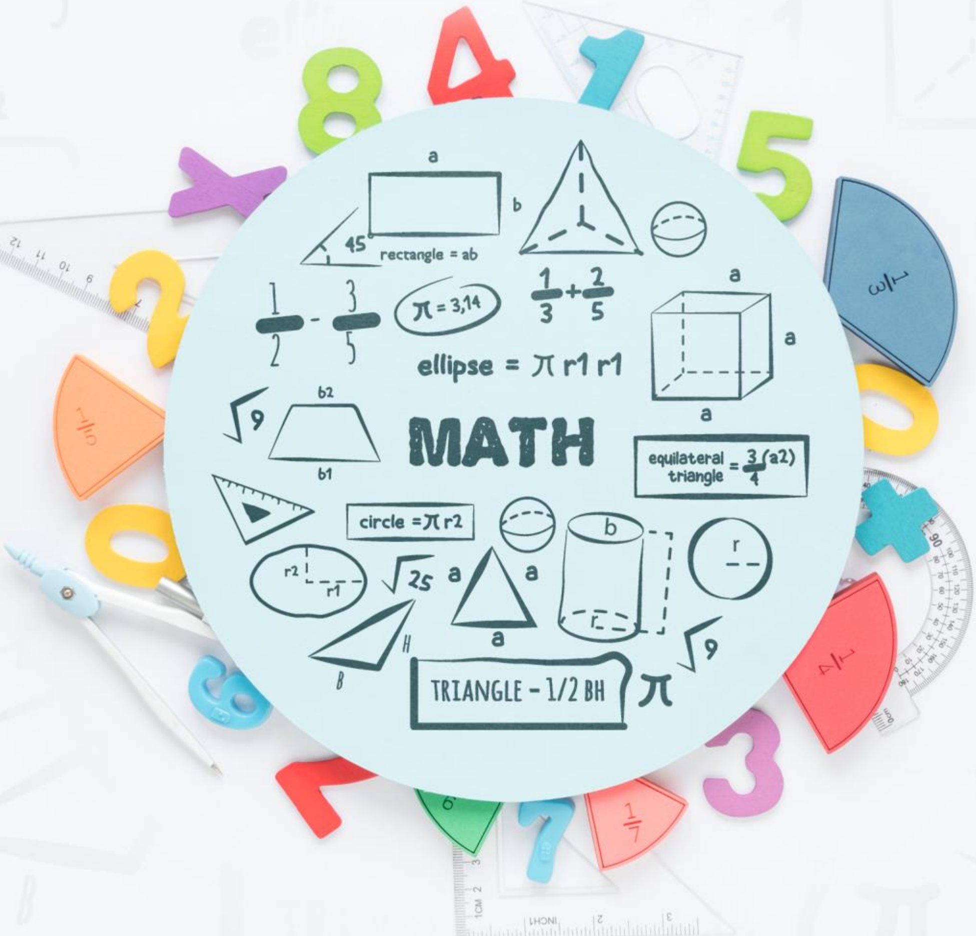
MATHEMATICAL CREATIVITY IN 5TH GRADE COMMON CORE CLASSROOMS

DR. ALI BICER



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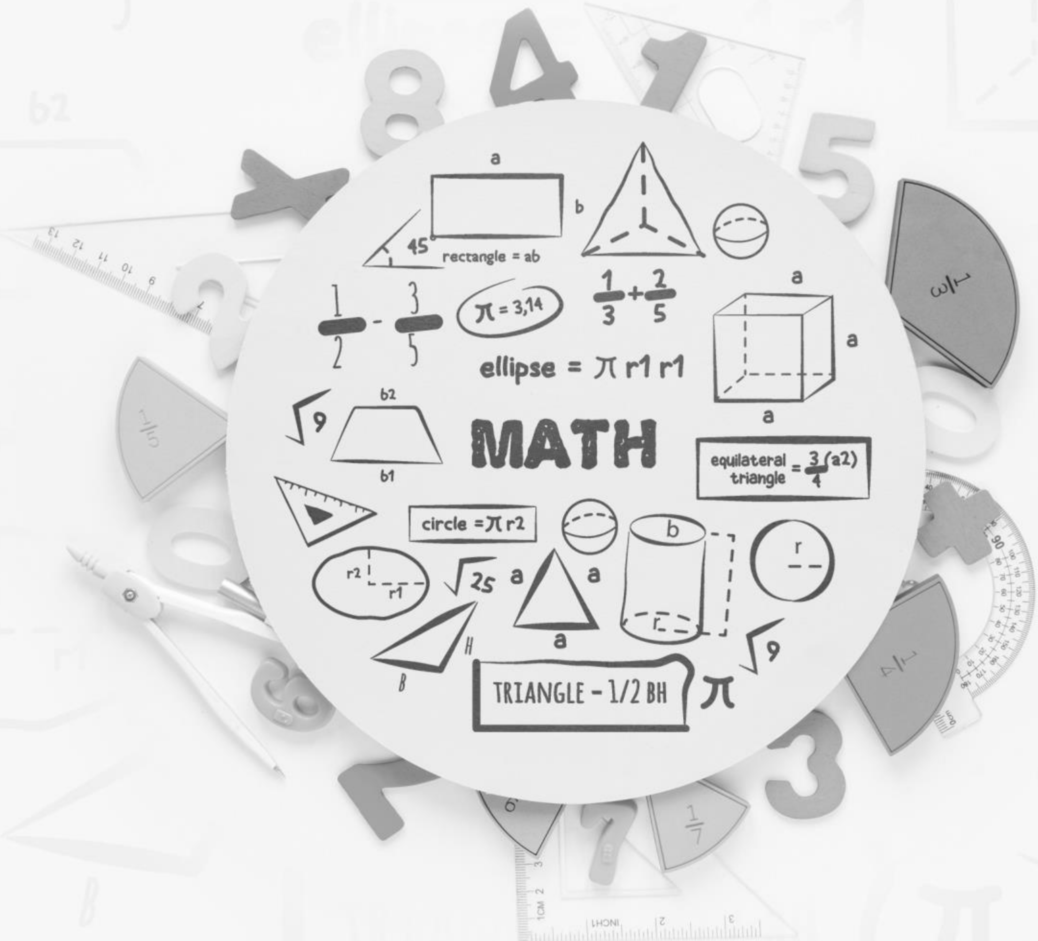

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Mathematical Creativity in 5th Grade Common Core Classrooms

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Author

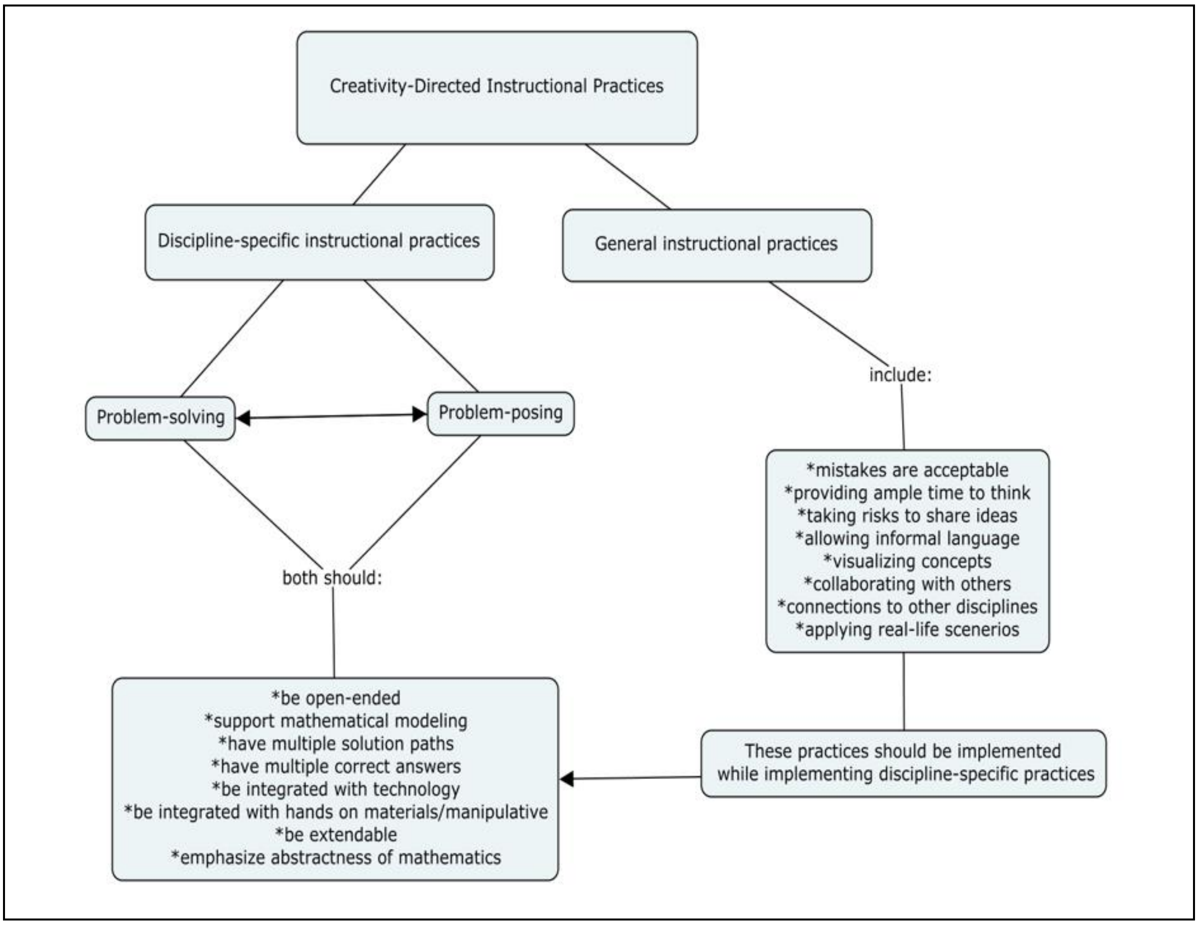
Dr. Ali Bicer is an assistant professor in the School of Teacher Education at the University of

Wyoming (UW), where he teaches undergraduate and graduate courses in mathematics education. His research interests have centered on mathematical creativity, creativity-directed tasks, STEM education, and writing in mathematics. Dr. Bicer received his Ph.D. in Curriculum and Instruction with a mathematics specialization from Texas A&M University in 2016. After receiving his Ph.D., he worked as a postdoctoral research assistant in Aggie STEM at Texas A&M University until 2018. Prior to starting his master's and doctoral program at Texas A&M University, he taught mathematics for three years in secondary and middle schools. Dr. Bicer currently received the Early Promising Faculty Award at the University of Wyoming. He published his research results in top-tier journals: Educational Studies in Mathematics (ESM), Thinking Skills and Creativity, Journal of Creative Behavior, ZDM-Mathematics Education, and Mathematics Education Research Journal. He is an international acquisition editor of the Journal of Urban Mathematics Education.

