tables, graphs, and equations; and (3) interpreting and applying mathematical findings, such as solving for unknowns, testing conjectures, and identifying functional relationships.

> *Kieran and Chalouh (1993):* Algebraic thinking involves the development of mathematical reasoning within an algebraic frame of mind by building meaning for the symbols and operations of algebra in terms of arithmetic

> *NCTM Standards (5-8) - Algebra (NCTM, 1989):* Understand the concept of variable, expression, and equation; represent situations and number pattern with tables, graphs, verbal rules, and equations, and explore the interrelationships of these representations; analyze tables and graphs to identify properties and relationships; develop confidence

in solving linear equations using concrete, informal, and formal methods; investigate inequalities and nonlinear equations informally; apply algebraic methods to solve a variety of real-world problems and mathematical problems.

NCTM Standards (5-8) - Patterns and Functions (NCTM, 1989): Describe, extend, analyze, and create a wide variety of patterns; describe and represent relationships with tables, graphs, rules; analyze functional relationships to explain how a change in one quantity results in a change in another; use patterns and functions to represent and solve problems.

Algebraic thinking involves understanding both how and why a set of rules is applied in a mathematical context. Over time, this type of thinking supports students' ability to solve problems using abstractions and to operate on mathematical entities logically and independently of the material world (Windsor, 2010). Algebraic thinking is not just about getting the answer (procedural), but also about understanding the reasoning and relationships behind the process (conceptual). Students who develop both aspects are better equipped to solve unfamiliar problems, justify their thinking, and transfer knowledge across contexts.

Procedural vs Conceptual Knowledge

Procedural and conceptual knowledge have a deep interrelationship. Procedural knowledge involves knowing how to carry out mathematical operations—such as solving equations, simplifying expressions, or applying formulas. Conceptual knowledge involves understanding why these procedures work—grasping the underlying principles, relationships, and structures in mathematics. According to Hurrell (2021), while many researchers argue that one type of knowledge should precede the other, the instructional methods they describe often begin with conceptual knowledge before introducing procedural knowledge.

Based on observations of students in algebra classes, most students tend to apply procedural approaches more frequently than conceptual ones, likely because conceptual knowledge is perceived as more abstract. Therefore, developing procedural fluency can sometimes help students access and understand conceptual knowledge more easily. Despite the strong connection between the two, important differences exist, as shown in Table 1.

Table 1. Example of Showing the Difference between Procedural and Conceptual Knowledge

Example - Problem: 3(x + 3) = 15	
Type of Knowledge	Application in Problem
Procedural	Distribute: 3x + 9 = 15 → Subtract 6 → Divide by 3
Conceptual	Recognize that multiplying and then adding (both sides) - dividing (both sides) and isolating the variable

(Source: Modification from ChatGPT)

Solving mathematical problems in high school algebra requires students to follow several steps to reach a final answer. Pólya's (1957) four-step process provides a model for teaching and assessing problem-solving in mathematics classrooms: understanding the problem, devising a plan, carrying out the plan, and looking back. These steps are repeated in many mathematical examples and often become routine for students. However, if students explain what rules they used and why they used them during these steps, they are effectively applying conceptual knowledge through their problem-solving process.

Key Characteristics of Algebraic Thinking

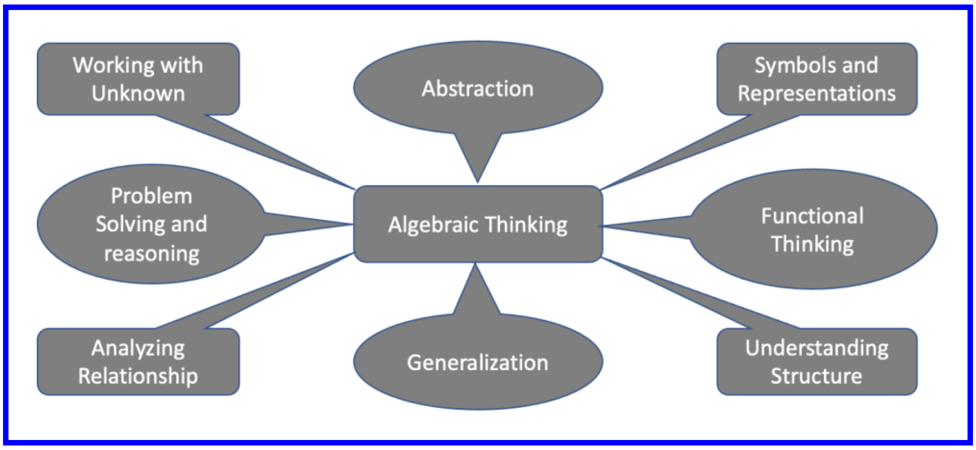

Figure 1. Key Characteristics of Algebraic Thinking

Key characteristics of algebraic thinking include generalization—recognizing patterns and expressing general rules, such as representing all odd numbers in the form 2n + 1, where *n* is a whole number—moving from specific examples to general principles through inductive reasoning. It involves the use of symbols and representations, using letters to represent numbers and relationships, and understanding how expressions, equations, and graphs convey mathematical ideas. Functional thinking focuses on relationships between variables and input-output structures, such as functions or cause-effect scenarios. Algebraic thinking also emphasizes understanding structure, such as recognizing that expressions like $3(x + 2)$ and $3x + 6$ are equivalent, and applying mathematical properties like the distributive, associative, commutative, inverse, identity, additive identity, multiplicative identity, reflexive, and symmetric properties. Problem solving and reasoning are central, involving logical thinking, making and testing conjectures, and justifying solutions. Comfort with unknowns is essential, including manipulating and solving for unknown values in equations and inequalities. Representing and analyzing relationships through tables, graphs, and equations—and translating among these forms—also plays a key role. Finally, abstraction, or the ability to move from specific numbers to general rules and symbolic representations, is fundamental to developing deep algebraic understanding..

Gaps or Limitations in Current Research

The current research does not include interviews with teachers, students, or administrators, nor does it use focus groups. Instead, researchers rely solely on observation and document analysis, which includes classwork, homework, exams, and classroom observations at the technology urban high school in New Jersey. While interviews could provide more detailed insights into algebraic thinking and enrich the study, the researchers believe that sufficient data can be obtained through observation and document analysis. Most importantly, protecting the privacy of students, teachers, and administrators remains a top priority for both the researchers and the school district.

Methodology

The study takes place within actual classrooms at a technology high school in New Jersey, allowing researchers to explore teaching and learning as they naturally occur. The classroom is, after all, where instructional decision-making takes root (Ho, 2021). The study observes and analyzes teachers' instruction, as well as students' classwork, homework, projects, and exams. Researchers examine these documents in detail. Document analysis is a form of qualitative research that uses a systematic procedure to analyze documentary evidence and answer specific research questions (Gross, 2018). This analysis focuses on the application of algebraic thinking as students solve mathematical problems, identify misconceptions, and reveal the effects of teachers' instruction on the development of algebraic thinking. Both observation and document analysis aim to answer the research questions.

Qualitative Classroom-Based Study

The research study was conducted in a natural classroom setting. A qualitative classroom-based study is a type of research that investigates teaching and learning as they naturally occur in educational environments, using qualitative methods such as observations, interviews, open-ended surveys, or analysis of student work. Furthermore, classroom-based research can help strengthen collaboration among teachers (Kostoulas & Lämmerer, 2015).

Observations - Watching What Happens in the Classroom

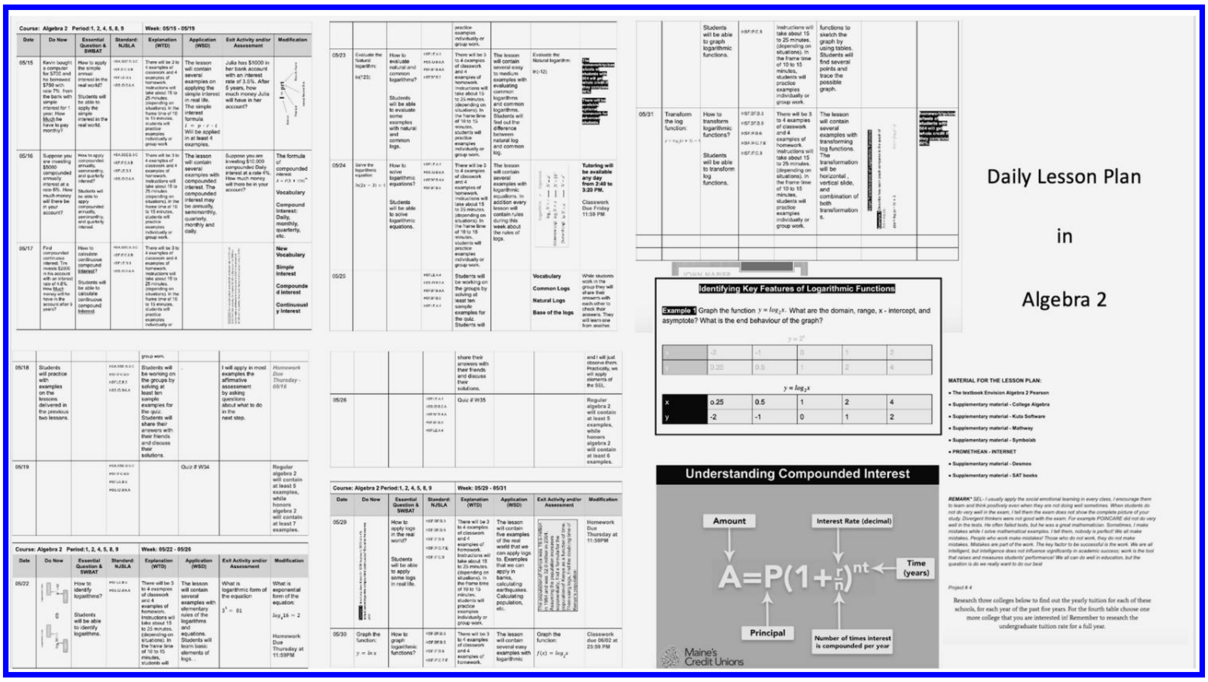

Figure 2. A Sample of the Actual Daily Lesson Plan Describes General Information of Lessons for Two Weeks (the document is from teachers' folder at the technology urban high school in New Jersey)

An observation of a teacher conducting a lesson on identifying key features of logarithmic functions took place in an Algebra 2 class at a technology high school in New Jersey. The mathematics teacher began by introducing logarithmic functions through related mathematical concepts. The lesson started with a comparison between logarithmic and exponential functions, using both logarithmic and exponential equations. Logarithmic functions are the inverse of exponential functions. The structure of the lesson, as described in Figure 3, emphasized this comparison.

The teacher briefly discussed the conditions that define logarithmic functions. For instance, the argument of the logarithm y must be greater than 0, the base b must be greater than 0 and not equal to 1, and the logarithmic expression is denoted as $log_b y$. This was followed by the explanation that the equation $log_b y = x$ is equivalent to the exponential form $y = b^x$.

After a brief introduction to logarithmic functions and their applications in mathematics, science, technology, engineering, economics, and psychology, the teacher explained how to convert between logarithmic and exponential forms. The lesson continued with practical examples. In the first two examples (1a and 1b), the teacher demonstrated how to use both general and specific equations.

For the remaining examples (1c, 2a, 2b, and 2c), students were given five minutes to think and collaborate with their peers seated nearby (to the left, right, front, or back). The teacher then invited students to come to the board and convert expressions from logarithmic to exponential form.

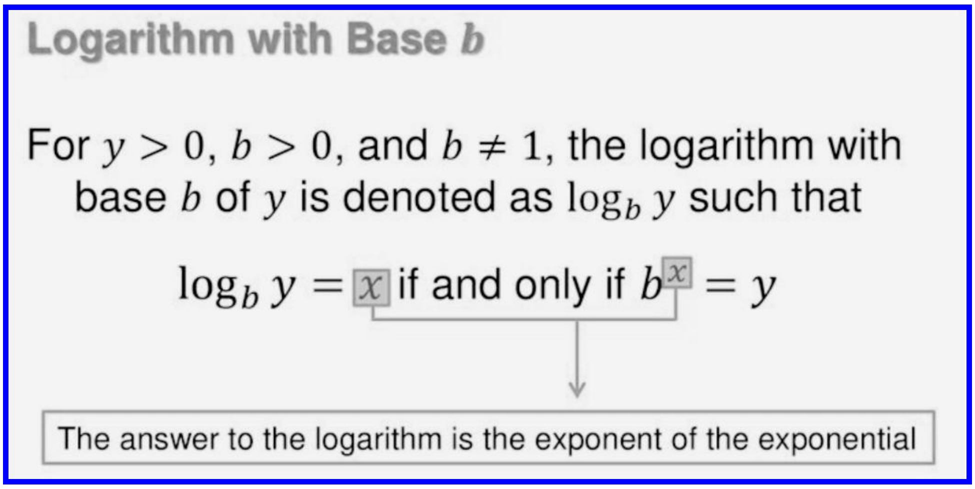

Figure 3. A Sample of Equations the Teacher Uses as a Big Idea to Introduce the Lesson

Examples 3 and 4 followed the same instructional pattern as the earlier examples, but they changed from exponential to logarithmic form. At the end of the class, the teacher addressed any lingering student questions and assigned homework with several related examples to reinforce the skill of converting equations between logarithmic and exponential forms.

Example 1. Changing from Logarithmic to Exponential Form. Write each equation in its equivalent form.

$a.\ 2 = log_5 x$ $b.\ 3 = log_b 64$ $c.\ log_3 = 7$

Example 2. Changing from Logarithmic to Exponential Form. Write each equation in its equivalent **Exponential form.**

$a.\ 2 = log_b 25$ $b.\ 3 = log_7\ 343$ $c.\ log_4 26 = y$

Example 3. Changing from Exponential to Logarithmic Form. Write each equation in its equivalent logarithmic form.

$a.\ 12^2 = x$ $b.\ b^3 = 8$ $c.\ e^y = 9$

Example 4. Changing from Exponential to Logarithmic Form. Write each equation in its equivalent logarithmic form.

$a.\ 2^5 = x$ $b.\ b^3 = 27$ $c.\ e^y = 33$

Figure 4. Sample of Examples Demonstrated in the Algebra 2 Class during the Class (the document is from teachers' folder at the technology urban high school in New Jersey)

The next day, the class began by reviewing homework questions on converting logarithmic equations into exponential equations and vice versa. Students were also asked to justify the validity of these conversions. The process of changing an equation from one form to the other primarily involved a procedural approach, where students strategically applied the conversion rule. They relied on the fact that 'b' signifies the base, 'y' signifies the argument, and 'x' signifies the exponent. Correctly placing the numerical values for these notations enabled students to arrive at the right answer.

However, the teacher wasn't merely interested in *how* the conversion worked; they went a step further to explore *why* it worked. Before explaining the underlying reasons, the teacher informed students they would first need to learn the properties of logarithms. Consequently, they postponed the 'mini-proof' for a few weeks.

The lesson plan was structured to complete all sections on logarithmic functions before introducing the forms of compound interest. On another day, the students' session focused on the relationship between inverse functions and logarithmic functions, as illustrated in Figure 5. When finding the inverse of an exponential or logarithmic function, we are essentially converting from one form to the other (Devlin, n.d.). The inverse of a logarithmic function—and likewise of an exponential function—is symmetric to its counterpart with respect to the line $y = x$. A graphical interpretation of inverse logarithms is shown in Figure 5.

In the lesson plan, identifying key features of logarithmic functions, specifically on inverse function of logarithmic functions the teacher explained the mini proof to validate why the logarithmic function is equal with exponential function. The teacher set up the proof by taking in consideration the restriction and started to prove the given equation.

Proof: Symmetry of Inverse Functions with Respect to the Line y = x

Let us consider a function $f(x)$ and its inverse $f^{-1}(x)$.

By definition, a function and its inverse satisfy:

$f(f^{-1}(x)) = x$ and $f^{-1}(f(x)) = x$

Now, graphically, if a point (a,b) lies on the graph of f, then the point (b,a) lies on the graph of f^{-1} .This reflection over the line y = x confirms that the graphs of a function and its inverse are symmetric with respect to that line.

Rigorous Proof of the Equivalence

$log_b y = x \Leftrightarrow y = b^x$

Prerequisites/Definitions:

We assume the existence and properties of the exponential function $f(x) = b^x \; for \; b > 0, b \neq 1$. Key properties include:

It's a continuous, strictly monotonic function (increasing if $(b > 1)$, decreasing if $(0 < b < 1)$. It maps $\Re \; to \; \Re^+ = (0, \infty)$.

It is therefore invertible.

Definition of Logarithm: The logarithm of y to the base b, denoted $log_b y$, is defined as the unique real number x such that $b^x = y$. In other words, $log_b y$ is the inverse function of b^x.

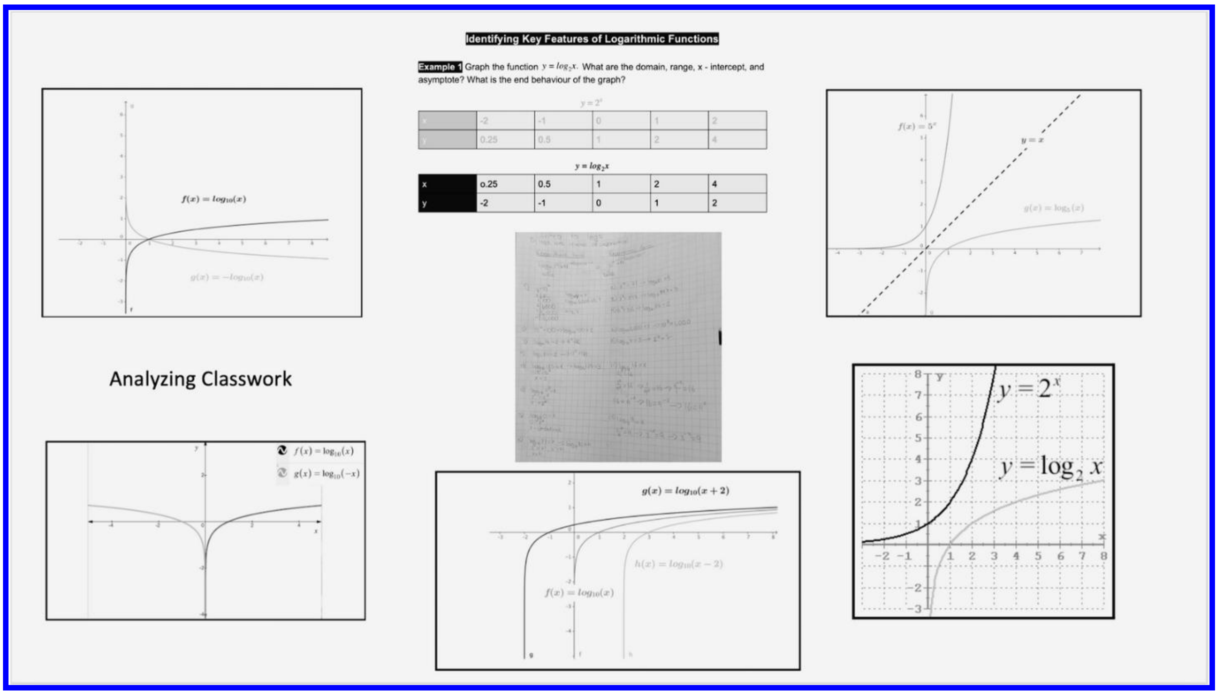

Figure 5. Sample of the Lesson Exploring Graphical Interpretation of Logarithms in Algebra Class (the document is from teachers' folder at the technology urban high school in New Jersey)

Formally, if $f(x) = b^x$, then $f^{-1}(y) = log_b y$.

Proof:

We need to prove the "if and only if" (⟺) statement, which means proving two implications:

1. Part 1: $If\ log_b y = x, then\ b^x = y.$
2. Part 2: $If\ b^x = y, then\ x = log_b y.$

Part 1: Prove that if $log_b y = x$, then $b^x = y$.

If $log_b y = x$, then by the definition of an inverse function, applying the inverse operation (the exponential function with base b) to both sides should yield the original input.

So, if $g(y) = log_b y\ and\ f(x) = b^x \ldots step\ (1)$

then $b^{log_b y} = y$ … Step (2)

According to conditions $log_b(y) = x$ substituting in step (2) on the left side.

Resulting in $b^x = y$ This is the proof of part 1.

Part 2: Prove that $if\ b^x = y, then\ x = log_b y.$

Given $b^x = y$ / take both sides log_b

So, if $log_b(b^x) = log_b y \ldots step\ (1)$

then $x = log_b y$ … Step (2) This is the proof of part 2.

Since we have proven both directions ($log_b y = x \Rightarrow b^x = y\ and\ b^x = y \Rightarrow log_b y = x$), we can conclude that:

$log_b y = x \Leftrightarrow b^x = y$

This proof is rigorous because it relies solely on the fundamental definition of the logarithm as the inverse of the exponential function, rather than assuming the equivalence it aims to prove. The proof involves very abstract thinking in algebra; however, it satisfies all conditions of algebraic thinking (refer to the key characteristics of algebraic thinking). This derivation was introduced after students had gained some experience with solving mathematical problems involving logarithms and felt more comfortable dealing with abstract mathematical concepts.

Document Analysis - Student Work

Usually, the introduction of a new chapter begins with easy examples involving two- to three-step mathematical problems, gradually increasing in number and complexity, and becoming more abstract. On the first day, the lesson introduces big ideas in a simple form, followed by easy examples and a few assigned as homework to reinforce students' learning.

On the second day, instruction continues with medium to difficult examples. Teachers need to ensure that students have mastered the concrete application before allowing them to move on to the semi-concrete or abstract phases (IRIS Center, n.d.).

On the third day, instruction includes word problems related to real-world examples. On the fourth day, students work in groups to collaborate and prepare for the exam. Homework is due on Friday, before students take the exam.

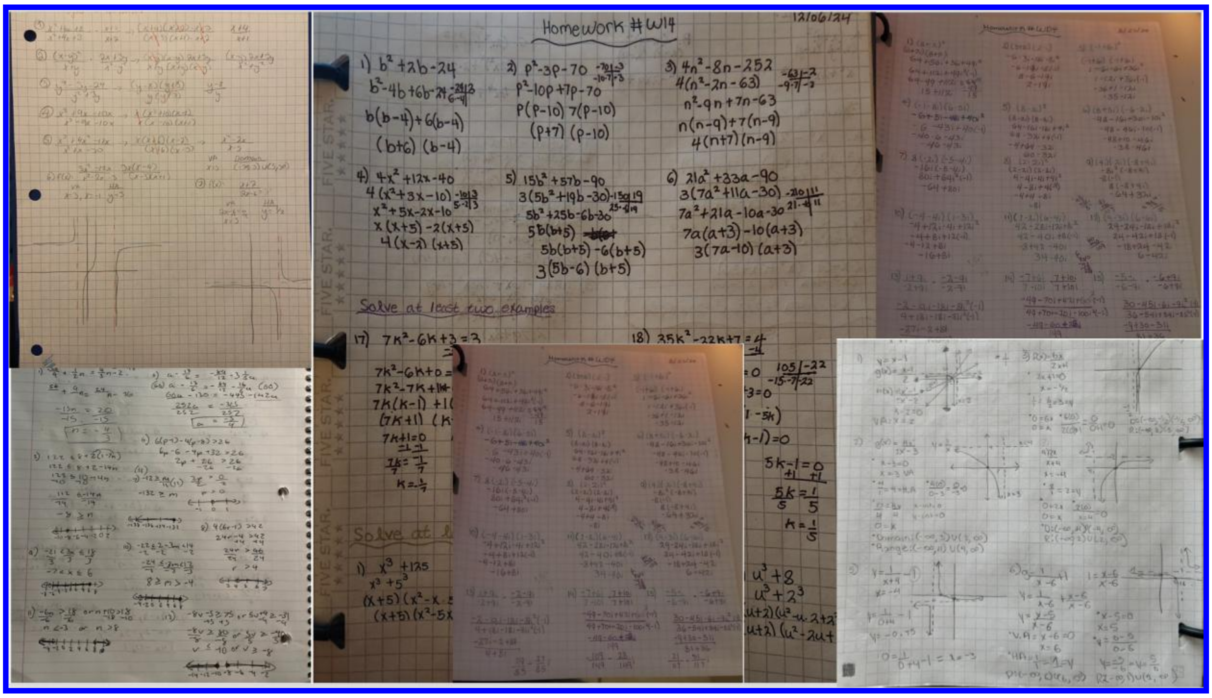

Figure 6. Analyzing Students' Homework of Algebra 2 in Different Algebraic Topics (the document is from teachers' folder at the technology urban high school in New Jersey)

Typically, exams are administered on Fridays so that the teacher can assess student understanding and reflect on future instruction. Every week, the teacher assigns homework. After each lesson, students receive a short assignment (2 to 3 examples), and on Wednesdays, they are given a longer homework assignment ranging from 10 to 20 examples, depending on its complexity. Homework is differentiated based on student needs. For instance, advanced students receive longer and more detailed assignments, while some students focus primarily on completing the problems. Students with inclusion needs are assigned fewer problems. However, all students engage in algebraic thinking to some extent. For example, factoring polynomials demonstrates algebraic thinking in action, as it builds and tests students' ability to reason symbolically and structurally. Interpreting functions in homework allows students to explore relationships through analytical examples and visual representations.

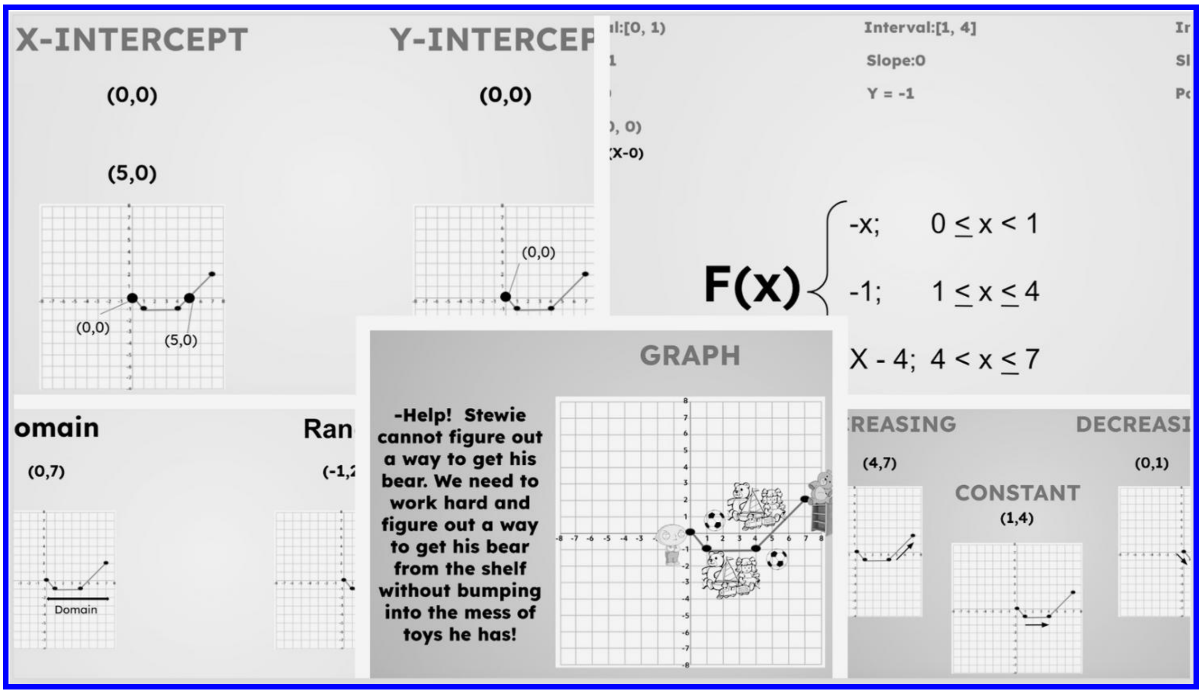

Figure 7. A Student's Project taken from Google Slides at the Technology High School New Jersey (the document is from teachers' folder at the technology urban high school in New Jersey)

According to Wilkie (2016b), teachers can support students in learning about functions and developing functional thinking by providing contextual problems. The study also reports that instruction on functions often emphasizes the operational rather than the structural conception of a function. Above all, students who performed well in algebra (factoring, functional thinking, generalizing, abstracting, reasoning, finding patterns) during grades 9 and 10 tended to do significantly better in later courses such as pre-calculus, calculus, and statistics.

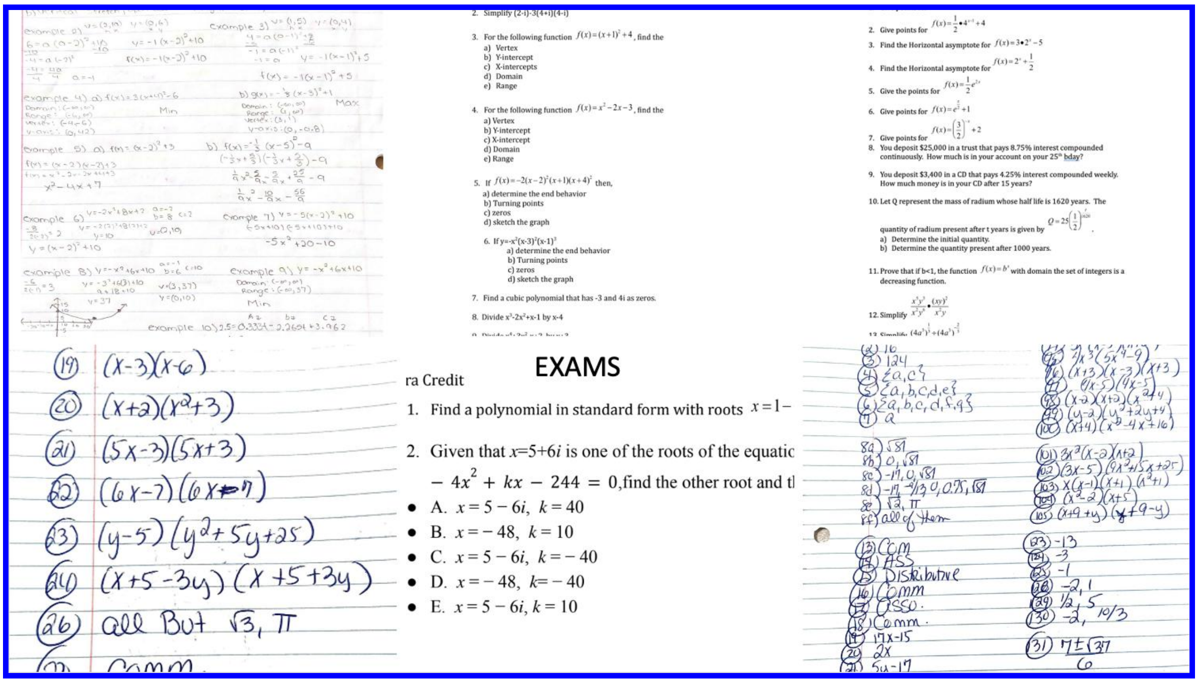

Figure 8. Several Exams that Describe Exams in Paper and Students Practice on Study Guides (the document is from teachers' folder at the technology urban high school in New Jersey)

In informal conversations with teachers, researchers asked whether homework has any influence on students' grades. The algebra teacher claimed, "In general, students who have good grades (above 90%) are very consistent with homework, and they complete all of it." Although the teacher did not conduct a formal analysis, they observed an apparent correlation between homework and students' grades based on intuition.

In the project presented in Figure 7, students solved a word problem that asked them to help Stewie (a cartoon character) find a path without bumping into his toys. Stewie's path is represented using linear and piecewise functions. Students used logical reasoning to identify positive, negative, and constant segments in Stewie's pathway, as well as to determine the increasing and decreasing intervals of the functions. *Explore a Real Cartoon Example* incorporates visually engaging figures, which help students as they search for answers or solutions to the given questions (Pllana, 2021). It is worth noting, however, that each individual project was expected to be a wholly distinctive product of creative endeavor, never seen before (Baez et al., 2024).

By solving mathematical problems—such as identifying x- and y-intercepts and determining the domain and range of piecewise functions—students engaged in functional thinking. More importantly, translating word problems and identifying the relationship between the narrative and the corresponding mathematical statements required active student involvement and encouraged mathematical thinking. Mathematical thinking involves the application of mathematical tools to solve real-world problems (Pllana, 2024).

Through this process, students developed skills for solving multi-step mathematical problems, as the algebraic project required the integration of several mathematical concepts into a single cohesive task. Additionally, students gained introductory experience in conducting research-related work.

Figure 8 shows few algebraic exams and few students practice on exam study guides. Some questions are multiple choice and others open ended questions. There are questions that ask students to solve mathematical problems, and through those questions students need to apply mathematical concepts. For example, the complex numbers example to find the other solution asks students to apply the concept that a complex solution might come as a conjugate complex pair solution.

This set of questions (see Figure 9) supports algebraic thinking, particularly in vocabulary, conceptual understanding, and the use of theorems. To make it even more effective, improve question clarity, encourage explanation, and add connections to real-world problems or graphs. Overall, this set of questions enables students to better understand mathematical concepts and perform more effectively on quizzes.

Exploratory and Interpretive

The goal is to understand meanings, perspectives, and experiences—for example, how students perceive a certain teaching method or how teachers manage classroom dynamics. The teacher in the algebra class taught several specific algebraic strategies. Gloag et al. (2025) claim that there are many methods for solving algebraic equations. Some students perceived these techniques positively and applied them in their learning. When the teacher asked students to solve mathematical problems on the board (a technological screen—Smart Board), some used slightly different methods they had learned in previous classes. The teacher encouraged them to use their preferred method if it led to the correct answer; otherwise, the teacher asked them to use a valid algebraic method that would produce the correct result.

H. Algebra 2 [redacted]

Questions for Class Work - Vocabulary Quiz

1. What is a monomial?
2. What is a binomial?
3. What is a polynomial?
4. What do we call a polynomial with the highest degree of 4?
5. What do we call a polynomial with the highest degree of 5?
6. What affects the leading coefficient and the even/odd degree?
7. If a function has an odd degree and a negative coefficient, what are the end behaviors? As x→∞what happens with f(x)?
8. If a function has an even degree and a negative coefficient, what are the end behaviors? As x→∞, what happens with f(x)?
9. Where are the turning points located on the graph of a function?
10. Is the maximum located on a graph that is concave up or concave down?
11. Is the minimum located on a graph that is concave up or concave down?
12. What are other names for the x-intercepts?
13. What does the y-intercept represent?
14. What do we call the roots of (x−a)^3 and (x−a)^4?
15. Show the difference between even and odd multiplicities.
16. What happens on the graph at the turning points?
17. At which points does the graph of the function change from positive to negative and vice versa?
18. What does the Remainder Theorem tell us about?
19. What does the Factor Theorem tell us?
20. What is Pascal's Triangle?
21. What numerical values correspond to the leading coefficient and constant in Pascal's Triangle?
22. What is the Binomial Theorem? (Answer: Expanding binomials into a simplified form.)
23. What is the Zero Factor Property?

Remark: Study the above vocabulary questions carefully for Friday Exam.

Figure 9. Study Guide for Students on Polynomial Equations - Study Math Vocabulary (the document is from teachers' folder at the technology urban high school in New Jersey)

Small Samples

Unlike large-scale surveys, qualitative studies often involve fewer participants, allowing for in-depth exploration. The study primarily took place in an Algebra 2 class, where the researchers had access to explore homework, classwork, projects, study guide samples, exams, and classroom sessions. In addition, they also observed, to some extent, other teachers and attended Professional Learning Community (PLC) meetings.

Participants

Participants in the study included students and teachers from 9th and 10th grades at a technology high school in New Jersey. The intensive group consisted of 10th-grade students enrolled in Algebra 2, while the extensive group included 9th-grade students enrolled in Algebra 1. The student demographics represented a diverse group, primarily from urban areas.

Data Collection

Data collection focused on classroom observations and student work. Classroom observations took place during sessions that included teachers' instruction on problem-solving, examples of algebra, group work as students prepared for exams, and group work on mathematical projects. In addition, the review of student work included classwork, homework, study guides for exams, and projects. The documents used for data collection are visually presented in Figures 2 through 9.

Analysis

The analysis focused on how teachers and students demonstrated algebraic thinking in problem-solving tasks, identified common misconceptions in students' algebraic reasoning and problem-solving, and examined classroom practices that promote deeper algebraic understanding. The analysis centered on the manifestation of key components of algebraic thinking through both procedural and conceptual approaches. Additionally, the study explored simple proofs in algebra and the derivation of mathematical formulas, which serve as foundational elements across all branches of mathematics.

Findings

The findings presented in the fourth section of the paper summarize major themes observed in examples from classwork, homework, projects, and exams, some of which are illustrated in figures. The paper identifies student misconceptions in understanding algebra and, more broadly, algebraic thinking. The transition in thinking progresses smoothly from arithmetic thinking to algebraic thinking, and then from algebraic thinking to mathematical thinking. This case study aims to generalize results to broader situations (Baez et al., 2025).

Summary of Major Themes Observed

The teacher in the algebra class presents examples ranging from easy to difficult. While all these examples serve as a foundation in mathematics, higher-order thinking questions provide a stronger basis for helping students master high school mathematics. The example in Figure 10 incorporates algebraic thinking and supports students in developing a deeper understanding of advanced mathematical concepts.

H. Algebra 2 [redacted]

6) Given that L *and* L^2 are the roots of the equation $4x^2 + bx + 32 = 0$,find the value of b.

Since we know solutions L and , we know the factors $(x - L)(x - L^2)$
Then we equate two quadratic equations as follows:

$$(x - L)(x - L^2) = 4x^2 + bx + 32$$

Let foil the first equation on the left side.

$$x^2 - Lx - L^2x + L^3 = 4x^2 + bx + 32$$

In order to equate the left equation with the right equation, we need to adjust the leading coefficient with 4.

$$x^2 - Lx - L^2x + L^3 / \textit{multiply by } 4 \Rightarrow 4x^2 - 4x(L + L^2) + 4L^3$$

Then we will equate the two equations together:

$$4x^2 - 4x(L + L^2) + 4L^3 = 4x^2 + bx + 32$$

Then, we will equate term by term from the highest degree to the lowest one.

H. Algebra 2 [redacted]

$$4x^2 = 4x^2 \qquad - 4x(L + L^2) = bx \qquad 4L^3 = 32 \ ...*$$

Let's start from the last term:

$$4L^3 = 32 \ / \textit{divided by } 4 \Rightarrow L^3 = 8 / \textit{take cubic root} \Rightarrow L = 2$$

So we found the numerical value of L.

$$L = 2$$

The other root is L^2, and it yields,

$$L^2 = 4$$

Now we can find the numerical value of b from the equation *.

$$- 4x(L + L^2) = bx \ / \textit{divided by } x \Rightarrow - 4(L + L^2) = b$$

Substitute L with numerical values then $- 4(2 + 4) = b$yields:

$$b =- 24$$

Figure 10. An Example of Higher Order Thinking (HOT) in Algebra Explored in the Algebra 2 Classroom (the document is from teachers' folder at the technology urban high school in New Jersey)

This is a strong example of a well-structured higher-order thinking (HOT) question in algebra, as shown in Figure 10, which supports students in mastering high school mathematics. Teaching HOTS is not only effective in improving students' academic performance but also in addressing their weaknesses (Heong et al., 2019; Sa'dijah et al., 2021). Higher-order thinking questions in algebra manifest algebraic thinking and prepare students to engage with more advanced mathematics in future classes and beyond. Figure 10 illustrates in detail the effectiveness of HOT questions in fostering algebraic thinking and enhancing mathematical understanding more broadly.

Algebraic Thinking in the Problem

Abstract Reasoning: Students are asked to work with variables and expressions (L and L^2) as roots, rather than just numbers. They must manipulate symbolic forms and relate factored and standard forms of quadratic expressions. Structure Recognition: The problem requires identifying and using the structure of a quadratic equation with specific roots (L and L^2). Students translate between the factored form and expanded standard form of a polynomial.

Connecting Representations

They go from a factored form (x−L)(x−L^2) to a standard form ax^2+bx+c, and compare coefficients. This helps them understand the relationship between roots and coefficients in a quadratic equation. The quadratic expression has roots at $x = L$ and $x = L^2$, meaning the graph of the quadratic (a parabola) will intersect the x-axis at those two points. The variable L could be any number (real or complex), and it controls the location of the roots.

Use of Algebraic Operations

Students perform algebraic operations such as FOIL, multiplying polynomials, combining like terms, and solving for variables. They are expected to equate cofficients across equivalent algebraic expressions. Applying algebraic operations such exponents rule in multiplication, zero property, and inverse operations to isolate unknown variables.

Higher-Order Thinking Skills Involved

Analysis: Students must break down expressions and identify patterns between roots and coefficients. Synthesis: They reconstruct a quadratic equation from given root relationships and compare it to a standard form. Problem Solving: The question requires multiple steps and use of different algebraic techniques in sequence. Justification: The method implicitly encourages students to justify why the steps (e.g., multiplying both sides by 4) are valid.

How this Helps with Advanced Mathematics

Prepares for Polynomial Function Analysis: Understanding how roots relate to coefficients is essential for studying polynomial functions in higher math. Builds Conceptual Understanding of Factoring and Expanding: These skills are foundational in calculus, linear algebra, and even abstract algebra. Encourages Symbolic Manipulation: Manipulating expressions with symbols rather than specific numbers builds mathematical maturity.

Examples of Teachers and Student Work Showing Varied Levels of Algebraic Understanding

Finite Geometric Series

Consider the following series, - ***Remark*- Formula of geometric sequence*** $S_n = a_1 r^{n-1}$

$S = a_1 + a_2 + a_3 + \ldots a_n \ldots$ (1)

Multiply the equation (1) replace $a_2 = a_1 r,\ a_3 = a_1 r^2,\ a_4 = a_1 r^3 \ldots$ *and so on*, then we obtain the equation (2).

$S = a_1 + a_1 r + a_1 r^2 + a_1 r^3 \ldots a_1 r^{n-1} \ldots$ (2)

Let multiply equation (2) with r both sides, and we will get the equation (3),

$S_n = a_1 r + a_1^2 r + a_1 r^3 + a_1 r^4 \ldots a_1 r^n \ldots$ (3)

Subtract equation (2) by equation (3) then we will get equation (4).

$S - S_n = a_1 + a_1 r + a_1 r^2 + a_1 r^3 \ldots a_1 r^{n-1} - (a_1 r + a_1^2 r + a_1 r^3 + a_1 r^4 \ldots a_1 r^n)$

H. Algebra 2 [redacted]

Substitute $S_n = Sr$ and after some algebraic work it yields,

$S - S\,r = a_1 - a_1 r \ldots$ (4)

Factorize S and divide by the GCF it follows,

$S = \frac{a_1 - a_1 r^n}{1 - r}$ ....(5) For $|r| > 1$

<u>*If*</u> $n \to \infty$ *for* $|r| < 1$ *then, Infinite Geometric Series*

$$S_n = \frac{a_1 - a_1 r^n}{1 - r}$$

$$S_\infty = \frac{a_1}{1 - r} - \frac{a_1 r^\infty}{1 - r}$$

As $n \to \infty$ the $\frac{a_1 r^\infty}{1 - r} \to 0$ *for* $|r| < 1$, *and* $r \neq 0$ *then,*

$S = \frac{a_1}{1 - r}$ ...(5)

Figure 11. Deriving Formulas for Infinite Geometric Series and Finite Geometric Series Demonstrated in Class (the document is from teachers' folder at the technology urban high school in New Jersey)

A significant number of the examples presented in the paper contribute meaningfully to the development of algebraic thinking. The example of the proof in Figure 11 demonstrates an algebraic equation that is part of Algebra 2 in high school. The teacher developed this proof collaboratively with their students. This derivation uses algebraic manipulation and logical reasoning, starting from a known definition (a geometric series) and concluding with a proven formula. Algebra is a branch of mathematics. According to Knuth et al. (2019), proving is central to the practice of mathematics and plays an important role in learning mathematics. In practice, the math teacher constructed several proofs during algebra class with students. Most of these proofs positively influenced students' algebraic thinking and enhanced their mathematical skills.

The derivations in Figure 11 and Figure 12 are considered informal proofs because they justify the validity of the equations. Typically, deriving formulas or equations is a function of algebraic thinking, as it involves manipulating and understanding algebraic components. Formula derivation is closely related to mathematical problem-solving in advanced mathematics and other subjects within the STEM field.

Deriving Formula for Axis of Symmetry

Another method - Quadratic Formula:

$x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$ Then, $x_1 = \frac{-b+\sqrt{b^2-4ac}}{2a}$ and $x_2 = \frac{-b-\sqrt{b^2-4ac}}{2a}$

$\frac{x_1+x_2}{2}$

Axis of symmetry is the midpoint of the x- intercepts use $\frac{x_1+x_2}{2}$:

$\left(\frac{-b+\sqrt{b^2-4ac}}{2a} + \frac{-b-\sqrt{b^2-4ac}}{2a}\right) \div 2 \Rightarrow \left(\frac{-b+\sqrt{b^2-4ac}-b-\sqrt{b^2-4ac}}{2a} \cdot \frac{1}{2}\right)$

After we clean up the equation,

$-\frac{2b}{2a} \cdot \frac{1}{2} \Rightarrow -\frac{b}{2a}$ Thus

$x = -\frac{b}{2a}$ **= *h***

Compare the general (standard) form of quadratic function and vertex form, then generate the formula for the axis of symmetry.

$f(x) = ax^2 + bx + c$ can be written in the vertex form such as $g(x) = a(x-h)^2 + k$

$f(x) = ax^2 + bx + c$

$$\begin{aligned} g(x) &= a(x-h)^2 + k \\ &= a(x-h)(x-h) + k \\ &= a(x^2 - 2hx + h^2) + k \\ &= ax^2 - a2hx + ah^2 + k \end{aligned}$$

$f(x) = ax^2 + bx + c \Leftrightarrow g(x) = ax^2 - 2ahx + ah^2 + k$

$ax^2 = ax^2$ $\quad bx = -2ahx$ $\quad c = ah^2 + k$

$bx = -2ahx$ /divide by $-2ax \Rightarrow h = -\frac{b}{2a}$ $\quad h = -\frac{b}{2a} = x$

Figure 12. Deriving Formula for Axis of Symmetry for Quadratic Functions into Two Methods (the document is from teachers' folder at the technology urban high school in New Jersey)

The captivating world of mathematics includes many fascinating concepts, and one such concept is the *axis of symmetry* (Caballes, 2024). In algebra, the axis of symmetry is a vertical line that divides the graph of a parabola into two equal parts. This axis has a specific formula that determines its position in quadratic equations. There are several ways to derive or justify this formula, and Figure 12 illustrates two such methods. These methods require the application of fundamental algebraic principles, including:

- Manipulating expressions and equations
- Combining and simplifying terms
- Solving for a variable
- Recognizing and applying algebraic properties (such as the distributive property)
- Understanding the relationships between different forms of an equation

These components reflect core elements of algebra and are clearly present in the proof, aligning with the concept of algebraic thinking. As mentioned in the previous paragraph, algebraic thinking in this context significantly influences students' learning in advanced mathematics.