Table of Contents

Introduction

This book offers creativity-directed tasks that aim to develop students' creative thinking skills while they construct new mathematical ideas, knowledge, and concepts. The creativity-directed tasks were adopted from Bicer's (2020) framework (see Figure below).



This book is for any teacher and parent who want to engage their fifth-grade students in creative thinking in mathematics by implementing a meaningful combination of some discipline-specific and general instructional practices within a mathematical task/lesson. My goal in this book is to combine creativity in mathematics and CCSS-M content and practice standards as I illustrated in each lesson plan. It is important to note that CCSS-M practice standards “do not dictate curriculum or teaching methods” (NGACPB & CCSSO, 2014c, p. 5). Rather, they provide the context for teachers so that they can apply whatever curricular approaches they follow. Beghetto and Kaufman (2014) argued that whether creativity is supported or suppressed in classrooms depends to a greater extent on how students experience the messages that are sent by the tasks and a particular classroom environment than on what curriculum their teachers follow. Several common instructional practices and the way teachers inadvertently implement mathematical tasks constrain student creativity in mathematics classrooms. This book offers several 5th-grade mathematics lessons that enable teachers to encourage their students to truly think creatively while they cover 5th-grade CCSS-M content and practice standards. Each lesson in this book emphasizes different practices (e.g., visualization, generalization, problem-posing) to foster 5th graders’ creative thinking in mathematics.

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Beghetto, R. A., Kaufman, J. C., & Baer, J. (2014). *Teaching for Creativity in the Common Core Classroom*. New York, NY: Teachers College Press.

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Task 1 - Write and Interpret Numerical Expressions: Cinemark Day

Mathematical Content Standards

CCSS.MATH.CONTENT.5.OA.A.1

Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols.

CCSS.MATH.CONTENT.5.OA. A.2

Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them. For example, express the calculation "add 8 and 7, then multiply by 2" as $2 \times (8 + 7)$. Recognize that $3 \times (18932 + 921)$ is three times as large as $18932 + 921$, without having to calculate the indicated sum or product.

Mathematical Practice Standards

1) Reason abstractly and quantitatively
2) Look for and make use of structure

Lesson Objective

Students will learn how to use parentheses, brackets, or braces in numerical expressions by creating: a) multiple expressions for given images, and b) multiple representations or images for given expressions. This task's aim is to develop students' mathematical creative thinking by challenging them to create as many images as possible they can for the given mathematical expressions and create as many expressions as possible they can for the given images. This task will be completed in two class times.

Engagement

(20 minutes) Ask students what animation movies are currently playing at the local movie theater. If they do not know, they can search online and see what animations are on display. After their search, ask them if the movies are on display are a continuation of a series. If it is a series of movies, ask them if they watched previous movies and ask if they are volunteering to share their feelings about it. If it is not a series, then talk about if they watched one of the currently display movies' trailer and if it is exciting. After a short whole-class discussion, show them a printed image that displays recently available seats on the three saloons for the animation movie that most of the class are excited to watch in the local Cinemark (see Figure 1).

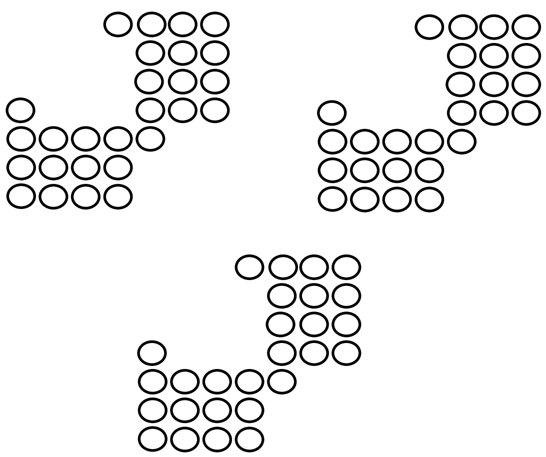


Figure 1. The Available Seats in the Local Cinemark

While you are showing the image, ask students how many seats are available and ask them how they counted. Encourage students not to count seats one by one, but employ any other strategies that help them find the number of seats flexibly. It is important to provide students think individually for three to five minutes before starting the number talk activity that they share how many seats are available in the Cinemark they found and how they counted or saw them. During the number talk activity, it is better for teachers to carefully listen to students' ideas so that they can either ask their students to draw how they counted the seats or they can draw what they hear from the students on the board to make each students' thought process visible to others. Using different colors can be a very helpful technique for students to understand how others see the available seats or circles on the image. For example, it is possible that a student or a group of students can see 4x3 with 2 more seats two times and one additional seat, then see that there were three of these saloons (see Figure 2).

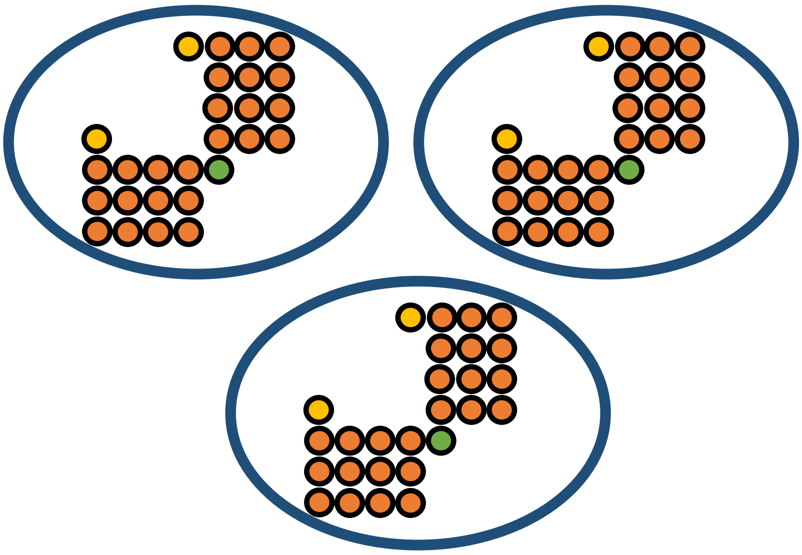


Figure 2. The Way a Student sees the Available Seats

After each student describes the way they saw the number of seats available at the Cinemark, students will record the mathematical expressions they did to find the numbers of seats by using parentheses, brackets, or braces. If possible, try to use the same colors to represent quantities, parentheses, and braces with the colors you used for the seats on the image or pictures you draw on the board. This will be helpful for students to create connections between the expressions and the corresponding images. For example, for the above example, a student can write the expression as: $[(3 \times 4 + 1) \times 2 + 1] \times 3$

If there are not many colors available, try at least to circle parts of the seats and label them with the corresponding quantities or numbers. After students see different ways the expressions could be written based on how they counted or saw the number of seats, ask them to get an expression from others that are not exactly the same as their expression and ask them to draw different images based on this expression. For example, a student may get an expression from his or her friend as $[(4 \times 3) \times 2 + 3] \times 3$ and draw different shapes (see Figure

3) based on how an expression was written give students an idea that the same mathematical expressions can present many different models or arrangements although the numbers of available seats are the same.

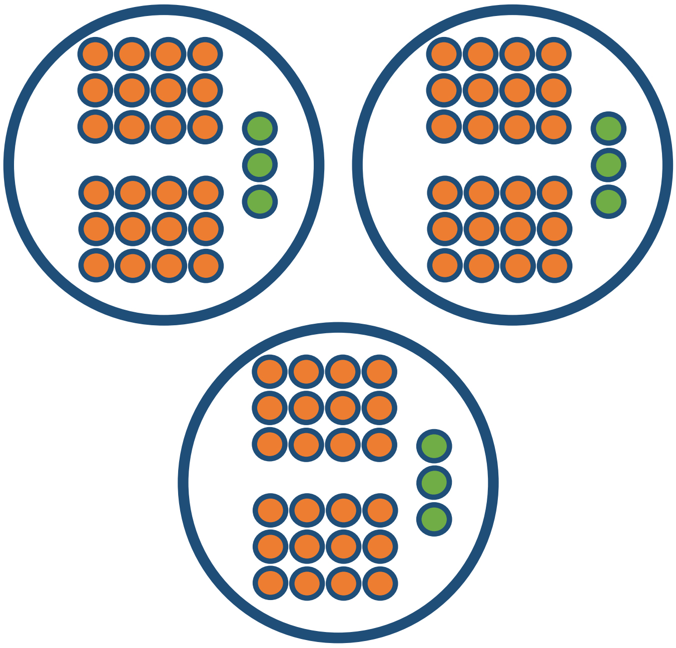


Figure 3. The Way another Student sees the Available Seats

Explore

(30 minutes) After this activity, provide students with more examples involving patterns and mathematical expressions that they can convert one to another (See Table 1 & 2). Ask students to work as a group of two or three. Each member needs printed pages of the images as they may have different representations or expressions than their group members that they like to show. Start asking students to describe what they see for each image (see Table 1) and create a number talk activity within their groups that they describe how they see the quantities to their group members. After students shared how they saw the quantities or images to their group members, ask them to write down the descriptions of how they saw the images in words. This follows up writing the expressions that match with their descriptions and the way they saw the total number of quantities. It is important to challenge each group and individual student to create as many expressions as possible for a given image so that they can feel creative in mathematics and realize that a single image can be represented or modeled with multiple expressions in mathematics. It is suggested that students should use as

many colors as possible so that they can make the relationship between the expressions and the images they created is visible to others. Students can also use circles, squares, snowflakes, or any other shapes to create their images for given expressions.

After engaging in finding the expressions for the given images, students now are expected to continue working on finding images that can represent the given mathematical expressions. Similarly, teachers ask students to have number talk within their groups as they describe how they imagine the expressions can be represented with images. Once students complete their number talk, they should write their descriptions of how they imagine the images matches the given expressions. The key is developing students' mathematical creativity by challenging them to find as many different images as possible for given expressions.

Explain

(20 minutes) Bring the class together to look at the various expressions and images they created. Ask each group to share the image or expression they think is the most creative or interesting. Once each group shared their solutions by describing or drawing on the board, ask students what similarities and differences each method has. Then, start a conversation about why and how using parentheses were useful. Ask students to explain what the expression they created in parentheses represents on the image. Then, it is suggested for teachers to find examples from their expressions leading to different answers and ask students to evaluate the expression if it didn't have parentheses. For example, how the expression that a student may write
$7 \times 7 - (3 \times 3 - 1 + 4 \times 4 - 2)$ or $49 - (8 + 14)$ is different than the following expression $49 - 8 + 14$.