Notable Misconceptions or Problem-Solving Behaviors

Analyzing assignments reveals the misconceptions or errors that students make during their work. The best opportunity for teachers to identify these mistakes occurs at the end of the week, when students take their exams. Understanding the errors made in tests is crucial, as they reveal underlying misconceptions (Stemele & Jina, 2024). The common mistakes students make are described in Table 2.

Based on our analysis, several student errors were observed, particularly during the early introduction of topics. Pournara et al. (2016) highlighted persistent errors in multiplication, exponents, letter evaluation, and mastering the basics of exponents among learners across various schools. Over time, as students engage more deeply with the material, these mistakes tend to decrease. However, persistent errors in components of algebraic thinking include difficulties in understanding structure (such as the distributive property, the multiplication property of equality, and the addition property of equality), as well as in using symbols and representations. Advanced students typically avoid these basic errors but may still struggle with aspects of functional thinking, generalization, and abstraction.

Table 2. Several Common Mistakes that Students Make in Solving Algebraic Equations or Inequalities

Algebraic Properties	Mistakes	Correct
Distributive Property	3(x + 4) = 3x + 4	3x + 12
Signs Errors (positive/negative)	-(6x + 1) = -6x +1	-(6x + 1) = -6x -6
Combining Like Terms Incorrectly	3x + 2 = 5x	3x + 2 different from 5x
Mistakes Solving Equations	x + 2 = 8 ⇒ x = 8	x + 2 = 8 ⇒ x = 6
Incorrect Use of Exponents	$(x^4)^3 = x^7$	$(x^4)^3 = x^{12}$

Algebraic Properties	Mistakes	Correct
Cross Multiplication Errors	$\frac{a}{b} = \frac{c}{d} \Rightarrow a = c \text{ and } b = d$	$\frac{a}{b} = \frac{c}{d} \Rightarrow ad = bc$
Forgetting to Flip Inequality When Multiplying by Negative Sign	$-4x < 12 \Rightarrow x < -3$	Flip the inequality $x > -3$

(Source: Several mistakes based on the homework and exams at the technology urban high school in New Jersey)

Students often make mistakes when applying mathematical concepts across multiple steps, and sometimes they lack mastery of the content knowledge. According to LibreTexts (n.d.), content knowledge involves the ability to reason mathematically and make logical connections between different mathematical ideas.

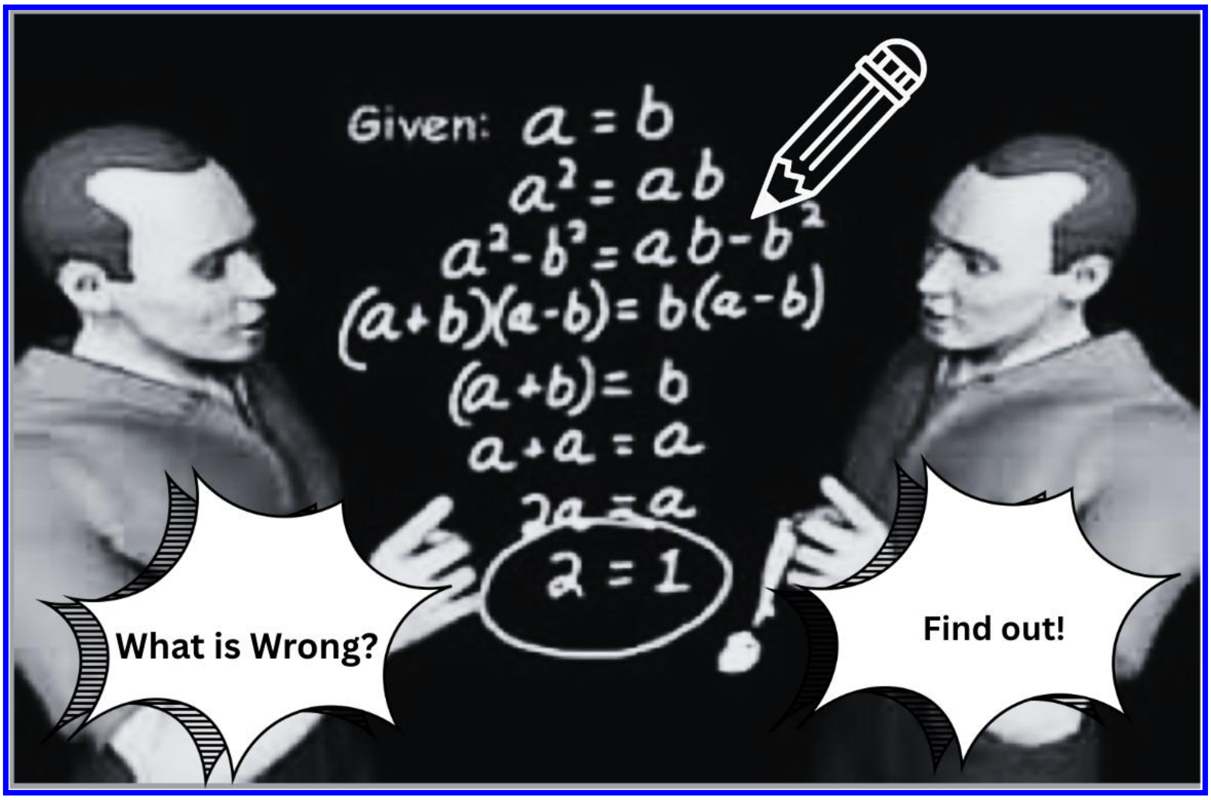

Figure 13. A Challenging Error in the Equation that Twins Do Not Understand their Mistakes

In the example shown in Figure 13, the solution may appear correct at first glance; however, the result—showing that 2 is not equal to 1—proves that the equation does not hold. In this type of question, the teacher did not provide students with the answer immediately but instead gave them time to research and think more deeply about the problem.

Insights into How Students Shift from Arithmetic to Algebraic Thinking

The shift from arithmetic to algebraic thinking is a crucial transition in students' mathematical development. It marks a move from concrete, number-based operations to abstract, generalized reasoning. In order to teach algebra, first of all, transition from arithmetic to algebra should be healthy, and misconceptions should be determined well and prevented in this process (Töman and Gökburun, 2022). As a misconception of students shift from arithmetic to algebraic thinking occurs misconception of variables when students might think variables have a fixed value, students might try to solve algebraic equation with arithmetic procedure, and arithmetic strategies lack the possibility to find patterns or develop general rules. Post, Behr, and Lesh (1988) indicated that "first-describing-and-then-calculating" is one of the key features that make algebra different from arithmetic. Comparisons between the arithmetic and algebraic approaches can highlight this unique feature.

Transitioning from Arithmetic to Algebraic Thinking Involves

1. Shifting from specific numbers to generalized representations. For example, the specific sequence 1, 3, 5, 7, … can be generalized by the expression $2n - 1$, where n is a natural number. Another example: the arithmetic statement $1 + 5 = 6$ (a specific case) can be generalized algebraically to describe the commutative property as $a + b = b + a$.

2. Understanding the equal sign as a balance, not just an operator. In arithmetic, the equal sign is often interpreted to mean "the answer comes next." In algebra, it signifies that two expressions are equivalent.

3. Moving from procedural to relational understanding. Algebra emphasizes understanding how quantities relate to one another, rather than simply computing answers as in arithmetic.

4. Supporting this transition through targeted instructional strategies and early exposure to algebraic ideas.This includes using patterns, visual models, and opportunities for generalization to help students develop algebraic thinking.

Discussion

Discussion will revolve around four sections: comparing the observation and students' work results with literature in journal articles; gaining insights into teaching methods and students' work in algebra, and how they incorporate algebraic thinking; classroom dynamics and student engagement; and the relationship between arithmetic and algebraic thinking, and respectively between algebraic and mathematical thinking. The discussion briefly explores all of these four sections.

Comparison with Literature

Simply put, algebraic thinking involves generalizing, representing, reasoning, and justifying—not just manipulating symbols. Teachers nurture it through thoughtful questioning and tasks, while students demonstrate it by exploring, explaining, and generalizing mathematical ideas. The Algebra Standard emphasizes relationships among quantities, including functions, ways of representing mathematical relationships, and the analysis of change (Kieran, 2004). In Figure 7, the students' project on piecewise functions illustrates a representation of quantities using variables expressed in different forms, such as mathematical equations.

Indeed, as the German philosopher Immanuel Kant suggested in the 18th century, unknowns, variables, and other algebraic objects can only be represented indirectly—through constructions based on signs (see Kant, 1929, p. 579; Radford, 2006). The analytical interpretation focuses on intervals where the function is increasing or decreasing, as well as identifying positive and negative values, along with the domain and range. The graphical interpretation represents the cartoon character's pathway using linear functions with restricted domains and ranges.

In the Algebra 2 class, the teacher and students explored examples involving two or more variables, such as the one shown in Figure 14. Among other concepts and skills recommended by the National Mathematics Panel (2008), multiple variables and parameters become essential when dealing with more advanced algebraic concepts. These advanced topics engage students in algebraic thinking by incorporating a variety of concepts and skills.
Researchers observed many examples related to complex topics in the algebra class. In

addition to problems involving two variables, students engaged with several formal and informal proofs. In fact, some researchers have pointed out that proofs are more than tools to verify the truth of a mathematical statement (Martinez, 2011). By proving algebraic statements or equations—such as those in Figures 5, 10, and 11—students gain valuable experience in mathematical reasoning.

Given that x=5+6i is one of the roots of the equation $-4x^2 + kx - 244 = 0$,find the other root and the value of k.

- A. $x = 5 - 6i,\ k = 40$
- B. $x = -48,\ k = 10$
- C. $x = 5 - 6i,\ k = -40$
- D. $x = -48,\ k = -40$
- E. $x = 5 - 6i, k = 10$

Solution

We know the complex solutions of quadratic equations are always pair conjugate, so the other solution is $x = 5 - 6i$

$x = 5 + 6i \text{ and } x = 5 - 6i \Rightarrow x - 5 - 6i = 0 \text{ and } x - 5 + 6i = 0$

$(x - 5 - 6i)(x - 5 + 6i) = 0$

$x^2 - 5x + 6i - 5x + 25 - 30i + 6ix + 30i - 36i^2 = 0$ / combine like terms

$x^2 - 10x + 25 - 36(-1) \Rightarrow x^2 - 10x + 25 + 36 \Rightarrow x^2 - 10x + 61 = 0$

$x^2 - 10x + 61 = 0$ *compare with the original quadratic equation* $-4x^2 + kx - 144 =$

H. Algebra 2 [redacted]

Let

$x^2 - 10x + 61 = 0$ *multiply by* -4 *and we will obtain the following equation*

$-4x^2 + 40x - 244 = 0$

compare with the original quadratic equation $-4x^2 + kx - 24$

Equate both equations

$-4x^2 + 40x - 244 = -4x^2 + kx - 244$ *Simplify further.*

$40x = kx$ *divide by x then* $k = 40$

So the answer is $x = 5 - 6i$ *and* $k = 40$

The correct answer is A

Figure 14. An Example with Two Variables (the document is from teachers' folder at the technology urban high school in New Jersey)

As students gain more experience, their understanding of what constitutes productive mathematical thinking is refined (Levin & Walkoe, 2022). In practice, engaging with complex examples and proofs in algebra naturally fosters algebraic thinking. Students who apply algebraic thinking in their algebra classes are on the right path to mastering high school mathematics.

Insights on how Teaching Methods Influence Algebraic Thinking

In general, teachers' interpretation of instructions plays a significant role in students' comprehension of lessons. The ideas a teacher focuses on, and how they interpret them, have important implications for classroom instruction (Luna & Sherin, 2017). The algebra teacher's teaching style begins with introducing big ideas and general concepts at a superficial level. Then, the teacher starts with simple examples, giving every student an opportunity to understand how to solve algebraic problems. The instruction continues with medium-level examples, and eventually, more complex problems are introduced, allowing students to explore algebraic concepts in greater depth.

By the end of each unit, real-world examples are used to apply previously learned algebraic concepts, helping students engage in algebraic thinking. In practice, both the teacher and students utilize algebraic thinking in the teaching and learning process. Algebraic thinking involves using algebraic strategies to solve complex problems and prove equations and statements, which encourages students to tackle more advanced mathematical challenges.

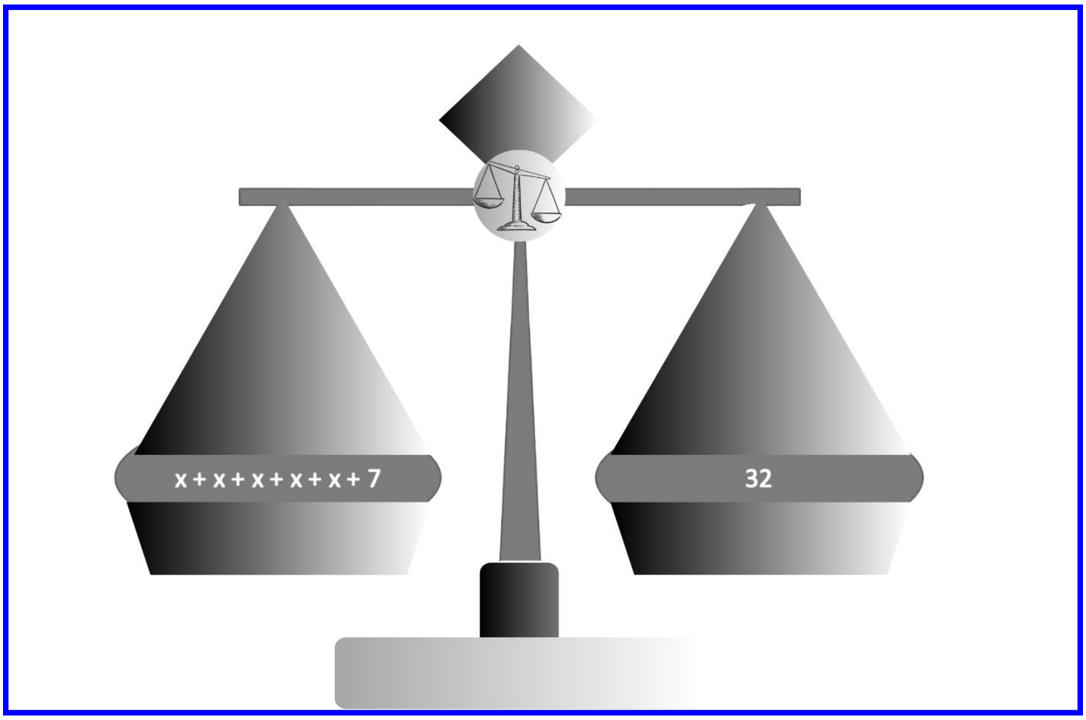

Figure 15. Modeling Algebraic Equations with Scale

Since the class consisted of diverse students with varying levels of mathematical ability, the teacher's instruction included a range of examples from easy to difficult. During the first three days, the teacher explained algebraic problems and concepts. On the fourth day, a student-centered teaching session took place, during which students worked in groups to prepare for the exam at the end of the week. These student-centered sessions occurred when students were preparing for quizzes, tests, or projects, providing them with opportunities to share their knowledge of algebra with one another.

The Difference between Algebra and Algebraic Thinking

In high school, algebraic thinking is closely related to algebra because as algebra becomes more sophisticated, algebraic thinking also advances. Algebra involves mathematical symbols and the rules for using them—in other words, algebra is a tool. In contrast, algebraic thinking is a way to reason with these rules and make sense of them. It helps students apply algebra in different contexts.

In its broadest sense, algebraic reasoning is about generalizing mathematical ideas and identifying mathematical structures (Ontario Ministry of Education, 2015). Algebraic thinking lays the foundation, while algebraic reasoning builds on it with more formal mathematical logic.

In this section, we are interested in exploring the difference between algebraic thinking and algebra, as shown in the table 3.

Table 3. The Difference of the Algebraic Thinking and Algebra in High School

Difference at the High School Level		
Aspect	Algebra (High School)	Algebraic Thinking (High School)
What it looks like	Solving complex equations and functions	Analyzing, modeling, reasoning, and generalizing
Example	Solve: $3x^2 - 3x + 1$	What happened to the graph of $f(x) = x^2$
Skill focus	Computation and symbolic manipulation	Pattern recognition and logical reasoning

(Source: Modification from ChatGPT)

Classroom Dynamics and Student Engagement

Some simple definitions of classroom dynamics and student engagement might include classroom dynamics refer to the interpersonal relationships, communication patterns, power structures, and overall atmosphere within a classroom, while student engagement is the level of interest, curiosity, and involvement that students show in the learning process.

In the algebra classes observed, the classroom dynamics between teachers and students were mutually respectful: the teacher respected the students, and the students respected the teacher. The teacher encouraged students to respond to open-ended questions used in formative assessment. They explained that there is no such thing as a "bad answer" when asking questions related to the lesson and encouraged students to either answer confidently or make educated guesses.

When students gave incorrect answers, the teacher did not judge or criticize them. Instead, they highlighted something positive from the response and praised the effort. After teaching two or three examples, the teacher asked students to solve similar problems on the board. While some students went up more frequently than others, the teacher aimed to give everyone an approximately equal opportunity.

If the teacher believed a problem was relatively easy, they would call on students who were struggling, giving them a chance to succeed and build confidence. In contrast, for medium or difficult problems, the teacher selected students based on their mathematical ability. Like in higher order thinking questions. Lama et al. (2024) writes Higher order thinking is beneficial for advanced students, while intermediate and struggling students typically use lower order thinking skills. When students encountered difficulties while solving problems at the board, the teacher provided assistance as needed, ensuring that all students felt comfortable participating.

Relationship between Arithmetic, Algebraic, and Mathematical Thinking

Based on the results, there is a close relationship between arithmetic thinking and algebraic thinking, with a smooth transition from one to the other. The teacher simplified algebraic problems into arithmetic ones to make them easier and more understandable for students. For example, when teaching how to multiply, divide, add, and subtract rational expressions, the teacher drew comparisons to similar operations with fractions. In other words, instruction on operations with algebraic expressions began with a review of fractions, helping students connect familiar concepts to rational expressions. Another example involved verifying systems of linear equations with two variables (e.g., x and y) by substituting specific numerical values. Sometimes, using only one method—either arithmetic or algebraic—may not lead to a solution. For instance, solving an equation of the form $ax + b = cx + d$ can be more tedious using only arithmetic methods (Filloy & Rojano, 1989; Pitta-Pantazi et al., 2025). Overall, connecting arithmetic and algebraic thinking allows for a fluid transition between the two, supporting students in solving algebraic problems more effectively.

The transition from arithmetic thinking to algebraic thinking in algebra classes typically occurred at the beginning of a chapter during the introduction. Usually, the teacher started with arithmetic examples that high school students were already familiar with and connected them to upcoming algebraic topics.

In the middle of the chapter, students deepened their understanding of algebraic thinking by exploring various algebraic examples and real-world applications. By the end of the chapter, the teacher introduced more advanced examples, including proving equations by deriving them and applying algebraic axioms.

Table 4. Key Aspects of the Difference between Arithmetic Thinking and Algebraic Thinking

Aspect	Arithmetic Thinking	Algebraic Thinking
Nature	Concrete	Abstract
Main Options	Basic operations with known numbers	Use of variables and expressions
Focus	Getting answers	Understanding relationships
Thinking Style	Procedural	Relational and generalization
Typical Problems	What is 12 - 4 ?	What number x makes $4x = 12$?

(Source: Modification from ChatGPT)

Table 5. Key Aspects of the Difference between Algebraic Thinking and Mathematical Thinking

Aspect	Algebraic Thinking	Mathematical Thinking
Scope	Narrow (subset of mathematics)	Broad (across all math areas)
Focus	Symbols, variables, functions, …	Reasoning, problem solving, abstraction
Skills Used	Pattern recognition generalization	Logical argumentation, strategy, proof
Curriculum Level	Mostly middle/high school	All levels of mathematics

(Source: Modification from ChatGPT)

Pllana et al. (2024) claim that analyzing the Spiral of Theodorus is primarily a geometric and mathematical problem that touches several areas of mathematics. Expanding the perimeter of the spiral using the Pythagorean Theorem begins with arithmetic, and generalizing the equation involves algebra. For example, it starts with an arithmetic expression: $P_{th} = 1 + \sqrt{2} + \sqrt{3} + \ldots + \sqrt{17}$ and then becomes generalized using algebraic thinking: $P_{th} = 1 + \sum_{n=1}^{17} \blacksquare \sqrt{n}$. In fact, analyzing the Spiral of Theodorus integrates multiple mathematical domains, including geometry, arithmetic, and algebra.

Proving algebraic statements, solving advanced algebraic problems, and working on real-world projects marked a smooth transition from algebraic thinking to mathematical thinking. Through this process of building mathematical knowledge—from the known to the unknown—algebra instruction effectively guided students from arithmetic thinking to algebraic thinking, and ultimately to mathematical thinking.

Conclusion

Instructions in algebra classes imply components of algebraic thinking based on the complexity of the algebraic problems, as reflected in the lesson plans. When teachers present simple examples, students have the opportunity to apply algebraic axioms such as the multiplication property of equality, addition property of equality, identity property of multiplication, and identity property of addition. As the problems become more complex, students begin to use combinations of these components of algebraic thinking.

There is a positive correlation between solving algebraic problems and the development of algebraic thinking: as the complexity of problems increases, students' algebraic thinking also advances. When instruction incorporates high-level analysis or the demonstration of proofs, especially in high school settings, it reflects elements of advanced mathematics.

When students encounter a new topic for the first time, they often cannot absorb all the information immediately. As a result, misconceptions are common at the beginning. Students who engage in more practice with algebraic problems tend to have fewer misconceptions, while those who practice less struggle more. However, even advanced students are challenged when multiple concepts are combined in a single problem or proof, such as in the example shown in Figure 13.

This paper contributes to mathematics education by supporting the idea that algebra is a fundamental branch of mathematics. Demonstrating more complex algebraic topics in class reveals more components of advanced mathematics. At the same time, components of algebraic thinking prepare high school students for higher-level math. Consequently, students may master high school mathematics more quickly.

The study has limitations, as it relies solely on classroom observations and student documents—such as homework, classwork, exams, and projects—to analyze the relationship between algebraic thinking and advanced mathematics. It does not include all elements of qualitative methodology due to the need to protect the privacy of staff and students. As a result, the study's scope is limited.

Future research and practice should expand this work by incorporating quantitative methods. Empirical approaches could enrich the study through additional documents and data, including interviews with mathematics teachers and students, focus groups, and statistical analysis. There is significant potential to develop this research further by exploring the topic from different perspectives.

References

Anthro_Poetry. (2022, June 5). *What is a definition? Medium*. https://medium.com/@anthro_poetry/what-is-a-definition-cb6c22364ac9

Baez, R., Sanchez, H., & Pllana, D. (2024). *Creativity in high school through drawing with polynomials*. *Advances In Social Sciences Research Journal, 11*(1), 318–332. https://doi.org/10.14738/assrj.111.16349

Baez, R., Sanchez, H., Sandeep, N., & Pllana, D. (2025). *Creativity and critical thinking in the AI age: A high School perspective*. *International Journal of Research in Education and Science (IJRES)*, *11*(2), 358–387. https://doi.org/10.46328/ijres.3656

Battista, M. T., & Brown, C. A. (1998). Using spreadsheets to promote algebraic thinking. *Teaching Children Mathematics, 4*(5), 470–478.

Caballes, J. (2024, January 3). *Axis of symmetry – Definition, formula, solved examples*. Brighterly. https://brighterly.com/math/axis-of-symmetry/

Devlin, P. (n.d.). *Finding the inverse of an exponential or logarithmic function* (16-week Lesson 32; 8-week Lesson 26). Purdue University. https://web.ics.purdue.edu/~pdevlin/Traditional%20Class/Lesson%2032/Finding%20the%20Inverse%20of%20an%20Exponential%20or%20Logarithmic%20Function.pdf

Digital Promise Global. (n.d.). *Algebraic thinking* [Learner Variability Project: Math 3-6 summary]. Learner Variability Navigator. Retrieved July 30, 2025, from https://lvp.digitalpromiseglobal.org/content-area/math-3-6/factors/algebraic-thinking-math-3-6/summary

Filloy, E., & Rojano, T. (1989). Solving equations: The transition from arithmetic to algebra. *For the Learning of Mathematics, 9*(2), 19–25.

http://www.jstor.org/stable/40247950

Gloag, A., Rawley, E., & CK-12 Editorial Team. (2025, August 1). *7.6 Comparing methods for solving linear systems*. In *CK-12 Algebra 1 Concepts* (FlexBooks 2.0). CK-12 Foundation.

https://flexbooks.ck12.org/cbook/ck-12-algebra-concepts/section/7.6/primary/lesson/comparing-methods-for-solving-linear-systems-alg-i/

Greenes, C. & Findell, C. *Algebra Puzzles and Problems (Grade 7*). Mountain View, CA: Creative Publications, 1998.

Gross, J. M. S. (2018). Document analysis. In B. B. Frey (Ed.), *The SAGE encyclopedia of educational research, measurement, and evaluation* (pp. 545–548). SAGE Publications. https://doi.org/10.4135/9781506326139.n209

Herbert, K., & Brown, R. "Patterns as Tools for Algebraic Reasoning." *Teaching Children Mathematics* 3 (February 1997): 340-344.

Heong, Y. M., Ping, K. H., Yunos, J. M., Othman, W., Kiong, T. T., Mohamad, M. M., & Ching, K. B. (2019).

Effectiveness of integration of learning strategies and higher-order thinking skills for generating ideas among technical students. *Journal of Technical Education and Training, 11*(3), 32–42. https://doi.org/10.30880/jtet.2019.11.03.005

Ho, J. E. (2021). What counts? The critical role of qualitative data in teachers' decision making. *Evaluation and Program Planning, 87*, 102046. https://doi.org/10.1016/j.evalprogplan.2021.102046

Hurrell, D. P. (2021, February). *Conceptual knowledge OR procedural knowledge OR conceptual knowledge AND procedural knowledge: Why the conjunction is important* *for teachers*. *Australian Journal of Teacher Education, 46*(2), 57–71. https://doi.org/10.14221/ajte.2021v46n2.4

IRIS Center. (n.d.). *Algebra-level A: Conceptual understanding of operations*. Vanderbilt University, Peabody College. https://iris.peabody.vanderbilt.edu/wpcontent/uploads/pdf_case_studies/ics_alg1.pdf

Kant, I. (1965). *Critique of pure reason* (N. Kemp Smith, Trans.). St. Martin's Press. (Original work published 1781/1787).

Kieran, C. & Chalouh, L. "Prealgebra: the Transition from Arithmetic to Algebra". In *Research ideas for the Classroom: Middle Grades Mathematics* edited by Douglas T. Owens. Reston, VA: NCTM, 1993.

Kieran, C. (2004). Algebraic thinking in the early grades: What is it? *The Mathematics Educator, 8*(1), 139–151. Retrieved from https://www.researchgate.net/profile/Carolyn-Kieran 2/publication/228526202_Algebraic_thinking_in_the_early_grades_What_is_it/links/55895b8408ae2affe714d428/Algebraic-thinking-in-the-early-grades-What-is-it.pdf

Knuth, E., Zaslavsky, O., & Ellis, A. (2019). The role and use of examples in learning to prove. *The Journal of Mathematical Behavior, 53*, 256–262. https://doi.org/10.1016/j.jmathb.2017.06.002

Kortering, L. J., deBettencourt, L. U., & Braziel, P. M. (2005). Improving performance in high school algebra: What students with learning disabilities are saying. *Learning Disability Quarterly, 28*(3), 191–203. https://files.eric.ed.gov/fulltext/EJ725672.pdf

Kriegler, S. (n.d.a). *Just what is algebraic thinking?* WestEd. https://www.wested.org/online_pubs/just_thinking.pdf

Kriegler, S. (n.d.b). *Just what is algebraic thinking?* In *Algebraic concepts in the middle school: A special edition of Mathematics Teaching in the Middle School*. Retrieved July 30, 2025, from https://currikicdn.s3-us-west-2.amazonaws.com/resourcedocs/54d281a1d8bbd.pdf

Kostoulas, A., & Lämmerer, A. (2015). *Classroom-based research: Materials created for ELT Connect 2015*. University of Graz. https://achilleaskostoulas.com/wp-content/uploads/2015/12/classroom-based-research.pdf

Lama, A., Baez, R., Sanchez, H., & Pllana, D. (2024). *Creative approaches to teaching foreign languages: Case study and survey findings – Students of English Language Faculty at UBT*. In the UBT *International Conference*.

https://knowledgecenter.ubt-uni.net/conference/2024/education/18

Levin, M., & Walkoe, J. (2022). Seeds of algebraic thinking: A Knowledge in Pieces perspective on the development of algebraic thinking. *ZDM – Mathematics Education, 54*(6), 1303–1314. https://doi.org/10.1007/s11858-022-01374-2

LibreTexts. (n.d.). *1.2: Understanding mathematical content knowledge*. In *Math for Educators (MATH 010A and 010B CID120)*. Coalinga College. https://math.libretexts.org/Courses/Coalinga_College/Math_for_Educators_(MATH_010A_and_010B_CID120)/01%3A_Teaching_Elementary_Mathematics/1.02%3A_Understanding_Mathematical_Content_Knowledge

Luna, M. J., & Sherin, M. G. (2017). Using a video club design to promote teacher attention to students' ideas in science. *Teaching and Teacher Education, 66*, 282–294. https://doi.org/10.1016/j.tate.2017.04.019

Martinez, M. V., Brizuela, B. M., & Castro Superfine, A. (2011). Integrating algebra and proof in high school mathematics: An exploratory study. *The Journal of Mathematical Behavior, 30*(1), 30–47. https://doi.org/10.1016/j.jmathb.2010.11.002

National Mathematics Advisory Panel. (2008). *Foundations for success: The final report of the National Mathematics Advisory Panel*. U.S. Department of Education. https://www2.ed.gov/about/bdscomm/list/mathpanel/report/final-report.pdf

Nurrahmawati, N., Sa'dijah, C., Sudirman, S., & Muksar, M. (2021). Assessing students' errors in mathematical translation: From symbolic to verbal and graphic representations. *International Journal of Evaluation and Research in Education (IJERE)*, *10*(1), 115–125. https://doi.org/10.11591/ijere.v10i1.20819

Ontario Ministry of Education. (2015). *Paying attention to algebraic reasoning: Support document for paying attention to mathematics education*. https://www.projectmaths.ie/workshops/seminar20142015/OntarioReportOnAlgebraicReasoning.pdf

Pitta-Pantazi, D., Chimoni, M., & Christou, C. (2025). Revisiting the relationship of arithmetical thinking and letter-symbolic algebra. *International Journal of Science and Mathematics Education, 23*, 689–711. https://doi.org/10.1007/s10763-024-10493-z

Pllana, D. (2021). *Explore a real cartoon example*. *International Journal of Inspired Education, Science and Technology (IJIEST),* *3*(1), 8–20. https://www.stsir.org/Journals/IJIEST/publications/Vol_3_1_Papers/IJIEST_Paper_2.pdf

Pllana, D. (2024). *The presence of mathematical tools in STEM: A mixed method study* (Publication No. 32116382) [Doctoral dissertation, New Jersey City University]. ProQuest Dissertations & Theses. https://www.proquest.com/openview/379a895e4f8042e8ba99df04ddc94d83/1?pq-origsite=gscholar&cbl=18750&diss=y

Pllana, D., Baez, R., Sanchez, H., & Sandeep, N. (2024). *Technology inspires mathematical creativity in high school. Futurity Education, 4*(3), 309–333. https://doi.org/10.57125/FED.2024.09.25.18

Pólya, G. (1957). *How to solve it: A new aspect of mathematical method*. Garden City, NY: Doubleday. National Council of Teachers of Mathematics. *Curriculum and Evaluation Standards for School Mathematics.* Reston,VA: The Council, 1989.

Post, T. R., Behr, M. J., & Lesh, R. (1988). Proportionality and the development of prealgebra understandings. In A.

F. Coxford & A. P. Shulte (Eds.), *The ideas of algebra, K–12: 1988 Yearbook of the National Council of Teachers of Mathematics* (pp. 78–90). Reston, VA: National Council of Teachers of Mathematics.

Pournara, C., Sanders, Y., Adler, J., & Hodgen, J. (2016). Learners' errors in secondary algebra: Insights from trackinga cohort from Grade 9 to Grade 11 on a diagnostic algebra test. *Pythagoras, 37*(1), Article a334. https://https://doi.org/10.4102/pythagoras.v37i1.334

Quinn, J. (2022, August 26). *What is mathematics?* Mathematical Association of America. https://maa.org/math-values/what-is-mathematics/

Radford, L. (2006). *Algebraic thinking and the generalization of patterns: A semiotic perspective*. In S. Alatorre, J.

L. Cortina, M. Sáiz, & A. Méndez (Eds.), *Proceedings of the 28th Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education* (Vol. 1, pp. 2–21). Mérida, México: Universidad Pedagógica Nacional.
https://www.pmena.org/pmenaproceedings/PMENA%2028%202006%20Proceedings.pdf

Sa'dijah, C., Murtafiah, W., Anwar, L., Nurhakiki, R., & Cahyowati, E. T. D. (2021). Teaching higher-order Thinking skills in mathematics classrooms: Gender differences. *Journal on Mathematics Education, 12*(1), 159–180. https://files.eric.ed.gov/fulltext/EJ1295012.pdf

Stemele, B. P., & Jina Asvat, Z. (2024). Exploring Learner Errors and Misconceptions in Algebraic Expressions

With Grade 9 Learners Through the use of Algebra Tiles. *African Journal of Research in Mathematics, Science and Technology Education*, *28*(1), 153–170. https://doi.org/10.1080/18117295.2024.2334989

Utami, N. S., Prabawanto, S., & Suryadi, D. (2023, May 18–21). *Students' learning obstacles in solving early Algebra problems: A focus on functional thinking*. Paper presented at the 13th International Congress on Mathematical Education (ICME), Cappadocia, Nevşehir, Türkiye. https://files.eric.ed.gov/fulltext/ED655429.pdf

Texthelp. (n.d.). *The importance of teaching math for lifelong success*. https://www.texthelp.com/resources/math/the-importance-of-teaching-math/

Töman, U., & Gökburun, Ö. (2022). What were algebraic thinking skills at different education levels? *World Journal of Education, 12*(4), 72–81. https://doi.org/10.5430/wje.v12n4p8

Weintraut, S. (2021, October 18). *Pre-calculus: Should You Take It?* The MA Chronicle. https://machronicle.com/pre-calculus-should-you-take-it/

Wilkie, K. J. (2016b). Learning to teach upper primary school algebra: Changes to teachers' mathematical Knowledge for teaching functional thinking. *Mathematics Education Research Journal, 28*(2), 245–275. https://doi.org/10.1007/s13394-015-0151-1

Windsor, W. (2010, July). *Algebraic thinking: A problem solving approach*. In L. Sparrow, B. Kissane, & C. Hurst (Eds.), *Shaping the future of mathematics education: Proceedings of the 33rd Annual Conference of the Mathematics Education Research Group of Australasia* (pp. 665–672). Fremantle, Western Australia: MERGA. Retrieved from https://files.eric.ed.gov/fulltext/ED521033.pdf

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Chapter 6 - New Trends in Music Education: Technological Approaches from a 21st-Century Perspective

Volkan Burak Kibici

Chapter Highlights

- Music education is undergoing a significant evolution in the 21st century, driven by rapid changes in global society, technological advancements, and shifts in educational paradigms. Traditionally focused on one-on-one instrument instruction and Western art music theory, music education is now moving toward a more inclusive, multifaceted, and student-centered structure.
- This review aims to examine prominent new trends in contemporary music education within the framework of technological integration, interdisciplinary approaches, STEM, and artificial intelligence.
- STEM and artificial intelligence (AI) offer revolutionary innovations in music education, personalizing learning processes, automating performance analysis, and supporting creative compositional processes.

Introduction

In recent years, the influence of constructivist, experiential, and sociocultural learning theories has increased in music education (Vygotsky, 1978; Kolb, 1984). Within this framework, learning processes are based on student active participation, collaboration, and the construction of meaning. Furthermore, students are encouraged to develop their musical identities through social interaction through online communities of practice (Partti & Karlsen, 2010).

Arts education in general, and music education in particular, have undergone a significant transformation in the last two decades due to technological advances, shifts in learning theories, and increasing cultural diversity. There has been a shift from traditional instrument-centered learning to digital and immersive learning environments (Özdemir, 2022a; Webster, 2017). This transformation is creating radical changes not only in teaching methods but also in areas such as music teacher training, musical creativity, and digital music production (Biasutti, 2021).

Digital technologies offer new pedagogical tools that support interactive learning and individual creativity in education (Çakır et al., 2018; Öztürk et al., 2021). Virtual reality (VR) and artificial intelligence (AI) applications enhance students' performance analysis and composition processes (Zhang, 2023; Burnard, 2020). Furthermore, with the STEAM (Science, Technology, Engineering, Arts, Mathematics) approach, music education fosters creative thinking and interdisciplinary interaction (Bequette & Bequette, 2012).

Music education has evolved from an approach based solely on Western-centric repertoires to a multicultural and inclusive structure (Campbell, 2018). This contributes to students' recognition of their own cultural musical heritage and respect for diverse musical traditions. Furthermore, accessible music technologies developed for individuals with special needs increase inclusivity in music education (Abramo & Pierce, 2021).

New trends in music education are shaped by the principles of digital transformation, interdisciplinary collaboration, cultural diversity, and lifelong learning. Priority goals for music educators in the future include developing digital pedagogical competencies, creating learning environments that support musical creativity, and promoting multicultural approaches.

Technology Integration and Digital Transformation

Technology has radically transformed both the pedagogical tools and content of music education. Music production software (DAWs), digital audio workstations (DAWs), and applications enable students to actively participate in the composition, arrangement, and recording processes. This fosters creativity and democratizes the way we "make" music (Dorfman, 2013). Tools such as GarageBand, Soundtrap, and Ableton Live, in particular, enable students to work collaboratively on complex musical projects, thus enabling distance learning.

Artificial intelligence (AI)-based applications offer personalized learning experiences. Smart teaching systems (e.g., Yousician, Simply Piano) are being developed that analyze student performance in real time and provide compensatory feedback (Mongan & Philip, 2020; Tekin, 2025). Additionally, Augmented Reality (AR) and Virtual Reality (VR) technologies have the potential to provide students with the experiences of playing instruments, conducting an orchestra, or performing in a concert hall in an interactive and immersive environment (Han et al., 2025).

The integration of digital technologies in arts education has pioneered the development of new pedagogical approaches (Kılınçer, 2025; Ozturk et al., 2023; Özdemir, 2022b; Turgut & Öztürk, 2025). Digital audio processing software, online music learning platforms, and mobile applications, in particular, support self-regulated learning styles tailored to students' individual learning pace (Ruthmann & Mantie, 2017).

Artificial intelligence (AI) and virtual reality (VR) technologies enable students to engage in interactive and experience-based learning (Ceran, 2025a; Zhang, 2023). These technologies enable the use of simulation-based experiences in music theory teaching, the automation of performance analysis, and the support of creative compositional processes (Burnard, 2020).

Cultural Diversity and Multicultural Music Education

The 20th century witnessed unprecedented human mobility and cultural interaction.

Classrooms became mosaics composed of students from diverse ethnic, linguistic, and religious backgrounds. In this context, one of the fundamental functions of education is to prepare individuals to live effectively and respectfully in this pluralistic society. Music, in addition to being a universal language, is one of the most powerful means of expressing cultural identity, values, and history (Nettl, 2015). Therefore, music education offers an ideal platform for reflecting and celebrating cultural diversity.

Traditional music education, especially in the Western world, has been largely built upon the Western art music canon and its standards of harmony, theory, and performance. However, this narrow focus ignores the world's vast musical diversity and can make the curriculum alienating for many students with diverse cultural backgrounds (Volk, 1998). Multicultural music education represents a pedagogical paradigm shift that has emerged to address this deficiency and make education more inclusive, equitable, and representative.In a globalizing world, shaping music education around the aesthetics and practices of a single culture (usually Western Art Music) is criticized and seen as inadequate.

The contemporary approach embraces a multicultural perspective that places world music at the center of the curriculum. This approach aims to help students understand and appreciate the musical traditions, instruments, performance styles, and contexts of different cultures (Campbell, 2018). This orientation requires a philosophy that permeates the entire curriculum, rather than simply adding an "activity." Students should listen to music from different parts of the world.

Pedagogical Approaches and Practices in Multicultural Music Education

A number of pedagogical approaches have been developed to bring multicultural music education into the classroom. These approaches emphasize in-depth and authentic learning, rather than a superficial "museum inventory" model.

World Music Approach: One of the most common methods involves introducing students to musical traditions from different parts of the world (e.g., drum rhythms from Ghana, Gamelan music from Indonesia, Sufi music from Türkiye). However, this approach requires caution in isolating music from its cultural, historical, and social context (Schippers, 2010).

Social Context and Critical Pedagogy: This approach emphasizes that music is not merely a series of sounds but also reflects power relations, resistance, identity struggles, and social change. For example, examining blues music within the context of the history of slavery and racism allows for an understanding of music's role as a social force.

Performance-Based Learning: Instead of simply listening to music, students engage in its performance (by playing simple instruments, singing, or dancing) to gain a deeper understanding and appreciation of that musical tradition (Campbell, 2018). This is one of the most effective applications of the principle of "learning by doing."

Curriculum Transformation: Rather than being a separate unit added to the curriculum, multiculturalism should be viewed as a lens that shapes the entire curriculum. This means including non-Western composers in composition history courses, examining different modal systems or microtonalities in music theory, and using instruments from various cultures in the classroom.

Challenges and Limitations in Implementation

Implementing multicultural music education can face a number of challenges:

(a) Educator Competence: Many music educators are trained in the Western music tradition and may lack the knowledge and skills to teach other musical traditions. *(b) Lack of Resources:* Access to authentic recordings, instruments, notations, and other teaching materials may be limited. *(c) The Danger of Cultural Stereotyping and Appropriation:* Music can be represented superficially or inaccurately, ignoring its sacred or cultural meanings. This raises the issue of cultural appropriation. *(d) Time Constraints:* Within busy curricula, it can be difficult to dedicate sufficient in-depth time to diverse cultures.

To overcome these challenges, it is essential for educators to invest in ongoing professional development, collaborate with culture bearers, and plan instruction in a respectful, contextual, and non-superficial manner. Ultimately, multicultural music education is not simply a curriculum change; it is a powerful educational commitment to building a more just, inclusive, and humane world. By leveraging the universal language of music, it lays the foundation for raising generations who celebrate differences as a source of enrichment.

Interdisciplinary and STEAM Approach

Today's education systems aim to help students acquire 21st-century skills such as analytical thinking, problem-solving, and creativity (Ozturk, 2023; Ozturk & Susuz, 2023). STEM education advocates the integrated teaching of science, technology, engineering, and mathematics (Bybee, 2013). However, this model has been criticized for not sufficiently incorporating artistic thinking processes, leading to the emergence of the STEAM (Science, Technology, Engineering, Arts, Mathematics) approach (Bequette & Bequette, 2012). Music education stands out as a unique field within the STEAM paradigm, combining both cognitive and emotional learning dimensions (Jensen, 2001). Musical learning is naturally linked to STEM components such as rhythm, acoustics, wave physics, sound technology, and programming (Ruthmann & Dillon, 2012). Therefore, music serves as an interdisciplinary bridge in a STEM-focused education. In this context, STEAM education, which brings together the disciplines of science, technology, engineering, arts, and mathematics, has emerged as an alternative to traditional educational models (Gülhan, 2022). The inclusion of arts education within the STEAM model allows students to develop not only technical knowledge but also aesthetic sensitivity and creative problem-solving skills (Bequette & Bequette, 2012). Arts education plays a critical role in helping individuals acquire the holistic literacy skills necessary to cope with the challenges of the 21st century. Research on the importance and positive effects of arts in STEAM education also supports this (Andreotti & Frans, 2019; Kara, 2021; Özkan, 2022; Phanichraksaphong & Tsai, 2021). This approach enhances students' cognitive flexibility, particularly in creative composition and music production, and bridges the gap between artistic production and technical skills (Koutsoupidou, 2018).

Another important trend in music education is the integration of music with other academic disciplines. The STEAM movement, specifically the inclusion of the arts in STEM (Science, Technology, Engineering, Mathematics) education, emphasizes music's inherent connections to these fields (González-Martín et al., 2024; Maltas, 2016):

- *Mathematics:* Rhythm patterns, meters, and interval ratios in music are directly related to mathematical concepts.
- *Physics:* The formation and propagation of sound, and the acoustic principles of instruments are subjects of physics.
- *History and Social Sciences:* Music reflects the history, values, and social structure

of the society in which it is produced.

• *Cognitive Science:* The positive effects of music education on memory, attention, and executive functions are supported by research (Hallam, 2010).

These interdisciplinary connections enrich the learning experience and provide students with the opportunity to view information in an integrated way (Vosough Matin, 2023).

In recent years, interdisciplinary approaches in education have become increasingly important, and the STEM (Science, Technology, Engineering, Mathematics) paradigm has been integrated into the arts. This integration allows creative disciplines like music education to merge with scientific thinking, technological literacy, and problem-solving skills. This article examines the use of the STEM approach in music education, its theoretical foundations, practical examples, and pedagogical implications. Findings show that repositioning music education within the STEM framework has positive effects on both cognitive and creative skills.

The Interaction Between STEM and Music Education

There are strong conceptual connections between music education and STEM disciplines. For example, the physical properties of sound waves (frequency, amplitude, resonance), mathematical ratios (rhythmic patterns, harmonic progressions), and engineering processes (instrument design, sound system construction) can be directly applied in music education (Bamberger, 2000; Miksza & Gault, 2014).

In this context, technology integration is the most obvious area of intersection between STEM and music. Digital audio processing software, music production platforms (e.g., GarageBand, Soundtrap, Ableton Live), and coding-based music applications (e.g., Sonic Pi) enable students to actively participate in creative musical production processes (Ruthmann & Hebert, 2012).

Examples of STEM-Based Applications in Music Education

Acoustics and Physics-Focused Applications

In secondary school music classes, students can experimentally learn the concepts of frequency, resonance, and harmonic structure by examining the physical properties of sound waves. For example, a simple "rubber band guitar" activity can demonstrate the relationship between vibration and wavelength (Gurgel, 2019). Such activities allow students to explore musical concepts using the scientific method.

Music Technology and Coding

In coding-based music education, students create algorithmic musical compositions using tools such as Python or Sonic Pi (Boon & Aziz, 2020). This approach develops students' computational thinking skills while also fostering creativity and aesthetic awareness.
Instrument Design and Engineering Skills

In STEM-based music projects, students experiment with design thinking by designing their own instruments using recyclable materials (Colucci-Gray et al., 2017). Such practices integrate engineering processes with music pedagogy.

Pedagogical Contributions and Challenges

STEM-focused music education develops students' cognitive flexibility, problem-solving abilities, and collaborative learning skills (Henriksen et al., 2015). Additionally, this approach enriches musical learning with concrete experiences and facilitates students' understanding of abstract concepts in music. Furthermore, teachers must possess STEM content knowledge and technology competencies (Crawford, 2017). Supporting music teachers in fields such as engineering or programming will enhance the quality of interdisciplinary education (Biasutti & De Luca, 2023).