Extend

(20 minutes) It is time for groups of students to create interesting images and expressions for other groups so that they can represent the images and expressions in multiple ways. Challenge the groups to create images and expressions that require using parentheses and as many operations as possible. Ensure that each group first has opportunities to test their images and expression out before handing them out to other groups. In the end, bring the

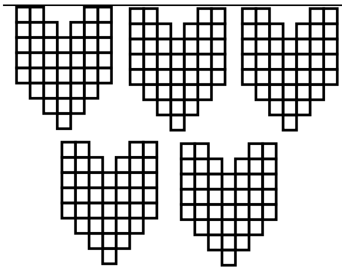
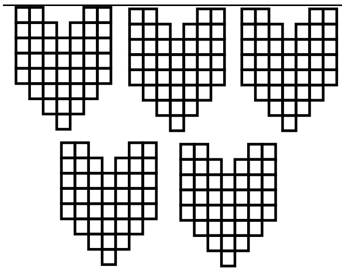
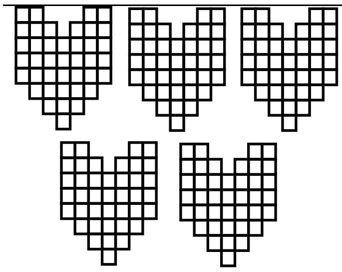
class together and ask them to share interesting and/or creative images they received from other groups to find expressions. Or, ask them to share the expressions they created that lead others to find quite interesting or creative images.

Evaluate

Teachers will have both formative and summative assessments. Formative assessment will take place as teachers closely observe each group while students describe their ideas to their group members. Formative assessment will also take during the whole-class number talk activity as the groups share their ideas, solutions, and questions. It is important teachers look for if students correctly use the symbols (e.g., parentheses, operation symbols). Students may forget to close the parentheses or overuse them as this lesson might be the first one they are introduced parentheses. Scaffold students' understanding of using parentheses in mathematical expressions by providing them feedback when they forget to use necessary parentheses and when they use parentheses redundantly. To emphasize creativity in this task, teachers need to look for if students are flexible and fluent on generating multiple images for given expressions or multiple expressions for given images. Teachers scaffold students by asking guiding questions if they cannot find more than one or two images or expressions. For example, teachers can ask what about considering all seats and then subtracting the unavailable ones at the end. Because each student work on the printed pages to generate expressions and images, teachers can collect them for summative assessment purposes at the end. This can enable teachers to address possible difficulties or misconceptions that a student may have that were not caught during the class.

Appendix. Task 1

Finding expressions for the given image

Image	How I see the image	Expression
		
		
		

Appendix. Task 2

Finding images for the given expression

Expression	How I picturize it	Image
$((2\times 5+4)\times 2)\times 3$		
$((2\times 5+4)\times 2)\times 3$		
$((2\times 5+4)\times 2)\times 3$		

Task 2 - Analyze Patterns and Relationships: What Patterns on an Ancient Roman City's Mosaics

Mathematical Content Standards

Focus Content Standard

CCSS.MATH.CONTENT.5.OA.B.3

Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane.

For example, given the rule "Add 3" and the starting number 0, and given the rule "Add 6" and the starting number 0, generate terms in the resulting sequences, and observe that the terms in one sequence are twice the corresponding terms in the other sequence. Explain informally why this is so.

Supporting Content Standard

CCSS.MATH.CONTENT.5.G.A.1

Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates. Understand that the first number indicates how far to travel from the origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond (e.g., *x*-axis and *x*-coordinate, *y*-axis and *y*-coordinate).

CCSS.MATH.CONTENT.5.G.A.2

Represent real world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation.

Mathematical Practice Standards

1) Construct viable arguments and critique the reasoning of others
2) Look for and make use of structure
3) Make sense of problems and persevere in solving them

Lesson Objective

The most important objective of this lesson is to develop students' creative thinking by challenging them to identify multiple ways of seeing the growth of two patterns. Students will investigate how each pattern's extension to the 5th, the 10^{th,} and later the 100th case using visual, words, tables, and graphs. Students will make connections among different representations by showing how mathematical knowledge/data/information represented with one form can also be represented with other forms (e.g., visual, words, tables, and graphs). Later, students will compare how the two given visual patterns are similar and/or different by identifying an apparent relationship between the corresponding terms of each case in two patterns. This task will be completed in three-class time.

Engagement

(20 minutes) Read the following poem written by Lycians about 3000 years ago:

> Do not despair if you cannot find me
> You will find my belongings
> Stones that I cut, roads that I open, statues that I carved, and you will feel that from beyond thousands of years, our handprints will touch each other.

Lykia is the region in Anatolia in what are now the provinces of Antalya and Mugla on the southern coast of Turkey, bordering the Mediterranean Sea, and Burdur province inland.

Then, create a short discussion about how they felt about being connected to people who lived 3000 years ago. After a short discussion about the poem, provide a short story to your students about Italy's submerged city of Baia. This submerged city of Baia was the most popular destination for wealthy Roman emperors and citizens. It is now the largest underwater museum in the world. A group of archeologists carefully moved some of the artifacts above the sea to protect the artifacts from rock-eating marine organisms and thieves. To keep the city safe as it is, they generated copies of the original artifacts and placed them underwater. Today, many divers can experience the city as it was 2,000 years ago, but it is an ongoing discovery that many artifacts such as mosaics, houses, and baths have not been excavated yet. Although the archeologists have applied advanced techniques to safely excavate the mosaics, some parts of the mosaics may have some important parts missing that we cannot completely picture the way the mosaics were designed. However, mosaics that include mathematical patterns are exceptions as we can use our mathematical imaginations to predict the extensions of the mathematical patterns on the mosaics. In this lesson, you will be working as mathematicians that you will decide how you will predict the extensions of visual patterns on the mosaics. Although the patterns on these mosaics are imaginary and each was created by Dr. Bicer in 2022, it is possible that some mosaics that will be excavated in the future may have similar visual patterns. In two different mosaics, we have three cases:

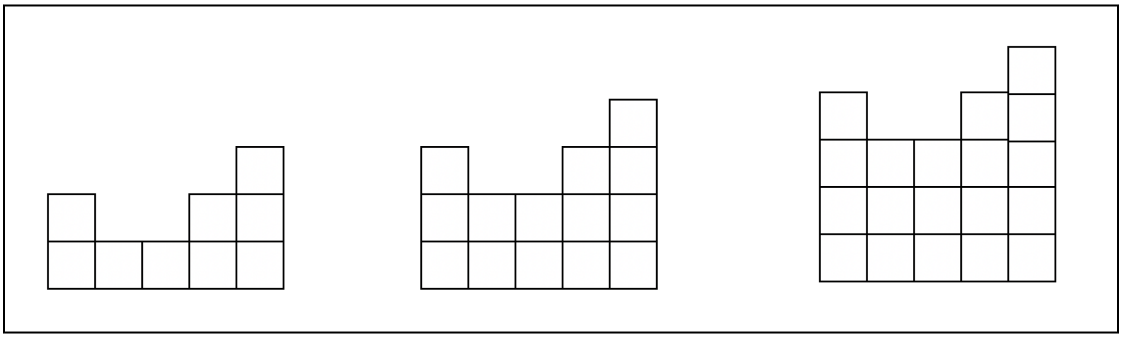


Figure 1. Three Cases of Pattern 1 on Mosaic 1

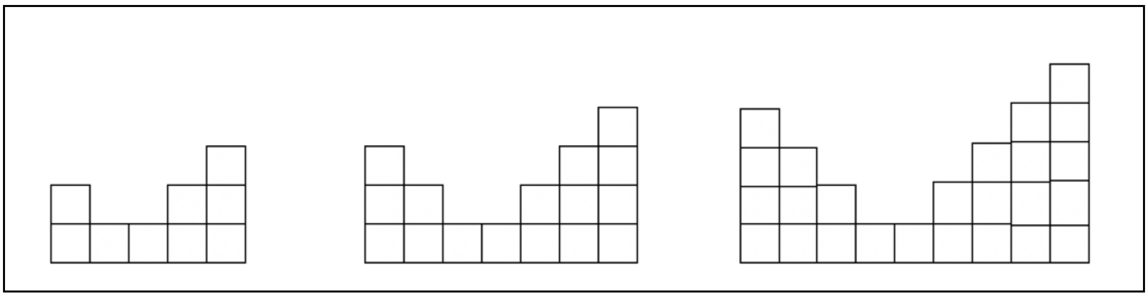


Figure 2. Three Cases of Pattern 2 on Mosaic 2

Ask your students how they see the patterns growing in both mosaics. Allow your students to look at the patterns individually about 3-5 minutes before they go to their groups and share how they see the growths of two patterns on the mosaics with their group members.

Explore

Teachers provide students with handouts (see Appendix A and Appendix B) and asked them to work in groups of three or four. Because one of the essential aims of this task is to develop students' creativity, each student individually should record the way of seeing the growths of the patterns. It is completely acceptable that students copy their group members' ways of seeing the growths of patterns as long as they understand them. Challenge individuals and groups to find multiple ways that they can identify how the patterns grow (see examples in Figure 3 & 4 examples) and ask them to use colors to make their ways of seeing the growths visible to others.