The STEM approach in music education offers a new teaching paradigm by combining artistic creativity with scientific thinking processes. This approach contributes to both the musical and cognitive development of students. Curriculum should include modules to enhance music teachers' STEM competencies, and universities should prioritize collaborative projects across art and science disciplines. In the future, AI-powered music applications and coding-based sound production tools will further strengthen the STEM-based aspect of music education.

The Role and Use of Artificial Intelligence in Music Education

The integration of artificial intelligence technologies into the educational field is redefining the 21st-century teaching paradigm (Ceran, 2025b; Kara, 2025). Music education has also been significantly impacted by this transformation; in particular, digital audio analysis, machine learning-based performance evaluation systems, and creative composition software have entered educational environments (Zhang, 2023).

While traditional music education methods focus on the emotional and intuitive aspects of the student, artificial intelligence supports these aspects with data-based learning models (Biasutti & De Luca, 2023). This has created a hybrid learning environment that redefines the roles of the teacher and student.

Artificial intelligence refers to systems that mimic human intelligence and can perform cognitive processes such as learning, perception, and decision-making (Russell & Norvig, 2021). AI-powered tools not only make educational processes more effective and efficient, but also offer flexible solutions that can be adapted to students' individual learning needs (Luckin, 2017). These technologies offer advantages such as personalizing teaching materials and adapting learning pace and style (Holmes et al., 2019). In this context, examining the effects of AI technologies on music education is crucial for the future application and development of this field. In their study titled "Artificial Intelligence and Music Education," Li and Wang (2023) state that the combination of AI and music education will make a significant difference in today's music education. AI-supported music education offers new opportunities for developing students' musical abilities. This technology provides an individualized learning process, analyzes performances, creates practical plans, and conducts musical analyses. It is believed that utilizing AI in music education can enable students to learn more effectively and efficiently.

The Use of AI in Music Education Generally Stands Out in Three Main Areas:

1. Teaching and learning analytics: Systems that measure student performance and provide feedback,
2. Musical creation and composition: Automatic or collaborative creative algorithms,
3. Pedagogical support: Tools that optimize teachers' lesson planning and evaluation

processes (Yang et al., 2022).

AI-powered music education creates a new paradigm in music pedagogy by making learning processes more personalized and interactive (Ruthmann & Mantie, 2017).

Application Areas in AI-Enhanced Music Education

Performance Analysis and Feedback Systems

Machine learning algorithms provide instant feedback by analyzing students' sound, rhythm, and intonation accuracy (Hung et al., 2020). For example, software such as "SmartMusic" and "Yousician" automatically evaluate student performance and offer improvement suggestions (Burnard & Murphy, 2020).

Composition and Creative Production

AI functions as a "co-composer" with students in creative processes. Systems such as Google's Magenta project or OpenAI's MuseNet model inspire students by generating melodic variations (Herremans et al., 2017). This process strengthens students' creative thinking skills (Pascoal & Correia, 2021). Learning Analytics and Personalized Education
AI-based analytics systems enable adaptive instruction by tracking students' learning styles, achievement levels, and progress (Ferguson, 2019). This allows teachers to develop differentiated learning strategies based on student needs.

Challenges and Ethical Aspects

The integration of AI applications into music education raises several ethical and technical issues. These include:

These include issues such as data privacy and copyright issues, the risk of automation of creativity, and the inability to represent human emotion through algorithmic measurements (Pascoal & Correia, 2021). Burnard (2020) states that the transformation of AI systems beyond mere tools into "aesthetic subjects" in art education has shifted pedagogical balances. Therefore, defining ethical boundaries and preserving a human-centered pedagogical approach is crucial.

The literature demonstrates that AI fosters musical creativity, transforms teaching processes, and strengthens learner-centered approaches. However, new ethical debates have also emerged regarding the transformation of the teacher role and data privacy. In music education, AI is a powerful tool that enables the personalization of learning processes, accelerates performance feedback, and fosters creativity. However, this technology should be viewed as complementary to the human experience.

It is recommended that future music teacher training programs include AI literacy and digital ethics. Furthermore, educational institutions should encourage students to bridge the gap between creativity and technology by establishing AI-supported music laboratories.

Practices Inspired by Inclusive Education, Social-Emotional Learning, and Music Therapy

Among the goals of music education, supporting not only an individual's musical skills but also their personal and social development is becoming increasingly important. Music is a powerful tool for expressing emotions, developing self-awareness, building empathy, and strengthening social bonds.

Inspired by music therapy, practices aimed at developing students' emotional intelligence are increasing in music education. Group improvisational activities, collaborative composition projects, and activities focused on expressing emotions through music help students develop communication skills, teamwork, and self-regulation (Saarikallio & Baltazar, 2018). This approach shifts music education from a focus on performance and technical skills to a model that focuses on the holistic well-being of the individual.

Music education has evolved from a static discipline into a dynamic and constantly evolving field. The creative use of technological tools, culturally diverse curricula, interdisciplinary collaborations, and pedagogies aimed at social-emotional development constitute the cornerstones of this new era (Perry, 2024). Future music educators are expected to adapt to these trends and design inclusive and transformative learning environments that address the needs, interests, and cultural backgrounds of all students. This transformation has the potential to further strengthen the profound value of music education for individuals and society.

Today, music education is moving away from an approach based solely on Western-centric repertoires and embracing a multicultural music education approach (Campbell, 2018). This approach helps students understand their own cultural musical heritage and respect diverse musical traditions.

Furthermore, accessible music technologies (e.g., adaptive instruments and touch-sensitive interfaces) for individuals with special needs are increasing inclusivity in music education (Abramo & Pierce, 2021).

Music Curriculum and Teacher Education and Competencies Based on New Trends

Pedagogical Impacts and Teacher Roles

STEM and artificial intelligence are transforming the role of teachers from knowledge transmitters to guides of the learning process (Biasutti, 2021). Music teachers are now using STEM and artificial intelligence tools not only as technical support but also as pedagogical partners.

This transformation brings the concept of digital pedagogical competence to the forefront. Crawford (2017) emphasizes that teachers' development of technology-integrated teaching practices improves the quality of learning. Digital tools and artificial intelligence also contribute to the measurability of musical creativity, making students' cognitive development processes visible (Ruthmann & Dillon, 2012).

Learning is enriched when the cultural context is more effectively processed by integrating music with disciplines such as history, language, visual arts, and sociology. STEAM/STEAM-like approaches are also suitable for evaluating cultural production processes from a technological and societal perspective (Bequette & Bequette, 2012).

From a 21st-century perspective, it is crucial that music curricula be planned to represent examples from diverse cultures. This requires a balanced repertoire encompassing both traditional folk music and contemporary popular practices. However, diversifying the repertoire should go beyond simply adding a superficial "variety of songs"; the origins, contexts of use, and cultural meanings of the pieces should be discussed in class (Abramo & Pierce, 2021).

Field studies, community collaborations, and performance-based projects deepen students' cultural learning. For example, engaging students in performances with local musicians using an ethnomusicology approach strengthens learning at both cognitive and emotional levels (Partti & Karlsen, 2010).

Music teachers' digital competencies are one of the key factors determining success in technology-supported music education (Crawford, 2017; Kara, 2020; Kilinçer, 2021). Developing positive attitudes toward technology is critical to implementing contemporary music pedagogy (Burnard & Murphy, 2020). Furthermore, reflective teaching, mentoring programs, and continuous professional development processes enable teachers to keep up with both pedagogical and technological innovations (Biasutti & De Luca, 2023).

Conclusion and Recommendations

New trends in music education are shaped around the principles of digital transformation, interdisciplinary collaboration, cultural inclusiveness, and lifelong learning. Priority goals for music educators in the future should include developing digital pedagogical competencies, creating learning environments that support musical creativity, and promoting multicultural approaches. Educational policies should support flexible and innovative curriculum designs that adapt to technological and cultural changes. To effectively use artificial intelligence technologies and STEM applications, teachers need training in these areas. Courses on artificial intelligence, STEM, and educational technologies should be offered in university education faculties. Additionally, in-service training programs should be organized to ensure that existing teachers gain knowledge and skills in these areas. Materials and software specifically designed for STEM and AI-supported music lessons should be developed. These materials should be designed to support students' creative skills in music education and enrich their learning processes. Furthermore, it is important that these materials be applicable to different musical genres and cultural contexts.

References

Abramo, J. M., & Pierce, A. (2021). *Accessibility and inclusion in music education: New technologies and approaches*. Oxford University Press. https://doi.org/10.1093/oso/9780190099373.001.0001

Andreotti, E., & Frans, R. (2019). The connection between physics, engineering and music as an example of STEAM education. *Physics Education*, *54*(4), Article 045011. https://doi.org/10.1088/1361-6552/ab246a

Bamberger, J. (2000). *Developing musical intuitions: A project-based introduction to making and understanding music*. Oxford University Press.

Banks, J. A. (2015). *Cultural diversity and education: Foundations, curriculum, and teaching* (6. baskı). Routledge.

Bequette, J. W., & Bequette, M. B. (2012). A place for art and design education in the STEM conversation. *Art Education*, *65*(2), 40–47. https://doi.org/10.1080/00043125.2012.11519167

Biasutti, M. (2021). Strategies and challenges for distance learning of music during the COVID-19 pandemic: Teachers' perspectives. *Music Education Research*, *23*(3), 240–258. https://doi.org/10.1080/14613808.2021.1932059

Biasutti, M., & De Luca, R. (2023). Digital competence development in music teacher education. *British Journal of Music Education*, *40*(1), 78–94. https://doi.org/10.1017/S0265051722000142

Boon, M., & Aziz, M. (2020). Coding in the music classroom: An introduction to algorithmic composition. *Journal of Music, Technology & Education*, *13*(1), 1–19. https://doi.org/10.1386/jmte_00014_1

Burnard, P. (2020). *Pedagogies of creating and performing music in digital environments*. Routledge. https://doi.org/10.4324/9781003014804

Burnard, P., & Murphy, R. (2020). Transforming music teacher education through technology. *Music Education Research*, *22*(5), 489–505. https://doi.org/10.1080/14613808.2020.1822302

Bybee, R. W. (2013). *The case for STEM education: Challenges and opportunities*. National Science Teachers Association Press.

Campbell, P. S. (2018). *Music, education, and diversity: Bridging cultures and communities*. Teachers College Press.

Ceran, S. (2025). Examining high school students' artificial intelligence literacy in visual arts subjects and metaverse-based digital art attitudes. *International Journal of Technology in Education*, *8*(4), 1245–1265.

Colucci-Gray, L., Burnard, P., Gray, D. S., & Cooke, C. (2017). STEAM: Science and arts through school projects. *International Journal of Science Education*, *39*(6), 803–819. https://doi.org/10.1080/09500693.2017.1324003

Crawford, R. (2017). Rethinking teaching and learning pedagogy for education in the twenty-first century. *Research in Comparative and International Education*, *12*(1), 13–25. https://doi.org/10.1177/1745499917698463

Çakır, E., Öztürk, M., & Ünal, M. (2019). Interpainting as a creating method in digital illustration: Reinterpretations from movie scenes. *Bilim, Eğitim, Sanat ve Teknoloji Dergisi (BEST Dergi)*, *3*(2), 78–88. https://doi.org/10.46328/seat.v3i2.32

Dorfman, J. (2013). *Theory and practice of technology-based music instruction*. Oxford University Press.

Ferguson, R. (2019). *The impact of learning analytics on education*. Springer.

González-Martín, C., Prat Moratonas, M., & Forcada Royo, J. (2024). Music and mathematics: Key components and contributions of an integrated STEAM teaching approach. *International Journal of Music Education*. Advance online publication. https://doi.org/10.1177/02557614241248267

Gurgel, R. (2019). Sound waves and strings: Using physics to teach music concepts in the classroom. *Physics Education*, *54*(3), Article 035011. https://doi.org/10.1088/1361-6552/ab03a8

Gülhan, F. (2022). Türkiye'de yapılmış STEAM/[STEM+ A (Sanat)] araştırmalarındaki eğilimlerin analizi. *Turkish Journal of Educational Studies*, *9*(1), 23–46.

Hallam, S. (2010). The power of music: Its impact on the intellectual, social and personal development of children and young people. *International Journal of Music Education*, *28*(3), 269–289. https://doi.org/10.1177/0255761410370658

Han, Y., Han, L., Zeng, C., & Zhao, W. (2025). The innovation path of VR technology integration into music classroom teaching in colleges and universities. *Scientific Reports*, *15*(1), Article 12200.

Henriksen, D., Mishra, P., & Fisser, P. (2015). Infusing creativity and technology in 21st century education: A systemic view for change. *Educational Technology & Society*, *18*(3), 30–41.

Herremans, D., Chuan, C. H., & Chew, E. (2017). A functional taxonomy of music generation systems. *ACM Computing Surveys*, *50*(5), Article 69. https://doi.org/10.1145/3108242

Holmes, W., Bialik, M., & Fadel, C. (2019). *Artificial intelligence in education: Promises and implications for teaching and learning*. Center for Curriculum Redesign.

Hung, C. C., Lin, S. H., & Chen, Y. L. (2020). Machine learning-based automatic assessment of instrumental performance. *International Journal of Music Education*, *38*(4), 619–633. https://doi.org/10.1177/0255761420941103

Jensen, E. (2001). *Arts with the brain in mind*. Association for Supervision and Curriculum Development.

Kara, S. (2020). Prospective visual arts teachers' innovation skills and attitudes towards computer assisted instruction. *International Journal of Technology in Education and Science*, *4*(2), 98–107.

Kara, S. (2021). An investigation of technological pedagogical and content knowledge (TPACK) competencies of pre-service visual arts teachers. *International Journal of Technology in Education*, *4*(3), 527–541.

Kara, S. (2025). The effect of artificial intelligence applications in 6th grade visual arts course on student attitudes and course outcomes. *International Journal of Modern Education Studies*, *9*(1).

Kılınçer, Ö. (2021). An investigation of pre-service music teachers' attitudes towards online learning during the COVID-19 pandemic. *International Journal of Technology in Education and Science*, *5*(4), 587–600.

Kılınçer, Ö. (2025). Investigation of music teacher candidates' technology integration self-efficacy and artificial intelligence literacy. *International Journal of Modern Education Studies*, *9*(2).

Kolb, D. A. (1984). *Experiential learning: Experience as the source of learning and development*. Prentice Hall.

Koutsoupidou, T. (2018). *Creativity in music education: Theory and practice*. Cambridge Scholars Publishing.

Li, P. P., & Wang, B. (2023). Artificial intelligence in music education. *International Journal of Human–Computer Interaction*, *40*(16), 4183–4192. https://doi.org/10.1080/10447318.2023.2209984

Luckin, R. (2017). Towards artificial intelligence-based assessment systems. *Nature Human Behaviour*, *1*, 1–3. https://doi.org/10.1038/s41562-016-0028

Maltas, W. (2016). *From STEM to STEAM: Integrating arts education into the STEM disciplines of science, technology, engineering and math* [Master's thesis, Drexel University]. Drexel University Repository.

Miksza, P., & Gault, B. (2014). Designing STEAM-based learning experiences in music education. *Music Educators Journal*, *101*(1), 25–32. https://doi.org/10.1177/0027432114540474

Mongan, W. M., & Philip, T. (2020). Leveraging artificial intelligence in music education: A review of the literature. *Journal of Music, Technology & Education*, *13*(1), 5–23. https://doi.org/10.1386/jmte_00011_1

Nettl, B. (2015). *The study of ethnomusicology: Thirty-three discussions* (3. baskı). University of Illinois Press.

Ozturk, M., Kinik, M., & Ozturk, M. (2023). Investigation of technological pedagogical and content knowledge (TPACK) competencies of university students. *International Journal of Technology in Education*, *6*(3), 418–433. https://doi.org/10.46328/ijte.524

Ozturk, M., & Susuz, M. (2023). The investigation of creativity levels of fine arts faculty students. *International Journal on Social and Education Sciences*, *5*(4), 929–944. https://doi.org/10.46328/ijonses.638

Ozturk, O. T. (2023). Examination of 21st century skills and technological competences of students of fine arts faculty. *International Journal of Education in Mathematics, Science and Technology*, *11*(1), 115–132. https://doi.org/10.46328/ijemst.2931

Özdemir, D. (2022a). Küreselleşmenin sanatın kültürel kimliği üzerindeki etkisi. *Bilim Eğitim Sanat ve Teknoloji Dergisi*, *6*(1), 100–114.

Özdemir, D. (2022b). A conceptual framework on the relationship of digital technology and art. *International Journal on Social and Education Sciences*, *4*(1), 121–134.

Özkan, Z. C. (2022). The effect of STEAM applications on lesson outcomes and attitudes in secondary school visual arts lesson. *International Journal of Technology in Education*, *5*(4), 621–636.

Öztürk, Ö. T., Türe, A., & Yaglici, S. C. (2021). *The relationship of futurism dynamism and today's technology*. International Society for Technology, Education, and Science.

Partti, H., & Karlsen, S. (2010). Reconceptualising musical learning: New media, identity and community in music education. *Music Education Research*, *12*(4), 369–382. https://doi.org/10.1080/14613808.2010.519381

Pascoal, D., & Correia, M. (2021). Artificial intelligence and music education: A conceptual analysis. *Frontiers in Psychology*, *12*, Article 719856. https://doi.org/10.3389/fpsyg.2021.719856

Perry, J. T. (2024). *An exploration into the social and emotional effects of K-12 music education* [Doctoral dissertation, Liberty University]. Liberty University Digital

Commons.

Phanichraksaphong, V., & Tsai, W. H. (2021). Automatic evaluation of piano performances for STEAM education. *Applied Sciences*, *11*(24), Article 11783. https://doi.org/10.3390/app112411783

Russell, S., & Norvig, P. (2021). *Artificial intelligence: A modern approach* (4. baskı). Pearson Education.

Ruthmann, S. A., & Dillon, S. C. (2012). Technology as music education. G. McPherson & G. Welch (Eds.), *The Oxford handbook of music education* (Cilt 2, ss. 567–584). Oxford University Press.

Ruthmann, S. A., & Hebert, D. G. (2012). Music learning and new media in virtual and online environments. *Contemporary Music Review*, *31*(1), 49–63. https://doi.org/10.1080/07494467.2012.759932

Ruthmann, S. A., & Mantie, R. (2017). *The Oxford handbook of technology and music education*. Oxford University Press. https://doi.org/10.1093/oxfordhb/9780199372132.001.0001

Saarikallio, S., & Baltazar, M. (2018). Music as a resource for psychological well-being: A systematic review of the empirical evidence. *Psychology of Music*, *46*(4), 459–472. https://doi.org/10.1177/0305735617734627

Schippers, H. (2010). *Facing the music: Shaping music education from a global perspective*. Oxford University Press.

Tekin, A. (2025). Examining high school students' attitudes towards contemporary and digital art and virtual exhibitions. *International Journal on Studies in Education*, *7*(4), 879–898.

Turgut, S. & Öztürk, M. S. (2025). Analysis of Posters Awarded in the Typography Category at Graphic Designers Professional Association Exhibitions. In M. S. Ozturk, O. Noroozi, & O. Bilen (Eds.), *Proceedings of ICSES 2025-- International Conference on Studies in Education and Social Sciences* (pp. 34- 42). ISTES.

Volk, T. M. (1998). *Music, education, and multiculturalism: Foundations and principles*. Oxford University Press.

Vosough Matin, M. (2023). Investigation of special education teachers' technology integration self-efficacy levels. *International Journal of Technology in Education and Science*, *7*(3), 290–305.

Vygotsky, L. S. (1978). *Mind in society: The development of higher psychological processes*. Harvard University Press.

Webster, P. R. (2017). *Music learning today: Digital pedagogy for creating, performing, and responding to music*. Oxford University Press.

Yang, Y., Luo, J., & Zhao, H. (2022). Artificial intelligence in music education: Trends and challenges. *Education and Information Technologies*, *27*(8), 11125–11141. https://doi.org/10.1007/s10639-022-11032-9

Zhang, L. (2023). Artificial intelligence in music education: Opportunities and challenges. *Frontiers in Psychology*, *14*, Article 1123489. https://doi.org/10.3389/fpsyg.2023.1123489

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Chapter 7 - Digital Transformation in Turkish Language Teaching: Integration of Technology and Use of Digital Tools

Dilek Ünveren

Chapter Highlights

- This research examines the integration of digital technologies into Turkish language teaching and the role of digital tools in education, focusing on pre-service Turkish teachers' perspectives from a qualitative standpoint.
- Using a case study design supported by in-depth interviews, the study explores how digital tools are integrated into Turkish lessons, their effects on teaching effectiveness, and their relationship with student motivation.
- Findings indicate that pre-service teachers show a strong tendency to use digital tools to enrich course materials, increase student engagement, and individualize learning processes, while students benefit from increased motivation and interest.
- However, challenges such as inadequate technological infrastructure and limited digital literacy among some teachers emerged as significant barriers.
- The study concludes that although digital transformation offers substantial opportunities for Turkish language teaching, it requires strategic, holistic approaches and supportive institutional policies to ensure effective and inclusive implementation.

Introduction

The Necessity of Digitalization in Turkish Language Teaching

The necessity of digitalization in Turkish language teaching has become a very important issue in today's educational world (Aksu, 2018). With the rapid development of technology and the integration of digital tools into all areas of life, this transformation has become inevitable in education systems (Alakoç, 2003). Digitalization in Turkish language teaching offers various advantages to students in developing their language skills (Alexander, Ashford-Rowe, Barajas-Murphy, Dobbin, Knott, McCormack, & Weber, 2019). First of all, digital platforms and resources provide students with the opportunity to diversify their language learning experiences by offering comprehensive and rich content (DEÜSEM, 2020). In this way, students can develop their language skills according to their own learning pace by making use of different resources (Ergüney, 2015).

Furthermore, thanks to digitalization, interactive learning environments can be created. Tools such as virtual classrooms, online discussions and video conferences allow students to interact more with each other and with their teachers (Erstad & Zounek, 2018). This encourages collaborative learning and increases students' motivation. Digitalization in Turkish language teaching also facilitates assessment and evaluation processes (Ertmer & Newby, 2013). Online exams, quizzes and automatic feedback systems allow teachers to assess student performance more quickly and accurately. In addition, thanks to digital tools, teachers can prepare personalized educational materials that are more tailored to the needs of their students (Şener & Gündüzalp, 2018).

Another important contribution of digitalization in Turkish language teaching is ease of access. Digital materials, which can be accessed from anywhere with an internet connection, eliminate geographical barriers and reach a wider student audience (Tapscott, 1998). However, in order for these advantages of digitalization to be effective, it is of great importance to develop the digital literacy skills of both teachers and students (McGraw, 2020). As a result, digitalization in Turkish language teaching can provide opportunities for the benefit of students and teachers and make significant contributions to the improvement of the quality of education. The correct and conscious adoption of digitalization in the field of education will play a key role in raising individuals with language skills of the future (Telli & Altun, 2020).

Digital Tools and Platforms Used

Digital Textbooks and E-Books: Digital textbooks and e-books used in Turkish education stand out as an important part of digitalization in education (Milli Eğitim Bakanlığı, 2018) These materials offer students the advantages of flexibility and accessibility in developing their language skills (Altın & Kalelioğlu, 2015). Unlike traditional printed books, digital textbooks and e-books can include interactive features and make learning more engaging by incorporating various multimedia content such as videos, audio files and animations (Bello, 2014). With these digital resources, students can learn topics at their own pace and access the materials they need at any time via the internet works (Bozkurt & Cilavdaroğlu, 2011). Digital textbooks and e-books enable personalized learning. Students can choose content according to their learning style and deepen their focus in the areas they want to focus on. In addition, teachers can provide different levels of content to guide students and prepare materials to suit the learning needs of each individual (Brooks, 2013). These digital books, which can be updated through online platforms, ensure that the content is always up-to-date. These digital resources developed for Turkish education offer students the opportunity to practice with various activities and applications. For example, interactive exercises on grammar rules or vocabulary can help students reinforce what they have learned (Kabakçı Yurdakul, 2011).