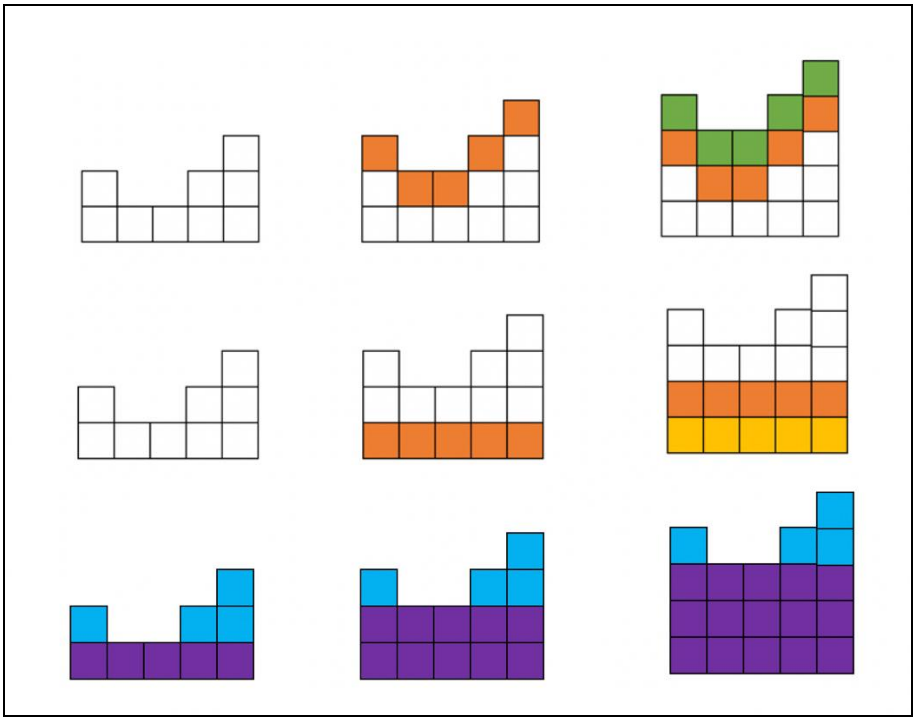


Figure 3. Three Examples from Students' Work for Pattern 1 on Mosaic 1

When students complete coloring and show at least two or three ways of seeing the growth of patterns, teachers can ask students to visit the other groups so that students can see how others identified the growths of the patterns. Then, challenge students to imagine the 5th and

10th case of each pattern and let them share their description by coloring, drawing, writing, and talking with their group members. Teachers should provide grid papers to students if they like to draw additional cases of each pattern to imagine the 5th and the 10th case. It is expected and suggested that students can describe their imaginations of the 10th case orally or picture a rough draft without drawing the complete picture of the 10th case. After students share their imaginations of the 10th case, ask them to explore the relationship between the case number and the total number of squares in the corresponding case for each pattern. Although it is possible that some students attempt to draw all cases up to the 10th case, some may generalize the growth they identified by expressing the 10th case with words, pictures, or numbers.

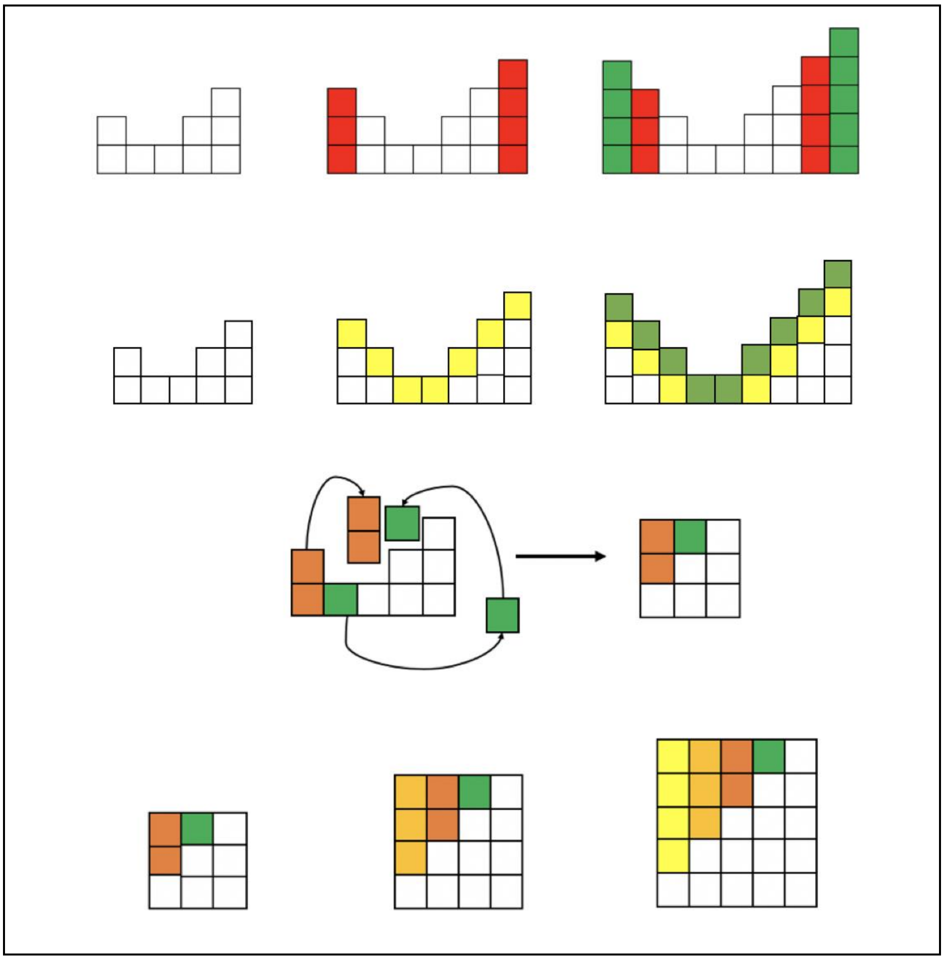


Figure 4. Three Examples from Students' Work for Pattern 2 on Mosaic 2

Explain

After students attempt to imagine the 10th case, ask them if it is possible to find the number of squares in the 100th case for each pattern. They may find drawing all cases up to the 100th

case is exhausting. So, let them share their ideas if it is possible to find the number of squares in the 100th case without drawing all cases. If students already have methods that work, let them explain their ideas to the whole class. After hearing students' answers, tell your students that they can use tables to investigate the relationship between two quantities and provide them pattern identification table (see Appendix C & D). Let students work on filling the tables by looking at their visual patterns. Ask students how they see the patterns' growths on the table by looking at the relationship between the numbers on the two columns. Let students discuss what numerical pattern they saw between the case number and the total number of squares and how this is related to the way they saw the pattern's growths.

Once students identified the numerical relationship between the case number and number of squares in the corresponding case, tell them graph is another tool that can be useful to identify the relationship between two numerical patterns. Then, briefly explain to students the first quadrant of the coordinate plan on a graph paper. First, put the origin and then get the two values from the table for mosaic 1 and write on the board as (case number 1, total number of squares in case 1). Ask students to write this by placing the values as (1, 9). Challenge students to show where this value of the pattern is represented on the graph. After hearing their ideas, indicate the point (1,9) on the graph and explain to them 1 represents case number 1 on the horizontal line and 9 represents the corresponding number of squares on the vertical line. Then, let students place all the paired numbers on the grid paper (see Appendix F).

Ask students to connect the points to see how they can model the growths and ask them to compare how they can explain the growths of the patterns on the tables and graphs. Towards the end, ask students to compare pattern 1 on mosaic 1 and pattern 2 on mosaic 2 by looking at the tables and graphs they generated. Ask students to find what commonalities and differences the patterns have. Since one pattern growth is linear and the other is quadratic, students will observe the growth is faster or larger in the second pattern and that is enough for students to visually comprehend as they do not know yet linear and quadratic models.

Extend

Ask students to create three cases of two patterns that are visually similar but have some differences. Ask them to test if their patterns are extendable to future cases by using colors, words, tables, and graphs before they hand out their patterns to other groups. Challenge the groups to identify multiple ways of seeing the growths of patterns by using colors, words, tables, and graphs. Ask students to share creative patterns that they receive from other groups or they generate by themselves. Although it is early for generalization, it is possible that teachers can scaffold students to generalize their patterns by bringing variables into their growth expressions if students are ready. For example, students can identify the following expression for pattern 2 on mosaic 2: Adding 2 to case number and squared it gives the total number of squares. Then, teachers can ask what if we represent the case number with n? Students, in that case, can come up with $(n+2)\times(n+2)=(n+2)^2$. Of course, the generalization can leave for the upper classes. It is important to bring examples such as $(case\ number+2)\ and\ (case\ numbeer)\times 2$ to create a discussion about how these are different. Since students were introduced to how to find the relationship between two quantities by using tables and graphs, they can use them to visuals how adding 2 and multiplying by 2 have different effects on a given quantity.

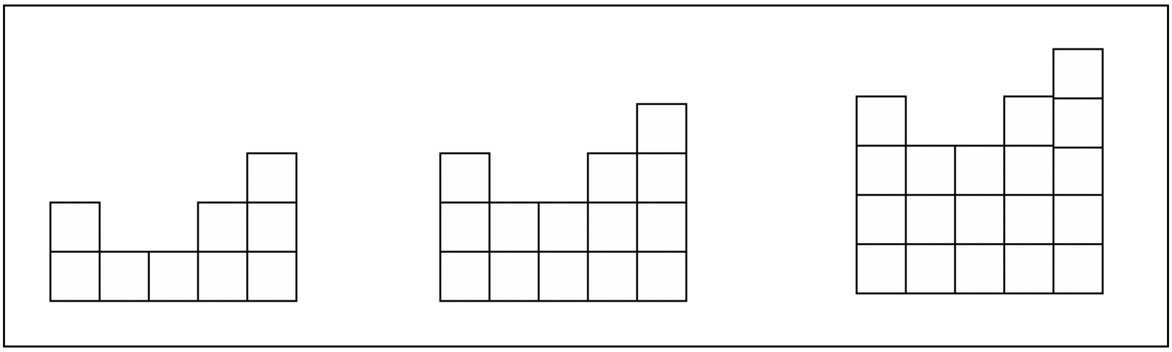
Evaluate

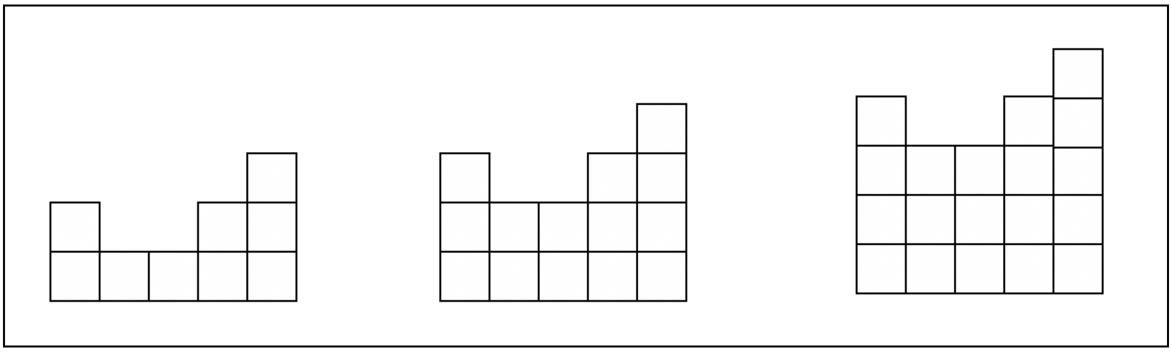
Since creativity is one of the most important dimensions of this lesson, it is important that teachers look for if students can come up with multiple ways of seeing the growth of given patterns. Because students work in groups and it is possible that all students in a group come up with the same model of growth, they may become stuck seeing the same model. Ask students to rotate their papers and look at the cases from different dimensions. This can help students identify the cases from different perspectives and this may help them find a new way of seeing.