E-books and digital textbooks also play a role in reducing paper use by providing an environmentally friendly alternative (Kayaduman, Sırakaya & Seferoğlu, 2011). Digitization of educational materials also offers a huge advantage in terms of portability. Students can access all their resources on a tablet or computer without carrying heavy bags (Kurt, 2013). However, in order to use these technologies effectively, the digital literacy levels of students and teachers need to be increased (Maddux & Johnson, 2006). It is also important that educational institutions provide the necessary infrastructure. In conclusion, digital textbooks and e-books in Turkish education play an important role in adapting to the future education system while making education more effective, accessible and interesting. The proper use of these digital resources enriches students' language learning processes and provides teachers with a more dynamic educational environment (Maddux & Johnson, 2006).

E-Learning Platforms: As one of the innovations of the digital age, e-learning platforms used in Turkish language education make language learning processes more accessible and flexible. E-learning platforms allow students to access course materials, interactive content and customized learning paths over the internet. These platforms offer many tools and resources for improving Turkish grammar, reading, writing, listening and speaking skills. Students have the opportunity to reinforce what they have learned through the platforms while following the content according to their own learning pace. These digital learning environments also offer various pedagogical advantages for teachers. E-learning platforms allow teachers to digitally organize lesson plans and easily track student progress. Moreover, teachers can make their lessons more effective by creating content enriched with various multimedia tools. Online discussion boards, live lecture sessions and feedback mechanisms for students encourage students to interact more and think more deeply about grammar (Mazman & Koçak-Usluel, 2011). The flexibility offered by e-learning platforms removes geographical constraints and allows students to learn at any time and place. This ensures continuity of education, especially in distance education processes or when face-to-face education is not possible (Cumhur & Çam, 2021). At the same time, e-learning platforms can also include gamification elements that increase student motivation by using technology. Students can be encouraged to learn by earning badges, certificates and rewards as they improve their level (Canbazoğlu Bilici, 2012). However, it is important to provide the necessary infrastructure for the effective use of e-learning platforms and to develop the digital competencies of both teachers and students. Educational institutions need to support adaptation to this digital transformation. In conclusion, e-learning platforms used in Turkish language education enrich educational processes with the multifaceted benefits they offer to students and teachers. These platforms bring a different perspective to language education and make Turkish learning more diverse and effective with the opportunities offered by technology (Çam, 2019).

Interactive Boards and Smart Classrooms: Interactive boards and smart classrooms have an important place among the opportunities offered by modern educational technologies in Turkish education. Interactive boards replace traditional blackboards and make lecturing more dynamic and effective (Johns, 2015). These boards offer teachers the opportunity to integrate various multimedia content into lessons, attracting students' attention and enriching the learning process. In particular, these technologies, which enable the use of audio-visual materials, provide a better understanding of the subjects and increase students' participation in the lesson. Smart classrooms are another innovation that offers students a more interactive learning environment (Jones, Johnson-Yale, Millermaier, & Pérez, 2009). These classrooms are structured so that students and teachers can interact with personal devices such as tablets and laptops. In smart classrooms, the dissemination of digital content and the ability to update this content in real time significantly increase the flexibility and effectiveness of lessons (Kabakçı-Yurdakul, Çoklar, Birinci, & Kılıçer, 2012). The integration of technology in Turkish language teaching also contributes to language learning processes. For example, grammar practice, vocabulary games and interactive storytelling can be carried out on interactive whiteboards. Through such activities, students have the opportunity to develop their language skills in a more fun and lasting way (Kabakçı-Yurdakul, Odabaşı, Kılıçer, Çoklar, Birinci, & Kurt, 2014). Furthermore, collaborative projects and group work in smart classrooms provide students with the opportunity to develop critical thinking, problem solving and communication skills. However, for the effective use of these technologies, teachers' digital literacy needs to be increased and the necessary technical infrastructure needs to be provided in schools. Training educators on how to integrate these tools into lesson plans will ensure that the potential of technology is fully utilized. The use of interactive whiteboards and smart classrooms in Turkish education transforms future educational practices by making students' learning experiences richer, more interactive and more productive (Kale, & Goh, 2014).

Educational Games and Applications: Educational games and applications in Turkish education are important tools that make students' language learning processes more interesting and effective. Compared to traditional teaching methods, these games and applications provide students with more interaction and motivation, allowing them to improve their language skills. In particular, these innovative approaches, which appeal to students of the digital age, increase interest in the learning process by making lessons more fun and engaging. Educational games are used to improve grammar, vocabulary, reading and writing skills. For example, word puzzles, grammar adventure games or interactive story apps provide students with fun opportunities to reinforce language skills (Altın & Kalelioğlu, 2015). Since these games provide frequent feedback to reinforce learning, they allow students to immediately see their mistakes and adjust their learning process. Furthermore, such games can also help students develop social skills by encouraging competition and cooperation. Mobile applications play an important role in personalizing course materials and allowing students to work independently. With these apps, students can practice Turkish at any time and place and learn at their own pace. Language learning apps are usually structured in multiple modules, with each level targeting different language skills, providing students with a comprehensive language learning experience. They can provide support in reading aloud, speaking practice, spelling and grammar (Maddux & Johnson, 2006). However, for educational games and apps to be effective, teachers need to make a careful assessment when integrating these tools into lesson plans. They should make sure that the content is appropriate, up-to-date and accurate for educational purposes. With the innovations brought by technology, educational games and applications in Turkish education offer the opportunity to develop language skills in a permanent way by enabling students to participate more actively. Educators and students can enrich language learning processes by using these tools consciously and effectively (Altın & Kalelioğlu, 2015).

Virtual Reality (VR) and Augmented Reality (AR): In Turkish language education, virtual reality (VR) and augmented reality (AR) stand out as innovative technologies that enrich and deepen the language learning experience. These two applications of educational technologies add a different dimension to the development of language skills by providing students with more immersive and interactive learning environments. Virtual reality offers students three-dimensional and fully fictionalized environments, encouraging them to engage in language learning activities independently from the real world (Ertmer & Newby, 2013). In virtual environments, learners can experience language and culture-related scenarios and perform tasks that require language use in a virtual world. These interactions can help them easily grasp the applications of language skills in everyday life, especially by increasing contextual learning opportunities. On the other hand, augmented reality makes learning more effective by adding digital information on top of the physical world (Brooks, 2013). AR applications in Turkish education can be integrated with textbooks and materials to provide audiovisual content for students (Maddux & Johnson, 2006). For example, characters or stories in textbooks can be animated through mobile devices, thus capturing students' attention and making the course material more engaging and understandable. AR also concretizes learning in grammar topics or vocabulary expansion activities by enabling words and sentences to be supported by visuals (Kabakçı Yurdakul, 2011).

Both technologies have the potential to increase student motivation and engagement (Ertmer & Newby, 2013). While enabling students to participate more actively in the learning process, the retention of information can be increased thanks to the interactive and visual advantages offered by technology. However, for VR and AR applications to be used effectively and appropriately in education, it is important that teachers receive training on these technologies and that the content is compatible with educational goals (Brooks, 2013). Virtual and augmented reality applications in Turkish education play an important role in shaping future educational approaches, making language learning experiences more engaging and effective (Kabakçı Yurdakul, 2011). In light of the above information, the following questions were sought in this study:

- In what ways is the integration of digital technologies into Turkish language teaching realized?
- How do teachers use digital tools in Turkish lessons?
- What are the alternative ways that Turkish teachers can use digital tools?
- How does digital transformation affect student motivation and participation?

Method

This study used the case study method as reseach design. If it is desired to understand how and why a phenomenon occurs, if detailed information is needed about a complex social phenomenon, if there are new or less researched topics where theoretical explanations are insufficient, a case study is an appropriate choice (Creswell and Plano Clark, 2011). For these reasons, case study was preferred as a design in this study.

Participant Selection

In qualitative studies, participant selection is made by selecting individuals who are rich in information and suitable for the purpose and nature of the research. This process is based on purposeful (purposive) sampling strategies instead of random sampling in quantitative studies. The number of participants is considered sufficient when data saturation (the point at which no new information is received) is reached. Therefore, this study was conducted with 16 participants.

Data Collection Tools

A semi-structured interview form was used as a data collection tool in this study. A semi-structured interview form is a data collection tool used in qualitative research, containing predetermined open-ended questions but also providing flexibility. The researcher can add new questions or skip some questions during the interview. This method is quite suitable for obtaining in-depth information.

Data Analysis

The analyses of this study were carried out using the content analysis technique. Content analysis is a systematic data analysis method used in qualitative research. The aim is to reveal hidden or explicit meanings in texts (interview transcripts, documents, written/visual materials, etc.), to determine themes and patterns, and to classify them systematically. This analysis method is suitable when interview/observation data is to be analyzed systematically, if the aim is to discover explicit and implicit themes, and if it is desired to organize a large amount of textual data (Creswell, 2007).

Results

As a result of analyses of the interviews with pre-service teachers, codes, cathegories and themes are shown in the table below.

Table 1. Codes, Cathegories and Themes

1- Integration of digital technologies in teaching Turkish	2-Use of digital tools by Turkish teachers	3-Digital tool suggestions that can be used in Turkish lessons	4-Effect of digital transformation on student motivation and participation
Education Information Network (EIN) (6) [The Education Information Network (EIN) is an online social education	During the process of motivation/ guessing/ intuition	Podcast, dijital story, blog (13)	Active participation (8)

platform offered free of charge to each individual by the General Directorate of Innovation and Education Technologies.]			
Digitalization of educational content and materials (e-book, audiobook, digital libraries, podcast, animation etc.) (9)	During Instruction: Smart board, Google classroom, video, presentation/slides , EIN (16)	Applications: games (5)	Entertainment (3)
Smart board/computer (5)	During skill development: Writing activities (Story board, storyjumper, digital games	Social interaction environments, social media, websites, virtual groups (goodreads, Discord) (5)	Interaction (3)
Fatih project (2)	During measurement and evaluation: Kahoot, Quizizz, Socrative, Wordwall (6)	Experience Environments: Virtual tours/museums (3)	Enthusiasm/desire/encouraging/motivating (7)
Web 2.0 Tools/ Applications: (Kahoot, wordwall, kotobee, storyboard, Turkish workshop,			Engaging (8)

lesson on screen, games, videos-, presentations (6)	
Digital interaction environments: (Blog, social media, zoom, Google classroom)	Individual learning opportunity (2)
	Adaptation to the age (8)

Results for the First Theme: 'Integration of Digital Technologies in Teaching Turkish'

Participants stated that the integration of digital technologies into Turkish language teaching both enriches the teaching process and increases student participation. It was emphasized that digital technologies have become an integral part of education during the pandemic.

> P1*:"Digital technologies in teaching Turkish help students both develop their language skills and use the digital world effectively. Distance learning and smart boards used in classrooms also help integrate digital technologies"*

> P5: "*When we look at the use of technology in education, we see that different tools are focused on in each period. For example, technology, which came to life in education with projection devices in the early 2000s, continues on its way with smart boards in the following years with the Fatih project. With this project, QR codes were added to our textbooks and tangible and inanimate textbooks were blended with technology and presented to teaching environments."*

The period when technology interacted most with education was actually the period when the world experienced the Coronavirus pandemic. The biggest supporter of students and teachers entering homes to receive education in an environment without schools was undoubtedly digital technology tools. With distance education channels, virtual classrooms and various video chat tools, efforts were made to minimize the negative effects of this pandemic.

P16*:"The use of digital technologies in teaching Turkish really adds a very different* *dimension to lessons. For example, thanks to educational applications, it has become much easier to develop grammar and reading skills. While vocabulary is expanded with mobile applications, we can learn subjects in more detail with digital books and online resources. Especially videos and multimedia content, both by listening and it also helps a lot in learning a language visually.”*

These statements show that digital technologies are not just tools, but have become a direct part of teaching processes. Especially extraordinary situations such as pandemics have clearly revealed the indispensability of technology in education. Smart boards, distance education platforms and digital content have provided permanent learning opportunities for both teachers and students.

P4: “*Online platforms are also very effective; teachers and students can communicate in a more interactive way with tools like Zoom or Google Classroom. In addition, developing writing skills becomes more fun thanks to social media and blogs. Learning as if playing with applications in the form of games is very useful both in terms of focusing attention and reinforcing grammar. Turkish lessons have become more fun and productive thanks to digital technologies.”*

P8: “*Digital technologies in Turkish lessons are used in various ways to make learning processes more effective, interesting and to make knowledge permanent. While these provide support with resources such as digital course materials, e-books and videos, platforms such as EIN make online learning more effective. In addition, smart boards and smart classrooms used in our schools increase student participation in the lesson with audio-visual content. In addition to these, gamification applications increase student motivation and provide an entertaining learning environment.”*

These types of content increase the permanence of learning because they appeal to multiple senses compared to traditional teaching materials. In addition, it is seen that technology supports the four basic language skills (reading, writing, speaking, listening) in a holistic way.

P11: “*In addition to what I wrote above; for writing skills, blogging and digital writing tools help students develop their written expression skills, while AI-powered tools provide practical support for grammar and spelling. These technologies* *increase interaction in teaching Turkish and offer a dynamic learning experience that is individualized and allows students to access information both more easily and more quickly."*

The prominence of educational use of social media, blogs and game-based applications enables students to establish a connection between their daily lives and education. This makes it easier for students to see Turkish as a means of communication and contributes to their development of a more positive attitude towards learning.

Results for the Second Theme: 'Use of Digital Tools by Turkish Teachers

Participants stated that Turkish teachers use digital tools for different purposes and in various ways. It was emphasized that these tools make teaching more effective and student-centered in both classroom and extracurricular processes.

P10: "*The method of guessing before starting to explain the subject is used, while the method of intuition is used during the subject explanation and while repetition is done after the subject explanation. When using the guessing method, animations related to the subject are watched and the student is asked to make a guess about the subject to be covered based on the animation. When a problem situation is given, news from various sources are watched and they are asked to present results regarding the solution of the problem and to prepare this presentation in a digital environment. In the speaking activities carried out to develop speaking skills, they are asked to make a speech by researching from digital sources and preparing a presentation on the computer. At the end of the lesson or at the end of the subject, games and activities related to the subject are opened from various applications to ensure the permanence of what is learned, students perform these activities on the smart board, thus making the lesson both more fun and more permanent."*

This structured teaching approach refers to the planned integration of digital content with pedagogical purposes. Participants stated that they do not only use digital tools, but also diversify these tools according to their teaching strategies.

P15: *"Instead of textbooks, teachers can use presentations that they have prepared in a digital environment and planned in accordance with their own narrative style and* *grade level. In addition, videos, animated visuals, digital programs, and games are also used in Turkish lessons. We can also say that teaching is effective thanks to this planning, which makes the lesson flow dynamic and increases motivation. Smart boards, which are found in almost every school, also fully support digital learning and are mostly preferred in teaching in the schools where they are located."*

This view suggests that teachers become digital content producers and personalize instruction, which increases both student interest and teacher creativity.

P3: "*Teachers use digital tools in Turkish lessons in different ways to help students learn more effectively. First, they teach grammar and spelling rules through digital software and applications. These tools allow students to practice at their own pace and see their mistakes immediately. In addition, teachers enrich their course content with digital books and online resources, thus providing students with a wider range of material. Multimedia content such as videos and audiobooks are frequently used to help students develop their reading comprehension skills. Teachers provide students with instant feedback and help them solve their questions in a digital environment by teaching on online platforms (e.g. Google Classroom or Zoom). They also provide students with the opportunity to develop their written expression skills through tools such as social media and blogs."*

P5: "*When examined under today's conditions, it is possible to say that the most frequently used method by teachers in using digital tools is interactive boards and projections. We can say that teachers use digital tools in Turkish lessons in this way with activities such as using videos and visual materials offered by interactive boards in the classroom environment, teachers' reflecting the coursebook they use in the teaching process in e-book format on the interactive board, presenting the materials used for listening/watching skills to the student together with visuals, using various online games in grammar teaching in a supportive way, creating tests and homework through the EIN system and managing the student's learning process."*

P11: *"Most teachers can also produce content in their fields of expertise through their own social media accounts on social media platforms. The subjects that Turkish teachers are generally active in are; content production for students preparing for the* *exam process, content production for grammar teaching, developing tactics for questions in the exams."*

P7: "*Teachers can organize quizzes and competitions using applications such as Kahoot, Quizizz, Socrative, or they can conduct the lesson interactively by using platforms such as EIN and YouTube. They can use applications such as Zoom and Google classroom for online lessons. They can also use digital libraries to find books."*

P6: *"They (teachers) use it to show videos for students, deliver listening exams, and explain topics through slides."*

This statement shows that digital tools are actively used in the evaluation process. Individual learning monitoring draws attention as an important indicator of student-centered education.

P3*: "Turkish teachers use digital tools to make their lessons more interactive and productive. While lessons are visually enriched with smart boards and digital materials, students are provided with individual homework and feedback via digital platforms. Students' participation in lessons is increased with gamification applications (such as Wordwall), and writing and speaking skills are supported with voice recording and video projects."*

According to the views of the participants, technological tools help students develop their language skills interactively by ensuring their active participation in the lesson. These digital tools allow teachers to present lesson content in a more creative and student-focused way.

Results for the Third Theme: 'Digital Tool Suggestions that can be Used in Turkish Lessons'

Participants offered suggestions for digital tools that could be used in Turkish lessons. These suggestions aim to facilitate students' understanding of abstract concepts, develop productive language skills, and provide an enjoyable learning environment.

P7: "*Since the subjects in Turkish courses are mostly abstract, their retention is low in students. It is necessary to provide experiences that will make the subjects concrete and transfer them to daily life. Using various visual and auditory materials increases* *retention. In addition, students often have problems with their narrative skills (speaking, writing). Students can be encouraged to write and speak with digital applications. Learning can be achieved by having fun by repeating the subject with digital games. While developing reading skills, importance can be given to visual reading and activities can be carried out through various visuals presented in the digital environment. Especially grammar subjects can be made more permanent with fun competitions such as find the correct one, pop the right balloon, fill the right apples in the basket, guess the answers and save the family held in the digital environment."*

This proposal highlights how multimedia elements support learning and the positive effects of gamification on students. Teaching abstract and challenging subjects, especially grammar, through games both concretizes learning and makes it more accessible to students.

P6: *"Most Turkish teachers have definitely done homework such as "writing compositions, essays" at some point in their lives. Instead of writing these on papers, collecting them or throwing them in a corner without looking at them, a blog page (which can be personalized) can be created and written. In this way, students' digital literacy will develop and they will have the chance to reach a wider audience. Perhaps many students will discover their hidden talents (writers, poets, etc.) in this way."*

P13: "*Course content channels can be created on social media and short information about the subject can be shared on these channels every day. Pages can be opened in the name of our valuable poets and writers and blogs can be created. Poetry websites can be created and students can be asked to share their poems here. Listening activities can be done on podcast websites."*

P11: *"Students can use digital storytelling to develop their written expression skills. Students can write their own stories using digital tools and present them with video or audio narration. Another option is to organize out-of-class interactions through social media and forums. Students can write blog posts on topics determined by their teachers, and observe the use of Turkish in social media language by sharing short texts and quotes on platforms such as Twitter or Instagram. In this way, students can* *better understand how the language is used in daily life."*

P9:*"Turkish teachers can also instantly measure students' knowledge levels and identify their deficiencies more quickly by using interactive exams and quizzes. Such digital exams allow students to track their own performance."*

P2*: "Students can improve their pronunciation with activities of reading aloud and language analysis applications. Audio books or speech recognition applications can help students learn words with correct pronunciation. Thus, an effective environment in which practices are made is provided for speaking and writing Turkish correctly. All these methods allow digital tools to be used in a versatile and creative way in teaching, and attract students' attention and ensure their active participation in the lesson."*

P5: *"Blogging activities can be organized to develop writing skills. Teachers can control their students, and in this sense, writing skills can be developed with various discussions. Writing activities can be done for creative writing studies, and these studies can be supported visually with tools such as canva. It can be used for storytelling. Literary works taught in Turkish lessons can also be listened to in audiobook format. Along with the development of reading skills, audiobook formats can also be used to develop listening skills of these texts. In the use of digital tools to develop students' speaking skills, students can have video presentations. The speaking skills of students who give speeches on certain topics can be evaluated."*

This suggestion points to the importance of providing students with a learning environment based on production and sharing. Blogging activities develop not only writing skills but also critical thinking and digital expression skills.

P13: "*Diagnostic measurement and evaluation activities for speaking skills can also be done in the classroom environment with powerpoint presentation files."*

P2: "C*oncept maps can be prepared in a digital environment while teaching vocabulary."*

P4: *"By creating virtual book clubs, students can read a certain work together and* *manage their discussions in a virtual environment. Blogging can also be used in this sense; character analyses, plot analyses, and studies on the language and narration of the text can be supported with blogging activities."*

P5: "*Digital applications provide meaningful learning by bringing the language into daily life. They also allow students to develop their language skills naturally within the digital culture."*

P12: "*They can organize virtual tours of important places related to literature or establish a reading group through digital platforms (for example, Goodreads, Discord, etc.) and have students read and comment on certain books."*

P7: "*By creating social interaction groups, classroom communication can be transferred to the outside world and students can be socialized."*

These suggestions reveal the importance of collaborative spaces created in digital environments so that students can read critically and gain intellectual depth on literary texts.

Results for the Fourth Theme: 'Digital Tool Suggestions that can be Used in Turkish Lessons'

Participants agree that digital transformation increases student engagement and positively affects their motivation to learn. The flexibility provided by digital tools allows students to manage their own learning processes.

P2*: "Touching a book, browsing through the pages of a book is always beautiful, romantic and nostalgic. Without denying this, we must also say this: We are now in an age where technology is rapidly developing and sometimes we cannot keep up with this speed. For this reason, considering today's requirements, using digital accessibility is important in terms of saving time, money and usability of technology. We can find the answers to many of the information we are looking for in the digital environment, this is a motivation in itself and an attractive element for reuse. We can also look at it from this perspective: we can easily have meetings that need to work together but are far from each other or cannot be together at the same time, and that* *will cause a certain financial burden, in the digital environment. In other words, we can say that the digital environment is completely economical. We can say that it has become attractive especially for students in this respect."*

According to participants, feedback given in a digital environment allows students to observe their own development and helps them develop a positive attitude towards learning.

P3: *"Digital transformation provides many advantages that increase students' motivation and encourage their participation in lessons. However, in order for this process to be successful, digital tools must be selected in accordance with pedagogical purposes and used with effective planning. Teachers who have sufficient knowledge and skills in this regard are an effective element in the success of the process. Digital transformation encourages students to be responsible for their own learning processes. At the same time, digital transformation facilitates students' access to materials to be used in the learning process. Students can easily access the materials."*

This statement shows that the flexibility offered by digital learning environments has a positive effect on students. In particular, access to resources that are suitable for individual learning pace encourages students to actively participate in the process and take responsibility.

P10: *"In digital environments, students engage in an activity aimed at sharing information and solidarity. It is beneficial in increasing motivation. Online learning environments offer students independence of time and space. This flexibility makes it easier for students to work according to their own schedules. Digital tools make students' progress visible."*

This statement shows that digital transformation offers not only an individual but also a social learning environment. Students both collaborate and learn from each other.