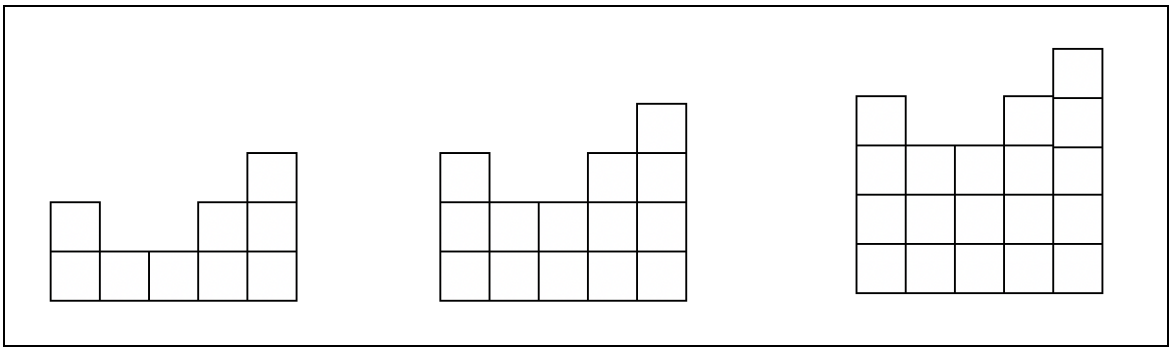
Another important assessment part is to observe if students are counting the number of squares for each case. It is because counting may help them identify different ways of seeing the growth. For example, it is possible that a student may not see that pattern 2 on mosaic 2 grow as squared numbers by only looking at the visuals of cases, but they can see that the shapes can be formed as squares if they count and see the number pattern as 9, 16, 25, 36. Saying this, it is important for teachers to examine if students attempt to draw all the cases once they were asked to find the number of squares in the 10th case. Help students see how they can find the forthcoming cases without drawing all cases as they can identify the relationship between the case number and the corresponding number of squares. This can be achieved by teaching them how to use tables and graphs (grid paper) to investigate the relationship between two quantities.

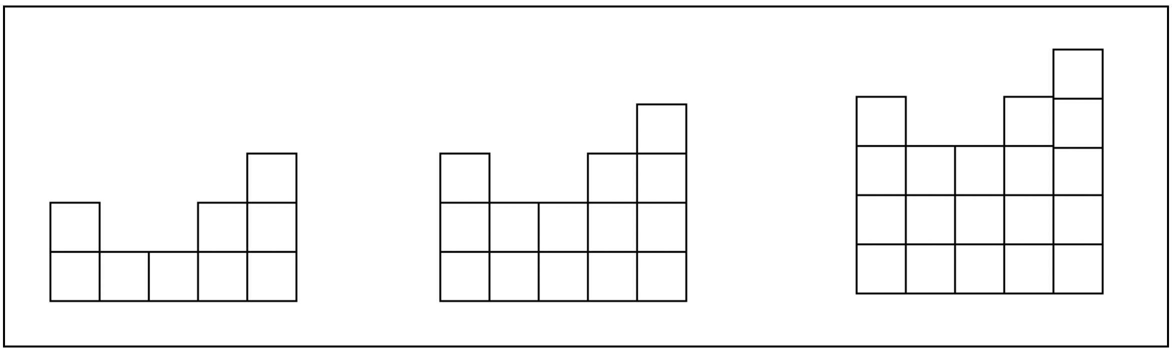
It is important for teachers to look for if students can translate several representations of pattern one to another (e.g., words to pictures, pictures to expressions) and connect them when they were asked about the relationship. Ask students informally to share how their tables, words, visuals, and graphs show the same information. In addition, teachers can specifically ask about the relationship between numerical expressions (or generalizations) and the visuals they came with. For example, ask students in pattern 2 on mosaic 2: Why the numbers are squared numbers? They may then organize the images by moving the squares and seeing that each case can be represented as a square. After these formative assessments during whole-class discussion and group work time, teachers can do summative assessments as they can collect students' works from Appendices and provide feedback to individual students.

Appendix A

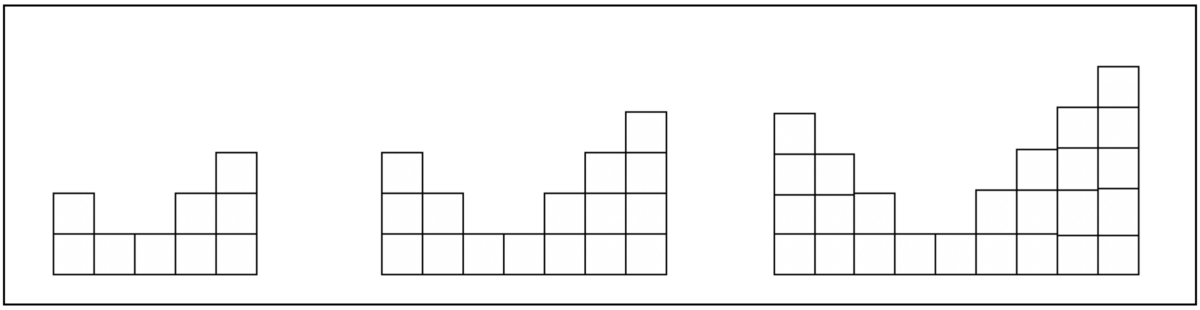


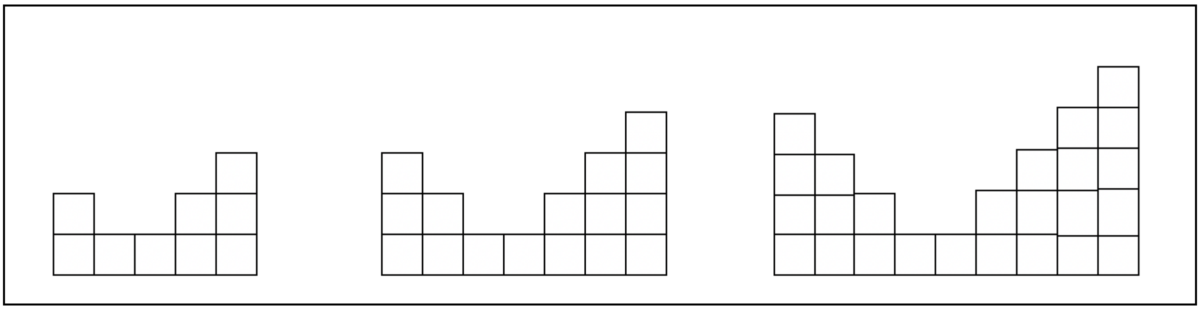


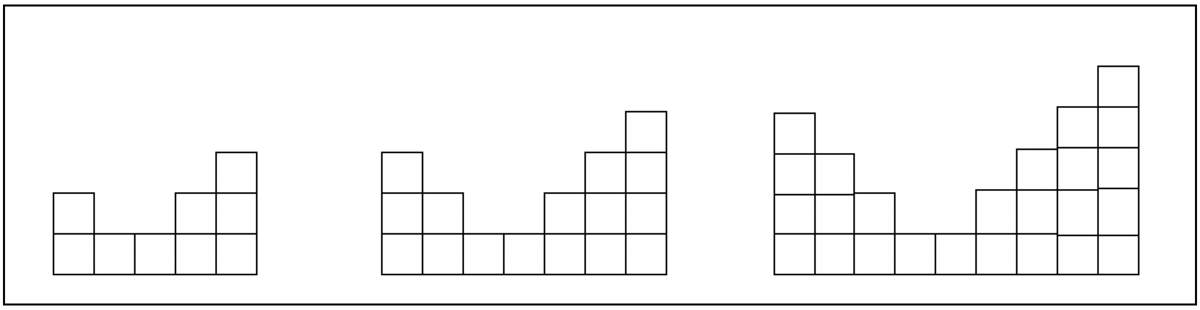


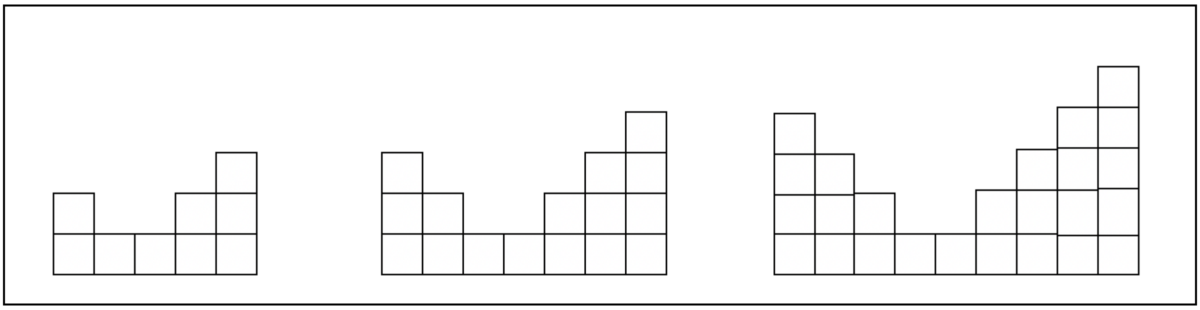


Appendix B









Appendix C

Pattern 1 on Mosaic 1	
Case Number	Number of Squares

Appendix D

Pattern 2 on Mosaic 2	
Case Number	Number of Squares

Appendix E

Task 3 - Understand the Place Value System: 1K Additional Steps

Mathematical Content Standards

CCSS.MATH.CONTENT.5.NBT.A.1

Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left.

CCSS.MATH.CONTENT.5.NBT.A.3

Read, write, and compare decimals to thousandths.

CCSS.MATH.CONTENT.5.NBT.A.3.A

Read and write decimals to thousandths using base-ten numerals, number names, and expanded form, e.g., $347.392 = 3 \times 100 + 4 \times 10 + 7 \times 1 + 3 \times (1/10) + 9 \times (1/100) + 2 \times (1/1000)$.

Mathematical Practice Standards

CCSS.MATH.PRACTICE.MP2

Reason abstractly and quantitatively.

CCSS.MATH.PRACTICE.MP4

Model with mathematics.

CCSS.MATH.PRACTICE.MP5

Use appropriate tools strategically.

Lesson Objective

Students will investigate the relationship between the power of 10 from 1/1,000 to 1,000 through applying length, area, and set models. This task's aim is to develop students' mathematical creative thinking by challenging them to first understand and then generate three different models (i.e., area, length, and set) showing the relationship between the power of 10 from 1/1,000 to 1,000.