P8: *"The curiosity and sense of learning that digital tools create positively affect the learning process."*

P15: *"Digital transformation has a very positive effect on student motivation and participation. Adapting technology to the educational process increases students' interest in lessons and encourages them to participate more actively in the lesson.*

> *Gamification applications and fun and competitive elements increase students' motivation in lessons and direct them to put more effort into the lesson. In addition, project work and interactive activities carried out on digital platforms contribute more to students' learning by increasing their interaction within the group and participation in the lesson. Technological tools reinforce students' motivation by providing customized feedback and instant performance evaluations for students' individual needs. As a result, digital transformation makes the educational process more efficient by ensuring that students participate more willingly and effectively in learning."*

This view emphasizes the motivational power of digital tools. Fun and competitive elements add a gamified structure to the learning process and increase student engagement. As a result of the interviews, it was seen that the majority of the prospective teachers agreed that digital tools support the learning process in teaching Turkish and increase the motivation of the students.

Conclusion and Recommendations

The use of digital technologies in teaching Turkish plays an important role in making educational processes more effective, interesting and student-focused. In addition to improving grammar, reading and writing skills, digital tools increase students' motivation and ensure their active participation in learning processes. Many digital tools such as e-books, multimedia content, interactive boards and online platforms increase students' interest in the course by offering them rich and diverse learning materials. Thanks to gamification applications and digital software, students are more involved in the learning process and their motivation in the course increases. According to the data obtained from student responses, it is seen that teachers develop different strategies in the use of digital tools and that these strategies are suitable for the individual learning speeds of students. Enriching lessons with videos, animations and other digital materials increases the permanence in the learning process and enables students to concretize abstract topics. In addition, lessons conducted through online platforms provide students with the opportunity for simultaneous feedback, making their learning experiences more individualized.

However, teachers' pedagogical knowledge and skills gain importance in the digital transformation process. Effective integration of digital tools is possible when teachers choose these tools in accordance with pedagogical purposes and integrate them into their learning processes. Therefore, in order to successfully continue the digital transformation in education, teachers' digital competencies need to be increased. Digital transformation positively affects student motivation and participation, making educational processes richer and more efficient. However, for this process to be successful, it is of great importance that teachers have the competencies to use digital tools effectively. Digital transformation in education encourages students to be responsible for their learning processes, while making them more active participants in a technology-based learning environment. This indicates that the digitalization process of education will become even more important in the future.

Recommendations

In line with the results obtained in this study, the following recommendations can be made in order to carry out the digital transformation process in Turkish language teaching more effectively and efficiently:

- Relevant courses on the pedagogical use of digital tools should be added to teacher training programs in education faculties; it should be ensured that teacher candidates not only get to know technology but also use these technologies in line with teaching goals. Moreover, in-service trainings with continuous and up-to-date content should be provided to Turkish language teachers on the effective use of digital tools for educational purposes. These trainings should cover both technical skills and pedagogical compliance.
- Additionally, in order for the digital transformation to be successful, basic infrastructure deficiencies such as internet connection, smart boards, and projectors should be eliminated; technological inequalities should be prevented.
- Preservice and in service teachers should be guided so that applications such as Kahoot, Quizizz, Socrative can be used not only for motivational purposes and in teaching processes but also for qualified measurement and evaluation purposes.
- Lastly, research conducted in the field of educational sciences should be reflected in digital applications; continuous improvement should be ensured by supporting the data obtained in applications with academic studies.

Notes

This study is an expanded and improved version of the presentation entitled "Digital Transformation in Turkish Language Teaching: Integration of Technology and Use of Digital Tools" given orally at the "International Conference on Research in Education and Science" held on April 28-May 1, 2025.

References

Aksu, H. (2018). *Dijitopya: Dijital dönüşüm yolculuk rehberi*. Pusula.

Alakoç, Z. (2003). Matematik öğretiminde teknolojik modern öğretim yaklaşımları. *The Turkish Online Journal of Educational Technology*, 43–49.

Alexander, B., Ashford-Rowe, K., Barajas-Murphy, N., Dobbin, G., Knott, J., McCormack, M., & Weber, N. (2019). *Educause horizon report: 2019 higher education edition*. Educause.

Altın, H. M., & Kalelioğlu, F. (2015). Fatih projesi ile ilgili öğrenci ve öğretmen görüşleri. *Başkent University Journal of Education*, *2*(1), 89–105.

Bello, A. A. (2014). *Impact of technology interventions on student achievement in rural Nigerian schools* (Publication No. 3632420) [Doctoral dissertation, Walden University]. Walden University ScholarWorks.

Bozkurt, A., & Cilavdaroğlu, A. K. (2011). Matematik ve sınıf öğretmenlerinin teknolojiyi kullanma ve derslerine teknolojiyi entegre etme algıları. *Kastamonu Eğitim Dergisi*, *19*(3), 859–870.

Brooks, S. (2013). *Making technology standards work for you*. ISTE. http://www.iste.org/docs/excerpts/MATEC3-excerpt.pdf

Cabı, E., & Ergün, E. (2016). The impact of instructional technologies and material development course on the teacher candidates' concern about using educational technologies. *Başkent University Journal of Education*, *3*(1), 37–43.

Canbazoğlu Bilici, S. (2012). *Preservice science teachers' technological pedagogical content knowledge and self-efficacy* [Unpublished doctoral dissertation]. Gazi University Institute of Educational Sciences.

Creswell, J. W., & Plano Clark, V. L. (2011). *Designing and conducting mixed methods research* (2. baskı). Sage.

Creswell, W. J. (2007). *Qualitative inquiry and research design: Choosing among five approaches* (2. baskı). Sage.

Cumhur, F., & Çam, S. S. (2021). Digital transformation in assessment and evaluation course: The effects of Web 2.0 tools. *Journal of Pedagogical Research*, *5*(3), 15–39.

Çam, Ş. S. (2019). *A professional development program proposal for the development of technological pedagogical content knowledge of the teacher educators* [Unpublished doctoral dissertation]. Gazi University Institute of Educational Sciences.

DEÜSEM. (2020). *Endüstri 4.0 ve dijital dönüşüm uzmanlığı*. http://desem.deu.edu.tr/tr/endustri-4-0-ve-dijital-donusum-uzmanligi-sertifika-programi/

Ergüney, M. (2015). Uzaktan eğitimin geleceği: MOOC (Massive open online course). *Eğitim ve Öğretim Araştırmaları Dergisi*, *4*(4), 15–22.

Erstad, O., & Zounek, J. (2018). Digital youth and their ways of learning. *Studia Paedagogica*, *23*(4), 5–10.

Ertmer, P., & Newby, T. (2013). Behaviorism, cognitivism, and constructivism: Connecting yesterday's theories to today's contexts. *Performance Improvement Quarterly*, *26*(2), 65–71.

Johns, K. (2015). Engaging and assessing students with technology: A review of Kahoot! *Delta Kappa Gamma Bulletin*, *81*(4), 89–91.

Jones, S., Johnson-Yale, C., Millermaier, S., & Pérez, F. S. (2009). US college students' Internet use: Race, gender and digital divides. *Journal of Computer-Mediated Communication*, *14*(2), 244–264. https://doi.org/10.1111/j.1083-6101.2009.01439.x

Kabakçı Yurdakul, I. (2011). Öğretmen adaylarının teknopedagojik eğitim yeterliklerinin bilgi ve iletişim teknolojilerini kullanımları açısından incelenmesi. *Hacettepe Üniversitesi Eğitim Fakültesi Dergisi*, *40*(1), 397–408.

Kabakçı-Yurdakul, I., Çoklar, A. N., Birinci, G., & Kılıçer, K. (2012). *Öğretmen adaylarına yönelik teknopedagojik eğitimin etkisi ve teknopedagojik eğitime yönelik bir model önerisi* (Proje No. 109K191). TÜBİTAK.

Kabakçı-Yurdakul, I., Odabaşı, H. F., Kılıçer, K., Çoklar, A. N., Birinci, G., & Kurt, A. A. (2014). Constructing technopedagogical education based on teacher competencies in terms of national standards. *Elementary Education Online*, *13*(4), 1185–1202.

Kale, U., & Goh, D. (2014). Teaching style, ICT experience and teachers' attitudes toward teaching with Web 2.0. *Education and Information Technologies*, *19*(1), 41–60.

Kayaduman, H., Sırakaya, M., & Seferoğlu, S. S. (2011). *Eğitimde FATİH projesinin* *öğretmenlerin yeterlik durumları açısından incelenmesi* [Bildiri sunumu]. Akademik Bilişim 2011, İnönü Üniversitesi, Malatya.

Kurt, A. A. (2013). Eğitimde teknoloji entegrasyonuna kavramsal ve kuramsal bakış. I. K. Yurdakul (Ed.), *Teknopedagojik eğitime dayalı öğretim teknolojileri ve materyal tasarımı* içinde (s. 1–38). Anı Yayıncılık.

Maddux, C. D., & Johnson, D. L. (2006). Type II applications of information technology in education: The next revolution. *Computers in the Schools*, *23*(1-2), 1–6.

Mazman, S. G., & Koçak-Usluel, Y. (2011). Bilgi ve iletişim teknolojilerinin öğrenme-öğretme süreçlerine entegrasyonu: Modeller ve göstergeler. *Eğitim Teknolojisi Kuram ve Uygulama*, *1*(1), 62–79.

Milli Eğitim Bakanlığı. (2018). *TTKB öğretim programları*. http://mufredat.meb.gov.tr/Programlar.aspx

Şener, G., & Gündüzalp, S. (2018). Akademisyenlerin üniversitelerde dijital dönüşüm ile ilgili görüşleri [Bildiri özeti]. *1. Uluslararası Battalgazi Multi Disipliner Çalışmalar Kongresi* (ss. 178–182).

Tapscott, D. (1998). *Growing up digital: The rise of the net generation*. McGraw-Hill.

TEKNOFEST. (2020). *Havacılık, uzay ve teknoloji festivali*. https://teknofest.org/

Telli, S. G., & Altun, D. (2020). Koronavirüs ve çevrimiçi (online) eğitimin önlenemeyen yükselişi. *Üniversite Araştırmaları Dergisi*, *3*(1), 25–34.

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Chapter 8 - The Future of Learning: Embracing Technology and Human Connection

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Chapter Highlights

- Education is rapidly transforming due to technological advancements and evolving pedagogical understanding. This document examines the future of learning by exploring the crucial interaction between technology and human connection, both of which are vital for optimal outcomes.
- It examines how emerging technologies, including AI, VR/AR, personalized learning platforms, learning analytics, and IoT, are transforming education from high school to professional development. While these technologies offer personalized, adaptive, and scalable solutions (e.g., automated feedback, intelligent tutoring), this document emphasizes the enduring importance of human interaction, mentorship, collaborative learning, and social-emotional learning (SEL) for well-rounded growth.
- Crucial skills like critical thinking, creativity, communication, and collaboration thrive through human interaction. The document analyzes the benefits and challenges of technology integration, considering accessibility, equity, data privacy, AI ethics, and educators' evolving roles as facilitators and mentors.
- It argues for a balanced approach leveraging technology to enhance, not replace, human connection, creating engaging, personalized, effective, equitable, and human-centered learning experiences that prepare learners for a rapidly changing world.

Introduction

As society and technology continually reshape our world, education, too, finds itself in constant flux (Barber & Mourshed, 2009; Ozturk, 2023). The digital age has woven technology into every learning environment, from primary schools to corporate training (Ceran, 2022, 2025; Çakır et al., 2019; Kibici, 2025; Ozturk 2023; Öztürk et al., 2021; Tekin, 2025). This presents a fascinating paradox: immense opportunities for innovation alongside complex challenges to traditional teaching methods. Technology has undeniably revolutionized how we access and share knowledge (Khan et al., 2024; Turgut & Öztürk, 2025). Yet, a crucial question remains: how can we strategically use these digital advancements without diminishing vital human elements? These elements are fundamental to fostering deep understanding, critical thinking, compassionate connections, and effective collaboration.

The rapid advancement of technology has resulted in a wave of transformative tools poised to reshape the educational landscape. At the forefront of this revolution is Artificial Intelligence (AI). Platforms such as Duolingo, with its adaptive language learning exercises, and Coursera, offering a vast array of courses tailored to individual progress, exemplify AI's potential to create truly personalized learning journeys. By dynamically adjusting content, pacing, and feedback based on a learner's unique needs and performance, AI promises to optimize individual learning outcomes (Hussain et al., 2023). Furthermore, Virtual Reality (VR) and Augmented Reality (AR) technologies are creating increasingly immersive and engaging learning environments (Hussain et al., 2022; Khan et al., 2021). These tools offer the unique ability to simulate real-world scenarios, providing invaluable experiential learning opportunities across diverse disciplines such as surgical training in medicine, complex system design in engineering, and historical changes that promote a deeper connection with the past (Zhu et al., 2020). Beyond direct instruction, sophisticated data analytics is revolutionizing educational assessment. By providing educators with real-time, detailed insights into student performance patterns and identifying specific learning gaps, data-driven interventions can be implemented promptly and precisely, ensuring that support is targeted and effective (Siemens et al., 2013). Finally, gamification, the strategic integration of game-design elements and game principles in non-game contexts, has emerged as a potent strategy for enhancing learner motivation and sustained engagement by leveraging intrinsic rewards and interactive challenges (Deterding et al., 2011).

Despite the undeniably compelling advantages offered by these technological advancements, a growing body of scholarly work urges a cautious and critical approach to their wholesale adoption in education (Khan, Hussain, Al Siyabi, et al., 2024). Concerns are increasingly being raised regarding potential negative consequences that warrant careful consideration. These include the emerging phenomenon of digital fatigue, characterized by cognitive overload and diminished attention spans resulting from excessive screen time and constant digital stimulation. Furthermore, there is a significant risk of aggravating existing societal inequities, as disparities in access to reliable technological infrastructure and varying levels of digital literacy can create a "digital divide," leaving marginalized learners further behind. Perhaps most critically, anxieties are mounting a potential decline in essential interpersonal interactions, those face-to-face exchanges that stimulate social-emotional development, build strong relationships, and cultivate crucial communication skills (Selwyn, 2016). These well-founded cautions underscore the critical need for a refined and balanced perspective, one that thoughtfully considers the inherent limitations alongside the much-touted affordances of technology in the intricate process of learning.

Conversely, the indispensable role of human connection in ensuring effective education has long been a foundational principle within educational theory and practical application. Lev Vygotsky’s (1978) seminal sociocultural theory powerfully emphasizes the fundamental importance of social interaction and collaborative dialogue in driving cognitive development, suggesting that learning is inherently a social process (Cole & Scribner, 1978). Similarly, the increasing recognition of emotional intelligence encompasses the crucial abilities to accurately perceive, deeply understand, and effectively manage both one's own emotions and the emotions of others. This vital component of effective teaching and meaningful learning continued to gain momentum (Culver, 1998). Empirical studies across diverse educational contexts consistently demonstrate that strong, positive, and supportive teacher-student relationships are significantly correlated with enhanced student motivation, increased academic engagement, and ultimately, improved academic outcomes (Ciddi, 2025; Roorda et al., 2011). Moreover, collaborative learning among peers has been repeatedly shown to cultivate higher-order thinking skills, including sophisticated critical analysis, creative problem-solving, and effective communication, effectively preparing students to navigate the complex social and professional landscapes of real-world challenges (Johnson & Johnson, 1999). However, as technology increasingly mediates learning experiences, legitimate and pressing concerns arise about the potential erosion of these crucial interpersonal dynamics, the very foundation upon which holistic development and meaningful learning are built.

Recognizing the inherent and distinct value of both cutting-edge technological innovation and rich human interaction, the evolving field of educational research is increasingly focusing on the critical task of finding a harmonious and effective balance between these two essential domains. Hybrid learning models, which strategically and thoughtfully combine the unique advantages of online digital tools with meaningful, in-person face-to-face interactions, offer a particularly promising avenue for achieving this crucial equilibrium (Hussain et al., 2024). For instance, well-designed blended learning environments effectively leverage the inherent flexibility and scalability of digital technology to deliver content and facilitate independent learning, while simultaneously preserving valuable opportunities for in-person collaborative projects, direct mentorship from educators, and the crucial development of social-emotional skills through direct human connection (Garrison & Vaughan, 2008). Furthermore, a growing emphasis on “co-design”. It is a collaborative and iterative process that actively involves both experienced educators and skilled technologists in the development of user-centered learning tools and platforms that hold significant potential for creating truly effective and humanistic educational technologies (Laurillard et al., 2018). Such collaborative initiatives aim to ensure that technological solutions are not merely adopted for their novelty or efficiency but are thoughtfully and deliberately aligned with sound pedagogical principles, explicitly emphasizing the cultivation of empathy, the promotion of meaningful social engagement, and the promotion of holistic cognitive and affective outcomes in learners. This balanced and human-centered approach, one that prioritizes both technological advancement and the fundamental needs of human learners, is crucial for navigating the rapidly evolving landscape of education and for effectively preparing learners to thrive in the multifaceted and increasingly complex demands of the future.

Building upon the understanding that education's trajectory is intertwined with societal and technological shifts, the current digital era presents both unprecedented opportunities and complex challenges. The pervasive integration of technology across various educational settings, from formal schooling to professional training, has undeniably reshaped the landscape of knowledge acquisition and dissemination (Khan et al., 2022; Khan, Hussain, Al-

Siyabi, et al., 2024). However, the fundamental question persists: how can we harness the power of these technological innovations without diminishing the crucial human elements that develop deep learning, critical reasoning, empathetic understanding, and collaborative competence?

Studies reveal that strong teacher-student relationships enhance motivation, engagement, and academic outcomes (Roorda et al., 2011). Peer collaboration boosts critical thinking and problem-solving skills, preparing students for real-world challenges (Johnson & Johnson, 1999). As technology mediates more learning experiences, concerns arise about the erosion of these vital interpersonal dynamics. This document highlights the dual imperative of using technology while preserving and enhancing human connections in educational contexts.

Education in Transformation

Rapid technological advancements are fundamentally reshaping the educational landscape, driving a move from traditional, teacher-centric models to more dynamic, student-centered, and technology-centered learning environments (Halverson & Smith, 2009). This change is evident across various facets of education:

Greater Accessibility and Personalization: Technology has dramatically expanded access to knowledge and learning opportunities, breaking down geographical barriers through online platforms and digital resources. Artificial intelligence (AI) plays a crucial role in personalizing learning experiences, adapting content and pace to individual student needs, and providing tailored feedback.

Higher Engagement and Immersion: These technologies are revolutionizing engineering training and skill development by offering immersive, interactive experiences that simulate real-world environments. Applications of VR and AR are becoming essential components of engineering education and practice. Through the creation of immersive, hands-on experiences, these technologies not only benefit engineers in developing technical skills but also augment their understanding of three-dimensional visualization, safety protocols, and innovative problem-solving.

Encouraging Collaboration and Developing Critical Thinking: Digital tools facilitate seamless collaboration among students, allowing them to work on projects together regardless of physical location. While AI can assist with tasks like lesson planning and grading, it also presents opportunities for students to develop critical thinking skills by evaluating the accuracy and biases of AI-generated information (Hussain et al., 2023).

Evolution of Teaching Roles: Our understanding of effective pedagogy is also in a continuous state of evolution. This dynamic process is driven by ongoing research into how people learn, societal shifts, and technological advancements. What was once considered "best practice" in teaching is constantly being refined and expanded upon. Transformation of education from teacher-centered to student-centered, the teacher's role has evolved from a "sage on the stage" to a "guide on the side" with technology empowering students to take more responsibility for their learning. Educators are increasingly leveraging digital tools to create interactive learning environments, differentiate instruction, and provide personalized support.

It can’t be denied that technology is taking a central role in the delivery of educational programs and altering the role of educators; however, alongside these promises by technology, challenges persist, including ensuring equitable access to technology (the "digital divide") and addressing concerns around data privacy and security (Cloete, 2017). Future trends predict further integration of AI, the expansion of micro-credentials, and the continued evolution of hybrid learning models. Research continues to explore the effective integration of technology to optimize teaching and learning outcomes, emphasizing the need for robust methodologies to assess its true impact without any ethical issues. Thus, future learning hinges on a powerful synergy between technology and human connection. While technology offers unprecedented access to information, personalized learning paths, and engaging tools, it cannot replace the delicate understanding, compassion, and social development promoted through human interaction. Effective pedagogy in the future will leverage technology to enhance efficiency and reach, but prioritize the essential human elements of mentorship, collaborative problem-solving, emotional intelligence, and genuine relationships to achieve truly optimal learning outcomes.

Emerging Technologies Driving the Change

The "technological tide" is not merely a ripple but a powerful current fundamentally reshaping the very foundations of learning. Gone are the days when education primarily revolved around chalkboards and textbooks; today's classrooms, both physical and virtual, are dynamic spaces infused with digital tools and platforms. This shift is driven by rapid advancements in areas like artificial intelligence, virtual and augmented reality, and widespread internet connectivity, all of which are continuously expanding the possibilities for how knowledge is acquired, disseminated, and applied.

This transformative wave offers unprecedented opportunities for personalized learning, allowing educational content and pace to adapt to individual student needs and preferences. AI-powered systems can identify learning gaps, provide targeted feedback, and even generate customized lesson plans, empowering students to take greater ownership of their educational journey. Furthermore, technology amplifies engagement through interactive simulations, gamified learning experiences, and immersive virtual environments. The environment can be generated for any specialization of study, ranging from social sciences to most technical subjects in engineering. Students can explore historical sites in VR, conduct virtual science experiments, or collaborate on projects with peers across the globe, making learning more vibrant, relevant, and memorable.

On the other hand, this technological tide also brings with it significant challenges. The gap between those who have access to technology, the internet, and digital literacy and those who do not remains a pressing concern, as unequal access to devices and reliable internet can aggravate existing educational disparities. Furthermore, concerns about data privacy and security are paramount as more personal and academic information is collected and stored digitally. Beyond these practical considerations, educators deal with how to ensure that technology enhances, rather than diminishes, crucial human elements of learning, such as critical thinking, genuine interpersonal connection, and the development of empathy. The ongoing evolution of learning demands careful consideration of both the immense potential and the complex implications of this technological revolution.

Benefits of Technology Integration

The promise of technology in education lies in its ability to deliver both personalized and scalable learning, fundamentally transforming how individuals acquire knowledge and skills.

Both advancements have their advantages, which are further elaborated in the following paragraphs.

Personalized learning influences technological advancements, particularly in Artificial Intelligence (AI) and data analytics, to tailor educational experiences to the unique needs of each student (Saragih, 2024). Adaptive learning platforms adjust the difficulty, content, and pace of material in real-time based on a student's performance, identifying knowledge gaps and providing targeted support. AI can also act as a virtual tutor, offering immediate feedback, answering questions, and generating customized learning materials like quizzes and flashcards. Another advantage of this integration is that educators gain valuable insights into the learning patterns of their students. This allows them to make informed decisions and intervene proactively when students struggle or excel (Ayeni et al., 2024). This approach promotes greater student engagement and ownership over their learning journey.

Simultaneously, technology enables scalable learning, extending high-quality education to a massive audience, transcending geographical and logistical barriers. Cloud-based infrastructures and digital content allow access to educational resources anytime, anywhere, on any device. This significantly reduces the cost and time associated with traditional face-to-face training. While Massive Open Online Courses (MOOCs) demonstrate scalability by providing instruction to thousands, or even millions, of participants simultaneously, often from leading institutions, making education more accessible and affordable globally. Along with this, the repetitive tasks for educators are automated and streamlined through the use of AI applications (Igbokwe, 2023) e.g., grading and scheduling, freeing up their time to focus on direct instruction, mentorship, and fostering creativity.

Therefore, the capacity for personalized and scalable learning through technology creates a more efficient, engaging, inclusive, and equitable educational landscape. It also prepares students for lifelong success by adapting to their requirements and expanding access to knowledge for diverse populations.

Indispensable Human Element

While technology offers powerful tools for learning, the enduring power of human connection remains an essential element for achieving optimal educational outcomes. Learning is inherently a social process, and the quality of relationships within the learning environment significantly influences student engagement, well-being, and academic success. Researchers have time and again shown through their work that collective exploration brings together creativity, community, and openness (Nerantzi et al., 2021). There are several key aspects of human connection's enduring power in education, some of which are elaborated in subsequent paragraphs.

When students feel genuinely cared for, understood, and valued by teachers and peers. These students develop a sense of belonging and psychological safety. This environment promotes resilience, reduces stress, and encourages students to take intellectual risks, ask questions, and engage more deeply with the material (Wanless, 2016). Sympathetic teacher-student relationships are critical to learning. When teachers demonstrate compassion, it creates a more positive classroom climate, enhances communication, and helps students feel seen and supported. These interactions are vital for students to develop their social-emotional skills, including empathy, communication, and conflict resolution, which are essential for both academic achievement and life beyond school.

Positive educator-learner relationships can be life-changing, providing guidance, support, and motivation. Teachers who build strong rapport can increase student attendance, boost intrinsic motivation to learn, and improve self-regulation, ultimately contributing to better academic performance and a reduction in disruptive behaviors (Schut et al., 2020; Spitz, 2019). Human connection facilitates effective social learning. Through peer-to-peer interaction, group work, and co-teaching models, learners can share insights, build skills organically, and raise understanding. This collaborative dynamic enhances our distinctive human capacity for social harmonization, where shared experiences and mutual understanding boost collective engagement and learning. In spirit, while technology provides infrastructure and tools. The human touch, compassion, mentorship, and sense of community unlock a learner's potential and promote holistic development.

Crucial Skills Development

Human connection is a creative ground for the development of essential skills that are increasingly valued in all aspects of life, particularly in learning and work environments.

These "human skills" or "soft skills" are fundamentally interpersonal and cannot be effectively cultivated in isolation. These key skills that flourish through human connection are elaborated in the subsequent paragraphs.

Effective verbal and non-verbal communication, active listening, and the ability to articulate thoughts clearly and compassionately are polished through regular interaction. Engaging in discussions, asking clarifying questions, and understanding delicate indicators are all skills that develop in a social context (Levasseur, 2013). Emotional Intelligence (EQ) encompasses self-awareness, self-management, social awareness, and relationship management. Human connection provides the opportunity to recognize and manage one's own emotions while also understanding and responding appropriately to the emotions of others. Compassion, a crucial component of EQ, is cultivated by putting oneself in another's shoes and understanding their perspective (Majid, 2017).

Working effectively with others towards a shared goal is a hallmark of human connection. This involves delegating roles, managing responsibilities, negotiating, and contributing diverse ideas. Collaborative problem-solving, a highly sought-after skill, is significantly enhanced when individuals can pool their intellectual resources and build trust through interaction (Bridges et al., 2011).

Disagreements among team members are an inevitable part of human interaction. The ability to navigate conflicts constructively, listen to different viewpoints, and find mutually acceptable solutions is developed through direct engagement and practice (Lopes et al., 2015). In contrast, human connections provide a supportive network that helps individuals handle stress and change. Learning from failures, receiving constructive feedback, and adapting communication styles to suit different social situations are often facilitated by the presence of caring peers and mentors (Mashigo, 2014). These skills are not just useful; they are crucial for achieving academic success, maintaining personal well-being, and advancing future career opportunities in a rapidly interconnected and technology-driven world.

Navigating the Challenges of Technology Integration

Integrating technology into education offers promising potential, but it also brings a complex set of challenges that necessitate thoughtful planning and strategic approaches. Without addressing these hurdles, the full potential of educational technology may remain suppressed, and existing inequalities could even be aggravated.

Accessibility and Equity: One significant challenge in technology integration is the digital divide, which refers to unequal access to technology and internet connectivity among students and communities. Socioeconomic disparities, geographic location, and lack of proper infrastructure can prevent many learners from accessing the essential tools needed for effective participation in technology-based education, thereby increasing the divide between the privileged class of students and the deprived ones. Addressing this requires concerted efforts to ensure equitable access to devices and reliable internet for all students (Dean, 2020; Malik, 2018).

Data Privacy and Security: Data privacy and security are other critical concerns to be addressed while integrating technology. As educational technologies collect vast amounts of student data, from academic performance to behavioral patterns, safeguarding this sensitive information becomes an essential requirement. There are concerns about potential data breaches, misuse of information by third parties, and the ethical implications of data profiling. This demands that robust data governance policies, transparent consent mechanisms, and comprehensive security protocols are in place to protect student privacy and learners' trust in digital learning systems (Airaj, 2024; Rousi et al., 2024).

Educator's Evolving Role: Preparedness and professional development of teaching staff are key to the successful technology integration. Many educators may lack the necessary knowledge, skills, or confidence to effectively integrate digital tools into their teaching practices, often due to insufficient training and support. In order to address this challenge, ongoing and relevant professional development programs are critical. These programs should emphasize more than just the development of technical skills. They must also cover teaching strategies for using technology to supplement how students learn. Without sufficiently skilled educators, even the most advanced technologies will fail to deliver their intended benefits (Ottenbreit-Leftwich et al., 2018; Smadi & Raman, 2024).

AI Ethics: Ethical considerations in the AI-driven educational arena present new complexities. Issues such as algorithmic bias, transparency in AI decision-making, and maintaining student autonomy are key problems to address. Ensuring that AI tools are fair, unprejudiced, and support rather than replace human judgment necessitates careful design, continuous evaluation, and adherence to ethical frameworks. Navigating these challenges effectively is key to ensuring that technology truly serves to improve education for all learners (Borenstein & Howard, 2021; Eden et al., 2024; Foltynek et al., 2023).

Educators from Sage to Guide

Along with other norms, the rapid integration of technology into education is fundamentally altering the role of the educator as well. The educator's role is changing from the traditional "sage on the stage" to a more dynamic "guide on the side." This evolution reflects a pedagogical move away from passive knowledge transmission towards active, student-centered learning (Dankbaar & de Jong, 2014; Khan, Hussain, Al Siyabi, et al., 2024; Stoddart & Niederhauser, 1993). Conventionally, the "sage on the stage" model positions the educator as the primary source and dispenser of information, also known as the teacher-centered learning model. In this setup, teachers delivered lectures, and students primarily absorbed content. However, with readily available information through various sources such as the web and sophisticated digital tools, the exclusive role of information providers has diminished.

The emerging "guide on the side" role emphasizes facilitation, mentorship, and personalized support on the part of the educator (Faxriddinov, 2023). In this capacity, the educators are required to:

- *Curate and Create Learning Experiences:* Instead of simply presenting facts, teachers now design engaging activities and projects, leveraging technology to offer diverse learning paths and resources.

- *Coach and Mentor:* Teachers act as coaches, helping students navigate complex information, develop critical thinking skills, and solve problems independently. This involves asking probing questions, providing timely feedback, and promoting a growth mindset.

- *Facilitate Collaboration:* Educators encourage and manage collaborative learning environments, both online and offline, where peer learning is encouraged, thus students learn from each other and develop essential teamwork skills.

- *Focus on Deeper Understanding:* This change allows teachers to move beyond rote memorization, focusing instead on assisting students to make connections, apply knowledge in real-world contexts, and develop higher-order thinking abilities.

This transformation empowers students to take greater ownership of their learning journeys while positioning educators as indispensable facilitators of exploration, critical inquiry, and skill development in a technologically rich world.

Power of Balance

The future of optimal learning hinges on a balanced approach, recognizing that neither technology nor human connection alone can fully unlock a learner's potential. Instead, the most effective pedagogical models will strategically integrate the strengths of both to create holistic, engaging, and deeply impactful educational experiences (Nerantzi et al., 2021). This digital scaffolding can enhance efficiency, provide immediate data insights for educators, and expand access to specialized knowledge, democratizing education on a global scale (Ayeni et al., 2024). Imagine an AI tutor customizing lessons based on a student's performance, or a virtual reality simulation bringing complex scientific concepts to life for thousands simultaneously.

As discussed above, technology offers unparalleled opportunities for personalized and scalable learning. Adaptive platforms coupled with vast digital resources allow for the tailoring of content to individual needs, enabling learners to progress at their own pace (Saragih, 2024). They can access education regardless of geographical barriers, from online courses offered by universities to user-generated lesson plans. This digital framework can significantly enhance efficiency by automating routine tasks, providing immediate data insights for educators to track progress. They can identify learning gaps and expand access to specialized knowledge, ultimately democratizing educational opportunities on a global scale (Riaz, 2024).

However, the enduring power of human connection remains irreplaceable. Skills like critical thinking, complex problem-solving, emotional intelligence, compassion, and effective collaboration are best cultivated through direct interaction, mentorship, and social learning environments. Teachers provide crucial emotional support, develop a sense of belonging, and guide students through nuanced discussions that build deeper understanding and character (Nerantzi et al., 2021). Human connection ensures that learning is not just about the acquisition of information but also about personal growth, social development, and the cultivation of values.

Therefore, optimal learning in the future will involve a collaborative model where technology acts as a powerful enabler and accelerator. On the other hand, human educators and peer interactions provide the essential guidance, emotional intelligence, and collaborative environments necessary for comprehensive development. This balanced approach ensures that learners are not only well-informed but also well-rounded, capable of thriving in a complex and interconnected world.

Shaping the Future Together

Ultimately, the future of optimal learning hinges on a balanced approach, a collaborative model where technology and human connection complement each other. The pervasive influence of technology is undeniably reshaping the educational landscape, yet the future of optimal learning is not solely digital. Instead, it lies in embracing a human-centered approach, one that strategically integrates technological advancements with the irreplaceable value of human connection and interaction.

Technology can free up educators from administrative tasks and provide data-driven insights, allowing them to focus more on individual student needs and encourage deeper relationships. Simultaneously, human educators and peer interactions provide the essential guidance, emotional intelligence, and collaborative environments necessary for comprehensive development. By prioritizing human connection as the core of the learning experience and strategically employing technology to enhance it, we can ensure that future generations are not only well-informed but also well-rounded, compassionate, and capable of thriving in a rapidly evolving world.

The journey into the future of education is not a binary choice between digital dominance and traditional methods. As we have explored, the key takeaway is undeniable, and the true potential of learning hinges on our ability to strategically integrate technology while simultaneously preserving and, indeed, amplifying the crucial role of human connection.

Technology has unfolded a banner of unprecedented personalization and scalability, offering adaptive paths, real-time feedback, and a global repository of knowledge, making learning more accessible and efficient than ever before. However, these tools are most effective when viewed as enablers and accelerators for human learning, not as substitutes for human interaction. The challenge lies in leveraging these powerful tools without inadvertently diminishing the essential human elements fundamental to cultivating deep intellectual understanding. Yet, without the warmth of human interaction, the refined guidance of a sympathetic educator, and the enriching dynamics of peer collaboration, learning risks become a solitary, less profound experience. The vital "human skills", such as critical thinking, emotional intelligence, collaborative spirit, and resilience, are passively absorbed from screens but are actively forged in the crucible of human relationships.

Therefore, the path forward for optimal learning is a balanced approach. It is a collaboration where technology serves as a powerful enabler, streamlining processes, providing data-driven insights, and opening new avenues for exploration. This relieves educators to become more effective "guides on the side" mentors who cultivate curiosity, develop belonging, and facilitate the deeper cognitive and emotional development that algorithms cannot replicate. Conscious prioritization of the human element at the core of every technological integration can bridge the digital divide more equitably, navigate privacy concerns with greater ethical integrity, and empower both students and teachers to thrive in a constantly evolving world.

Embracing this human-centered future requires navigating several challenges. The digital divide remains a significant hurdle, as unequal access to technology can intensify existing educational disparities. Furthermore, concerns about data privacy and security are paramount as more personal and academic information is collected digitally. Crucially, teacher preparedness and professional development are vital; educators need ongoing training not just in technical proficiency but also in pedagogical strategies for effectively integrating technology while preserving human connection

This balanced future of learning promises not just smarter individuals but more compassionate, collaborative, and adaptable global citizens. It is an exciting frontier where innovation genuinely serves humanity, transforming education into a truly holistic and enriching journey.

References

Airaj, M. (2024). Ethical artificial intelligence for teaching-learning in higher education. *Education and Information Technologies*, *29*(13), 17145–17167.

Ayeni, O. O., Al Hamad, N. M., Chisom, O. N., Osawaru, B., & Adewusi, O. E. (2024). AI in education: A review of personalized learning and educational technology. *GSC Advanced Research and Reviews*, *18*(2), 261–271.

Barber, M., & Mourshed, M. (2009). *Shaping the future: How good education systems can become great in the decade ahead*. McKinsey & Company.

Borenstein, J., & Howard, A. (2021). Emerging challenges in AI and the need for AI ethics education. *AI and Ethics*, *1*(1), 61–65.

Bridges, D., Davidson, R. A., Soule Odegard, P., Maki, I. V., & Tomkowiak, J. (2011). Interprofessional collaboration: Three best practice models of interprofessional education. *Medical Education Online*, *16*(1), Article 6035.

Ceran, S. (2022). The relationships between secondary school students' visual creativities in art and their attitudes towards graphic design. *International Journal of Research in Education and Science*, *8*(3), 576–595. https://doi.org/10.46328/ijres.2962

Ceran, S. (2025). Examining high school students' artificial intelligence literacy in visual arts subjects and metaverse-based digital art attitudes. *International Journal of Technology in Education*, *8*(4), 1245–1265. https://doi.org/10.46328/ijte.5360

Ciddi, M. (2025). Analysis of attitudes of undergraduate art students toward painting workshop lessons. *International Journal on Social and Education Sciences*, *7*(2), 195–204. https://doi.org/10.46328/ijonses.764

Cloete, A. L. (2017). Technology and education: Challenges and opportunities. *HTS: Theological Studies*, *73*(3), 1–7.

Cole, M., & Scribner, S. (Eds.). (1978). *Mind in society: The development of higher psychological processes* (L. S. Vygotsky, Author). Harvard University Press.

Culver, D. (1998, November 4–7). *A review of Emotional Intelligence by Daniel Goleman:*

Implications for technical education [Paper presentation]. FIE'98. 28th Annual Frontiers in Education Conference, Tempe, AZ, United States.

Çakır, E., Öztürk, M., & Ünal, M. (2019). Interpainting as a creating method in digital illustration: Reinterpretations from movie scenes. *Bilim, Eğitim, Sanat ve Teknoloji Dergisi (BEST Dergi)*, *3*(2), 78–88. https://doi.org/10.46328/seat.v3i2.32

Dankbaar, M. E., & de Jong, P. G. (2014). Technology for learning: How it has changed education. *Perspectives on Medical Education*, *3*, 257–259.

Dean, M. (2020, July 21). *What are some barriers to technology integration in education?* Classcraft Blog. https://www.classcraft.com/blog/barriers-to-technology-integration/

Deterding, S., Sicart, M., Nacke, L., O'hara, K., & Dixon, D. (2011, May 7–12). *Gamification: Using game-design elements in non-gaming contexts* [Paper presentation]. CHI '11 Extended Abstracts on Human Factors in Computing Systems, Vancouver, BC, Canada.

Eden, C. A., Chisom, O. N., & Adeniyi, I. S. (2024). Integrating AI in education: Opportunities, challenges, and ethical considerations. *Magna Scientia Advanced Research and Reviews*, *10*(2), 006–013.

Faxriddinov, M. (2023). The impact of modern technology on education. *Integration of Science and Education*, *2*(2), 37–42.

Foltynek, T., Bjelobaba, S., Glendinning, I., Khan, Z. R., Santos, R., Pavletic, P., & Kravjar, J. (2023). ENAI Recommendations on the ethical use of artificial intelligence in education. *International Journal for Educational Integrity*, *19*(1), 1–4.

Garrison, D. R., & Vaughan, N. D. (2008). *Blended learning in higher education: Framework, principles, and guidelines*. John Wiley & Sons.

Halverson, R., & Smith, A. (2009). How new technologies have (and have not) changed teaching and learning in schools. *Journal of Computing in Teacher Education*, *26*(2), 49–54.

Hussain, T., Khan, A. A., Awan, M. A., Al Dairi, J., Al Saadi, F., Al Rashid, H., & Al Shibli, A. (2022, November 10–13). *A step forward in learning during pandemic: Emerging technologies and innovative pedagogies* [Paper presentation]. International Conference on Studies in Engineering, Science and Technology (ICSEST 2022), Antalya, Turkey.

Hussain, T., Khan, A. A., Pillai, J. R., & Al Siyabi, M. (2023). Adaptive learning in higher education: Implementation, challenges, and opportunities. In H. Polat, A. A. Khan, & M. D. Kaya (Eds.), *Studies on Education, Science, and Technology 2023* (Vol. 1, pp.

4–31). ISTES Organization.

Hussain, T., Khan, A. A., Pillai, J. R., Masud, S., Tashtoush, T., & Hussain, S. (2024). *Hybrid methods and technology interventions for transforming post-Covid teaching and learning*. Proceedings of International Conference on Studies in Engineering, Science, and Technology.

Igbokwe, I. C. (2023). Application of artificial intelligence (AI) in educational management. *International Journal of Scientific and Research Publications*, *13*(3), 300–307.

Johnson, D. W., & Johnson, R. T. (1999). Making cooperative learning work. *Theory Into Practice*, *38*(2), 67–73.

Khan, A. A., Al Siyabi, M., Ashraf, M. A., Hussain, T., & Fakhim, B. (2021). *Hands-on learning without laboratory* [Paper presentation]. Teaching and Learning Innovation Festival 2021, Universiti Teknologi PETRONAS, Malaysia.

Khan, A. A., Hussain, T., Al Siyabi, M., Al Shibli, A., & Hussain, S. (2024). Challenges and prospects of cutting-edge technology in education. In S. M. Ozturk & M. Koc (Eds.), *Studies on Education, Science and Technology 2024* (Vol. 1, pp. 104–121). ISTES Organization.

Khan, A. A., Hussain, T., Al-Siyabi, M., & Al-Shibli, A. (2024, April 27–30). *Innovative learning environment and experience* [Paper presentation]. International Conference on Research in Education and Science (ICRES), Antalya, Turkey.

Khan, A. A., Hussain, T., Pillai, J. R., Al Siyabi, M., & Al Shibli, A. (2024, October 20–23). *Use of adequate technology to attain engineering graduate attributes* [Paper presentation]. International Conference on Studies in Engineering, Science and Technology, Antalya, Turkey.

Khan, A., Hussain, T., Al Siyabi, M., Al Shibli, A., & Pillai, J. (2022). *Positive impact of pandemic on e-learning: A progressive narrative*. Proceedings of ICSEST 2022 - International Conference on Studies in Engineering, Science and Technology.

Kibici, V. (2025). Examining the digital literacy and artificial intelligence attitudes of university students studying arts. *International Journal of Education in Mathematics, Science and Technology*, *13*(5), 1283–1297. https://doi.org/10.46328/ijemst.5839

Laurillard, D., Kennedy, E., Charlton, P., Wild, J., & Dimakopoulos, D. (2018). Using technology to develop teachers as designers of TEL: Evaluating the learning designer. *British Journal of Educational Technology*, *49*(6), 1044–1058.

Levasseur, R. E. (2013). People skills: Developing soft skills—A change management perspective. *Interfaces*, *43*(6), 566–571.

Lopes, D. C., Gerolamo, M. C., Del Prette, Z. A. P., Musetti, M. A., & Del Prette, A. (2015). Social skills: A key factor for engineering students to develop interpersonal skills. *International Journal of Engineering Education*, *31*(1), 405–413.

Majid, N. A. (2017). The importance of teachers' interpersonal communication skills in enhancing the quality of teaching and learning. *World Applied Sciences Journal*, *35*(6), 924–929.

Malik, R. S. (2018). Educational challenges in 21st century and sustainable development. *Journal of Sustainable Development Education and Research*, *2*(1), 9–20.

Mashigo, A. C. L. (2014). *Factors influencing work readiness of graduates: An exploratory study* [Master's thesis, Stellenbosch University]. Stellenbosch University Repository.

Nerantzi, C., Chatzidamianos, G., Stathopoulou, H., & Karaouza, E. (2021). Human relationships in higher education: The power of collaboration, creativity and openness. *Journal of Interactive Media in Education*, *2021*(1), Article 26.

Ottenbreit-Leftwich, A., Liao, J. Y.-C., Sadik, O., & Ertmer, P. (2018). Evolution of teachers' technology integration knowledge, beliefs, and practices: How can we support beginning teachers use of technology? *Journal of Research on Technology in Education*, *50*(4), 282–304.

Ozturk, O. T. (2023). Examination of 21st century skills and technological competences of students of fine arts faculty. *International Journal of Education in Mathematics, Science, and Technology (IJEMST)*, *11*(1), 115–132. https://doi.org/10.46328/ijemst.2931

Öztürk, Ö. T., Türe, A., & Yaglici, S. C. (2021). *The relationship of futurism dynamism and today's technology*. International Society for Technology, Education, and Science.

Riaz, M. (2024). *A personalized learning system: Education by AI* [Bachelor's thesis, Centria University of Applied Sciences]. Theseus.

Roorda, D. L., Koomen, H. M., Spilt, J. L., & Oort, F. J. (2011). The influence of affective teacher–student relationships on students' school engagement and achievement: A meta-analytic approach. *Review of Educational Research*, *81*(4), 493–529.

Rousi, R., Alanen, H. K., & Wilson, A. S. (2024, October). *Data privacy, ethics and education in the era of AI–A university student perspective* [Paper presentation]. Conference on Technology Ethics 2024 (Tethics 2024).

Saragih, R. I. E. (2024). AI-powered education: Transforming learning through personalized and scalable solutions. *International Journal of Information System and Innovative*

Technology, *3*(2), 1–9.

Schut, S., van Tartwijk, J., Driessen, E., van der Vleuten, C., & Heeneman, S. (2020). Understanding the influence of teacher–learner relationships on learners' assessment perception. *Advances in Health Sciences Education*, *25*, 441–456.

Selwyn, N. (2016). Digital downsides: Exploring university students' negative engagements with digital technology. *Teaching in Higher Education*, *21*(8), 1006–1021.

Siemens, G., Dawson, S., & Lynch, G. (2013). *Improving the quality and productivity of the higher education sector: Policy and strategy for systems-level deployment of learning analytics*. Society for Learning Analytics Research (SoLAR).

Smadi, M. A.-L. M., & Raman, A. (2024). Skills and knowledge impact teachers technology integration in social studies teaching in Jordanian school. *Revista Iberoamericana de Psicología del Ejercicio y el Deporte*, *19*(4), 402–405.

Spitz, J. (2019). The teacher–learner relationship: Considerations for social work in the digital age. *Journal of Teaching in Social Work*, *39*(4-5), 402–413.

Stoddart, T., & Niederhauser, D. (1993). Technology and educational change. *Computers in the Schools*, *9*(2-3), 5–23.

Tekin, A. (2025). Examining high school students' attitudes towards contemporary and digital art and virtual exhibitions. *International Journal on Studies in Education*, *7*(4), 879–898. https://doi.org/10.46328/ijonse.5561

Turgut, S. & Öztürk, M. S. (2025). Analysis of Posters Awarded in the Typography Category at Graphic Designers Professional Association Exhibitions. In M. S. Ozturk, O. Noroozi, & O. Bilen (Eds.), *Proceedings of ICSES 2025-- International Conference on Studies in Education and Social Sciences* (pp. 34- 42). ISTES.

Wanless, S. B. (2016). The role of psychological safety in human development. *Research in Human Development*, *13*(1), 6–14.

Zhu, X., Chen, B., Avadhanam, R. M., Shui, H., & Zhang, R. Z. (2020). Reading and connecting: Using social annotation in online classes. *Information and Learning Sciences*, *121*(5/6), 261–271.

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