Students will pick a unit for each model and use physical or online materials to represent the relationship between the power of 10 of a selected unit from 1/1,000 to 1,000. Students can represent the power of 10 by taking photos or downloading pictures online as their 1/1,000 and 1,000 can be too small or too big based on their selected units. They can draw pictures or mark physical spaces around the classrooms or schools. This task will be covered within 4 class times.

Engagement

(50 minutes) Start the class by telling students a recommendation by doctors that individuals should walk 10,000 steps to control their weight and keep healthy. Start a conversation by asking if students think they walk 10,000 steps, more, or less each day. After hearing their ideas, tell them shooting that number is overwhelming or almost impossible for some people due to many reasons such as having office jobs, spending a lot of time driving, and prioritizing job or school-related work to meet the deadlines. Tell them another suggestion by doctors to those people who cannot take 10,000 steps as increasing your number of current activities by additional 1,000 steps.

It seems that almost anyone can achieve this goal that has numerous health benefits: lowering your heart rate, reducing the chance of having a stroke, lowering your cholesterol and increasing your good cholesterol, reducing your stress level, strengthening your bones and muscles, and losing weight. Today, we will investigate how far and how long you can go if you walk 1,000 steps without taking 1,000 steps. Ask groups of students to walk 10 steps from a designated space and ask them to record how far they move by taking pictures. Also, ask students to record time to see how long this activity (10 steps) take. Then, do the same activity by taking 100 steps.

To make it visible for everyone, students can mark the beginning point and endpoint or they can use strings, tapes, or any other materials. Bring all students together and let them share their groups' records. Their pictures (i.e., how far they move) and time (how long it takes) may not be identical and this is good to start a discussion about unit by asking them why they think their data for how far they move and how long it takes are not identical, but similar. We expect to hear from our students that their steps are not identical as their length of legs is not the same, so their unit is different.

However, since their lengths of legs are not hugely different from each other, the numbers (i.e., time) are expected to be quite similar. Then, tell your students that they can also google to see how many steps people take in 1 minute. Their search will have resulted in 100 steps. This number will be meaningful to them since their numbers will be either on 100 steps or quite close to that number. Then, ask how many steps in 10 minutes.

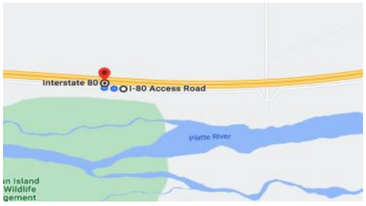
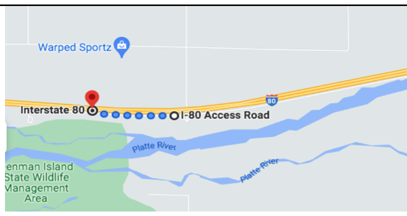
Students will google and come up with 1,000 steps. Inform students that these are for moderate intensity. Ask them to look at the relationship between the number of steps (how far they move) and how long this activity takes if the activity is moderate intensity. They can identify the relationship between time and the number of steps by creating tables (see Table 1) and observing the power of ten from 100 to 1,000.

Table 1. Relationship between the Number of Steps and How Long it Takes

Time	**How many steps**
1 minute	100 steps
10 minutes	1,000 steps
100 minutes	10,000 steps

It is possible that the school or school backyard may not have enough space for students to take pictures of 1,000 steps to compare it with the pictures of 100 or 10 steps. In this case, teachers can use Google Maps to identify how far they can go if they walk 1,000 steps from their current location. It is suggested that students can change time to 1 minute, 10 minutes, and 100 minutes to picture and compare how far they go from 10 steps to 10,000 steps (see Table 2).

Table 2. How Far You Go with 100 Steps, 1,000 Steps, and 10,000 Steps

How far you go		
100 steps	1,000 steps	10,000 steps
		

After the Google Map activity, let students watch a YouTube video titled “Cosmic Eye” and ask them to share their observations. The link for the video is: https://www.youtube.com/watch?v=8Are9dDbW24

After watching the video, ask students to share their observations. Since students already observed that they are multiplying by ten to go upward, ask them what we do when we go inward on the video. Students may already observe that they divide by 10 as they go inward. Then, ask them to share the picture showing how far they go with 10 steps and ask them as a group to decide how long walking 10 steps take. Students can show the picture they took and decide how long walking 10 steps took for them either by looking at the pattern on the table (see Table 3) or their own record.

Table 3. Relationship between the Number of Steps and How Long it Takes

Duration	**How many steps**
1/1000 minute	1/10 step
1/100 minutes	1 step
1/10 minute	10 steps
1 minute	100 steps
10 minutes	1,000 steps
100 minutes	10,000 steps

Ensure that students have similar estimations or calculations with the patterns they draw on the table. Students will observe that each time they divide the number of steps by 10 to find the next smaller number of steps. Create a discussion environment about comparing how far they go with 10 steps and how long it takes. Ask students to share their pictures (see Table 4) with 10 steps, 1 step, and 1/10 step.

Table 4. How Far They Go with 10 Steps, 1 Step, and 1/10 Step

How far you go		
10 steps	1 step	1/10 step
		

Explore

(60 minutes) After this observation, ask students to work in groups of three or four and choose a unit that they can compare the power of ten from 1/1,000 to 1,000. For example, if they pick their unit as a foot size of a member of their group, then they can say 10 times of their friend foot size is about a width of whiteboard or 1/10 of their group member's foot size is about a width of a regular pink eraser.

Ensure that students pick their units carefully as they may have a hard time finding very little or big objects. For example, if they pick a foot size, they may not find an object easily that is the same size as their group's member's 1/1000 of the foot size. You can encourage students to pick their unit bigger in their first attempts. For example, the height of teachers or doors can be a good start. When students find objects that represent the corresponding power of ten from 1/1,000 to 1,000 based on what unit they selected, they can share their examples with the whole class to see different ways of seeing the power of tens.

After students completed the first columns (see Table 5) by using the length model as we emphasized more during the engagement, start the following game or activity that students can learn and apply the area model.

Table 5. What Power of Ten of a Selected Unit Looks Like

Models	1/1,000 looks like	1/100 looks like	1/10 looks like	**Our unit or (1) is**	10 looks like	100 looks like	1,000 looks like
Length							
Area							
Set							

For the game, provide students a 10,000 grid (see Appendix C) and a die. Tell students that you will changed the numbers on the die randomly to play this game since you want them to use decimals. Provide the following table:

Table 6. Play with Area Model of Decimals

Numbers on a die	What decimal numbers
1	1/1,000
2	5/1,000
3	10,1000
4	1/100
5	5/100
6	1/10

If teachers have time, they can ask students to create their dices that these decimals on them. In this game, one student rolls the die and uses a marker to shade the amount rolled on the 20x50 grid. For example, if 1/1,000 is rolled, the student shades the smallest rectangle on the grid. While playing the game, students will understand the relationship between the power of ten from 1/1,000 to 1 and they will eventually note that 10/1,000 is equivalent to 1/100. The groups should try to cover the whole grid or 1. It is better if students can use different colors to visually see each decimal. After this game, teachers can use various manipulatives to develop students' conceptual understanding of the power of ten further.

Explain

(45 minutes) To develop students' conceptual understanding of using area models in

decimals, let them first play with base-ten materials. Since students were introduced to these materials at early grades, they can quickly use them to represent given numbers, especially whole numbers. For example, a teacher can ask students to represent 1624 by using tinies (known as singles), strips (known as longs), squares (known as flats), and super squares (known as a cube). After students represent 1624 by using base-ten materials (see Figure 1), teachers can ask if it is possible to represent this by only using longs and tinies.

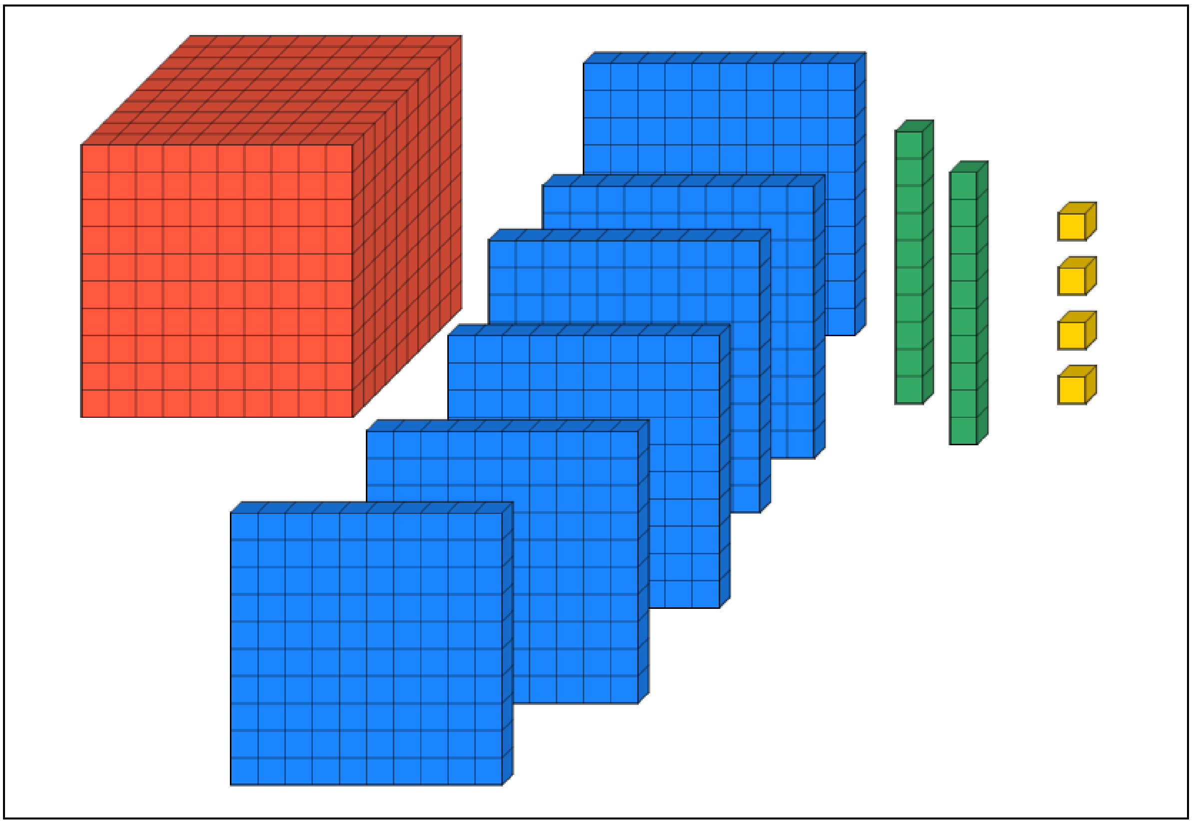


Figure 1. Representing 1624 with Base-ten

Let students share their ideas. We expect our students say “162 longs and 4 tinies”. Ask them if they can write this by using only longs. We can tell them to consider the relationship between tinies and longs. Then, students can say we have 162 longs and 4/10 longs. Then, introduce students to how we write it by using decimal as 162.4. Continue asking what about using squares (or known as flats), longs, and tinies?

Students will say 16 squares, 2 longs, and 4 tinies. Then, ask their guesses about how we write it down and write for them as 16.24 squares. Ensure that students see 1/10 the relationship. You can provide students the following table to fill:

Table 7. Translating Units to Each Other by Using Base-ten

Given Unit	**Conversion**	**How we read it**	**How we write it**
1624 tinies	162 longs and 4 tinies	162 and 4 tenths	162.4 longs
1624 tinies	16 squares, 2 longs, and 4 tinies	16 and 24 hundredths OR 16 and 2 tenths and 4 hundredths	16.24 squares
1624 tinies	1 super squares, 6 squares, 2 longs, and 4 tinies	1and 624 thousandths OR 1 and 6 tenths, 2 hundredths, and 4 thousandths	1.624 super squares

After filling out the table, ask students if they can find infinitely many pieces by multiplying the largest piece with 10. Hear students' understanding of place value for whole numbers and now ask them what about multiplying the smallest piece by 1/10. Is there ever the smallest piece? Let students discuss it with their group members and then ask the groups to tell their reasoning during the whole class discussion. Ensure that students use visuals to justify their thoughts. At the end of the whole class discussion, students will see that a 10-to-1 relationship extends infinitely in two directions with no smallest piece to largest piece.

Extend

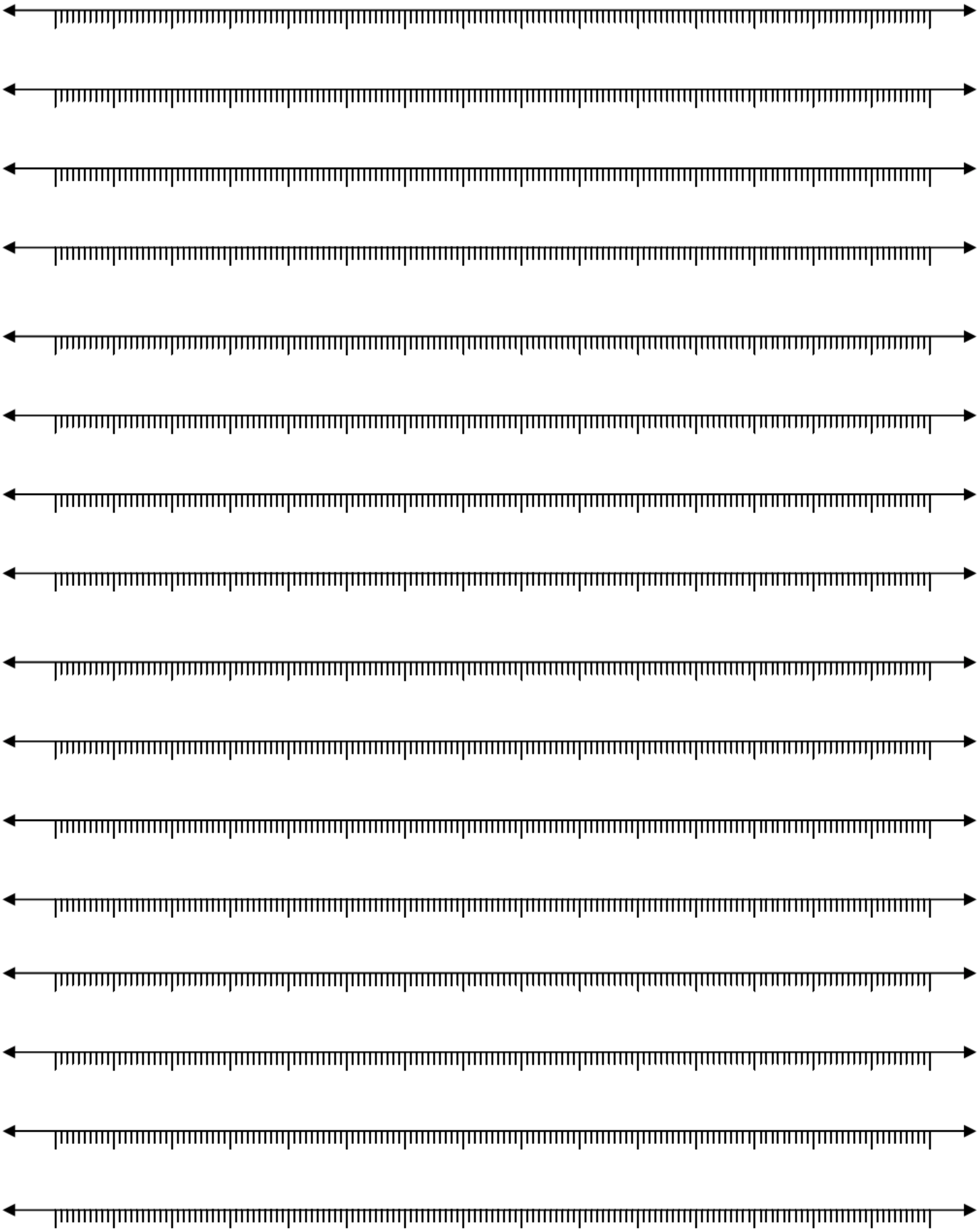
(45 minutes) Teachers can extend this by asking students to read and then write the extended form of 1624. We expect our students to write down 1x1000+6x100+2x10+4x1. After this, tell students to write the expanded form of 162.4. Ensure that they look at their record how they read it before writing it down. Remind them that we read it as 162 and 4 tenths. Students will then write this as 1x100+6x10+2x1+4x(1/10). Extend it by asking them to write the expanded form of numbers including the power of ten from 1/1,000 to 1,000. 4347.392 = 4 x 1,000 + 3 × 100 + 4 × 10 + 7 × 1 + 3 × (1/10) + 9 × (1/100) + 2 × (1/1000).

After students build a strong number sense of decimals, you can introduce money as a set model. Ask students if they can identify a similar relationship between the power of 10 from 1/100 to 100 by using monetary concepts. Students can build this relationship by using fake money. They can think of 1 dollar as their unit and identify the relationship as 1/10 of a dollar is one dime and 1/100 of a dollar is one cent, 10 times a dollar is 10 dollars, and 100 times a dollar is 100 dollars. After this investigation, ask students to represent 5 cents and 25 cents by using decimals. Since they know that they have 100 cents in a dollar. They will say it as 5 hundredths and 25 hundredths and ask them to write it down as 0.05 and 0.25. Right at this moment, ask students to represent one cent (penny), five cents (nickel), one dime, 25 cents (quarter), 50 cents, a dollar, and 1.5 dollars by using the area and length model and write them in decimal formats. This will allow students to connect multiple representations of monetary units by applying set, area, and length models. Use 10x10 grid (see Appendix A) and number line (based on tenths) sheet (see Appendix B).

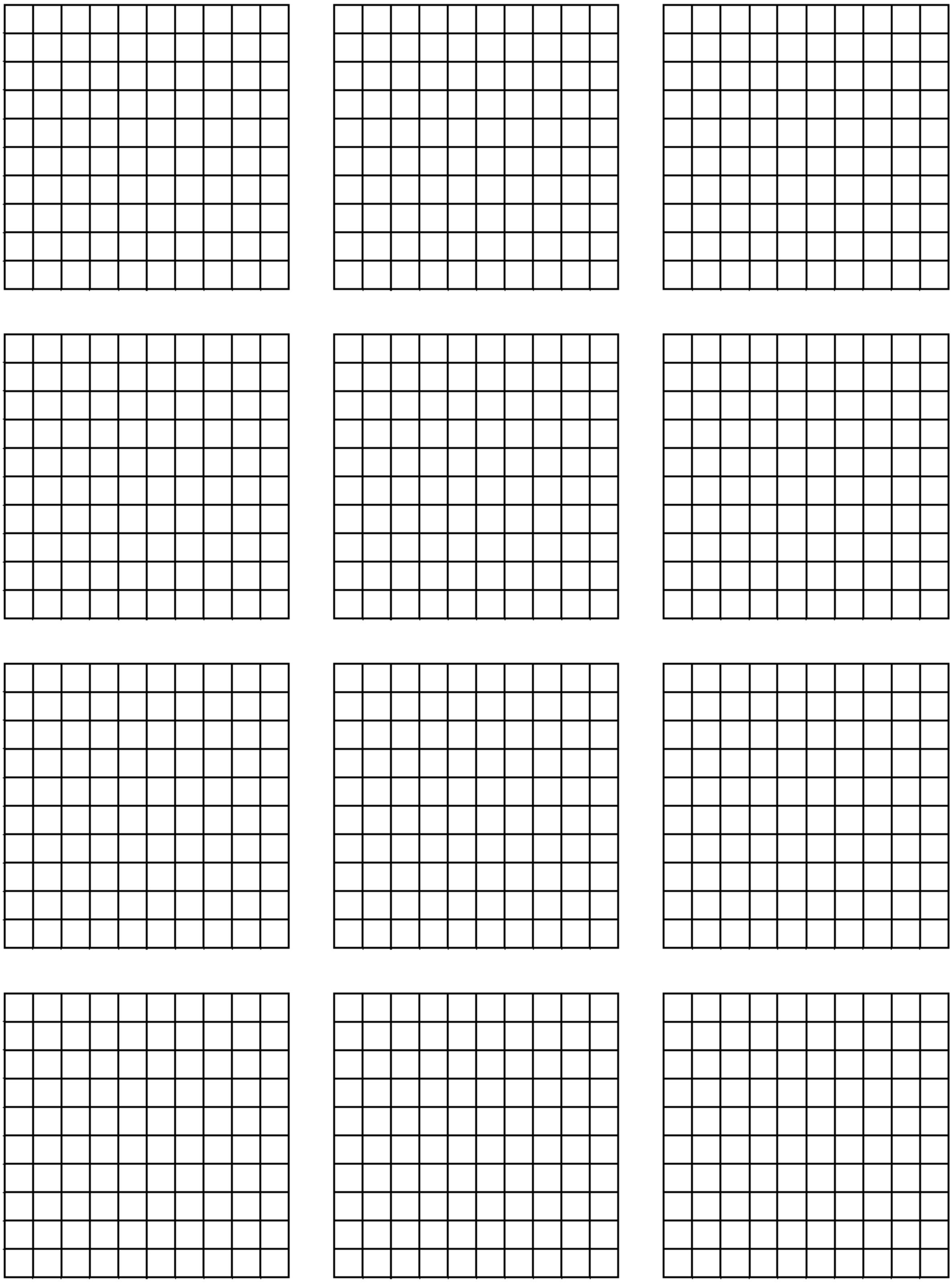
Evaluate

Teachers first should look for if students have strong number sense before introducing decimals. For example, we should ensure that students understand the multiplicative relationship as they multiply by 10 when moving from right to left or multiply by 1/10 when moving from left to right on a given whole number. If some have issues with whole numbers, it is better to play with base ten materials, grid papers, and number lines. Second, teachers should look for if students use units to explain their decimal sense while they work in groups. The numbers are only meaningful when we define what units we are talking about. Ask students constantly what is your unit and let them explain how 1,000 might be either too small or too big based on the selected unit. Teachers also look for if students read decimals correctly. For example, we read 1.15 as “one and fifteen hundredths. In contrast, we should not say “one hundred and sixteen when we read 116. Teachers also need to look for if students connect “two-tenths” with both 0.2 and 2/10 representations since it shows the connection between decimals and fractions. Provide a couple of examples if students struggle with this connection. A teacher can ask students to write the number has 4 tenths, 5 hundredths, and 7 ones. Some may directly write 45.7 as they assume they have to move from left to right rather than 7.45. This will assess if students grasp decimal place values. Teachers should stress or emphasize the way they pronounce the “ths” when they were reading decimals so that students can capture the differences in words such as tens and tenths. Lastly, teachers must look for if students can translate one representation of a decimal to another one. Translating one to another is important for students to develop their mathematical creative thinking as that kind of activity helps them gain different mathematical perspectives because multiple representations enable students to see different aspects of a mathematical object that is not directly available or visible for students with one representation. If they have difficulty translating one representation of a decimal to another, provide them more opportunities to engage in representing decimals by drawing or marking number lines, coloring grids, and representing with manipulatives or objects available in the classrooms.

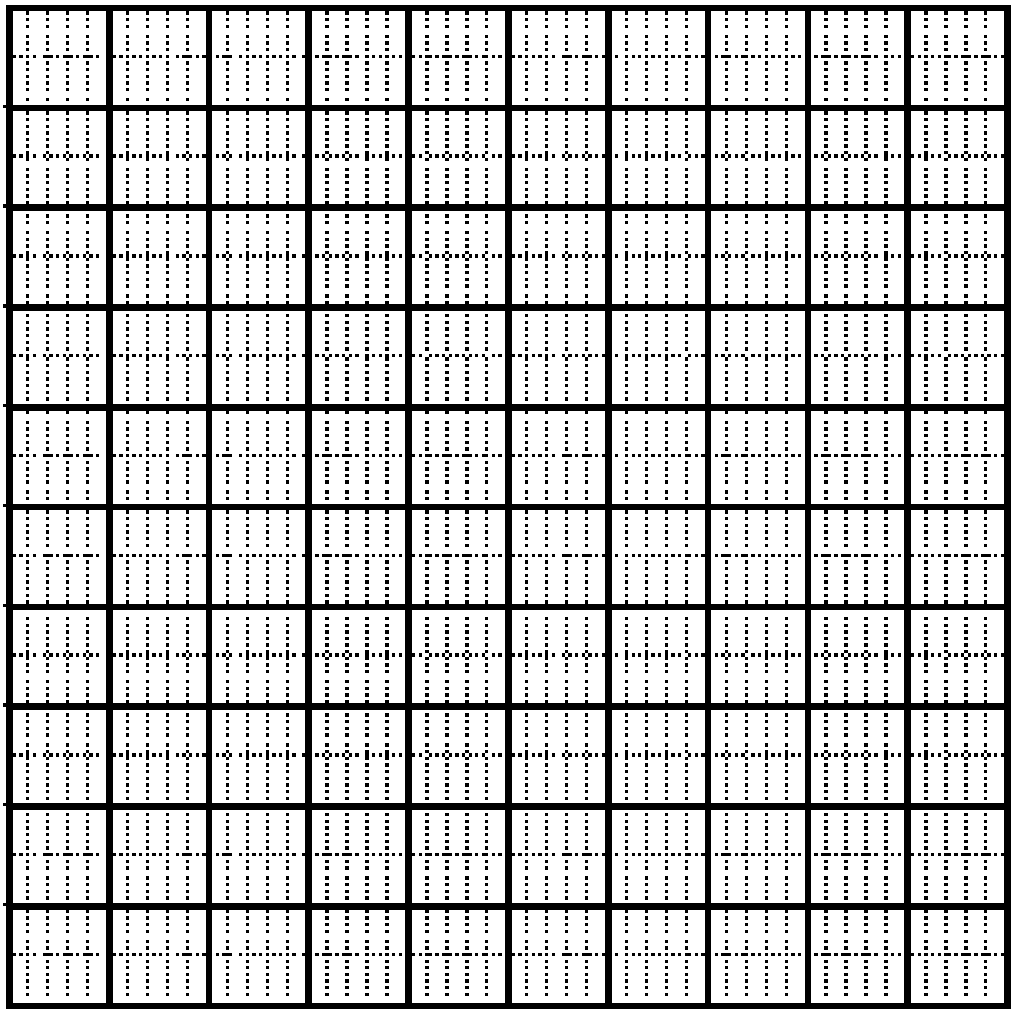
Appendix A



Appendix B



Appendix C



Task 4 - Perform Operations with Multi-digit Whole Numbers and with Decimals to Hundredths: Let's Design a Town to Welcome Refugees

Mathematical Content Standards

CCSS.MATH.CONTENT.5.NBT.B.7

Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

Mathematical Practice Standards

CCSS.MATH.PRACTICE.MP4

Model with mathematics.

CCSS.MATH.PRACTICE.MP6

Attend to precision.

CCSS.MATH.PRACTICE.MP7

Look for and make use of structure.

Vocabulary

Decimal addition, decimal multiplication, decimal modeling

Materials

Grid papers, base 10 blocks, Post-it Super sticky Easel Pad 25x30 inches, strings, glue, scissor, and colored pencils.

Lesson Objective

Students will investigate how they can add and multiply decimals to hundredths by applying length, area, and set models. This task's aim is to develop students' creative thinking skills by challenging them to find multiple representations of decimal addition and multiplication while designing a town that includes 10 features (e.g., house, hospital, school) and roads connecting these features. An important dimension of this lesson is to let: a) students see how mathematics is a connected field and, b) teachers understand how they can emphasize the connectedness of mathematical topics.

Engagement

Start class by asking students if they are familiar with people who were forcibly displaced from their homes due to many reasons including race, religion, nationality, political opinion, membership in a particular social group, natural disasters, population pressures, and economic recessions. After hearing their experiences, you can share current statistics available on the United Nations High Commissioner for Refugees' official website (https://www.unhcr.org/flagship-reports/). As stated on this website, a record number of people were forcibly displaced in 2020. There were 82.4 million people who were forcibly displaced worldwide and 91.9 million people were of concern to the United Nation Refugee Agency. These people were from many countries (see Table 1) and their status in their host countries is one of the followings: refugees, asylum seekers, and Venezuelans displaced abroad. About 42% of all forcibly displaced people were children and they were severely affected during forcibly displacement crises, especially if their displacement drags on for many years. Although a very limited number of people were able to return to their country of origin safely, many of them are looking for resettlement opportunities in third countries.

For example, the U.S. announced to admit more resettled refugees up to 62,500 in 2021 and up to 125,000 in 2022. Although this is great news to hear, we should work harder to provide an environment that can ease the refugees' adaptation to their resettled countries. Providing temporary shelter, jobs, language preparation, and other social participation should be carefully arranged before their arrival. Therefore, in this lesson, we will design a town for accepted refugees with a limited budget. Our job is to keep the budget as little as possible as we like to help more and more refugees to live in a town in which they can find their needs. Then, ask students:

- What kinds of places do they think refugees need in their town?
- What might be some problems that refugees may have in their resettled countries?
- How do they plan to solve possible problems?

Table 1. Number of People who forcibly displaced by their Countries in 2020

International Displacement Situations by Country of Origin	**Number of People**
Syria	6.8 million
Venezuela	4.9 million
Afghanistan	2.8 million
South Sudan	2.2 million
Myanmar	1.1 million
DRC	0.9 million
Sudan	0.9 million
Central African Republic	0.6 million
Eritrea	0.6 million

Explore-Part A

After hearing their responses, provide them grid papers so that they can sketch a rough draft of a town including all features. They need to include 10 features (e.g., house, school, hospital) and all features should be designed by using different shapes (e.g., triangle, square, hexagon). They can also use irregular shapes or a combination of two or more shapes. Tell them they have a budget of 100 dollars to design a map. they will design one feature every day and the cost of "1 square unit" is different each day. Provide Table 2 to students including the cost of each day and let them pick what shape(s) they like to pick for each day. Even they use a part of a square unit, it does cost "1 square unit" on a corresponding day. It is important to note that the computed areas of each shape need to range from 10 to 100 square feet and the area of each shape needs to be different from each other. Tell students that the company they purchase the grid papers decided to charge 1/100 of the area of the features in the first three days, 1/10 of the areas of the features on the second three days, and charge the full amount at the last four days. So, ask students to divide the number of squares or areas of their features by 100 for the first three days and by 10 for the second three days before they fill out the rows on Table 1 and apply the area model of decimal multiplications.

Table 2. Total Cost for Each Day and/or Feature

Days	Cost of "1 square unit"	Description of Features/Shape(s)	# Squares. Think about daily discount (e.g., divide by 100 first three days)	Total Cost
1st day	1.5			
2nd day	1.25			
3rd day	0.5			
4th day	0.25			
5th day	0.2			
6th day	0.1			
7th day	0.05			
8th day	0.04			
9th day	0.02			
10th day	0.01			
Total cost to design a map for refugees' town				

Let students work as a group of three or four and give them many grid papers and one poster paper or board (e.g., Post-it Super sticky Easel Pad, 25x30 inches). Ensure that students first design their features on the grid papers and fill out the rows on Table 2 for each day before they cut the features. It is essential to inform students that you let them cut their features when they compute their multiplication by using area models, but not applying standardized algorithms or calculators. Provide papers that include many hundred charts to students so that they can show their multiplication of decimals by applying area models. They can use calculators if they like to check their answers from the area model of multiplication.

Explain

If students are not familiar or fluent with the area model of multiplication, apply number talk activities. This can be helpful for students to refresh their minds and see both the connection of multiplication application as an operation across many number systems and the connection of fractions to decimals. For example, ask students to compute 18x15 by using the area model and represent their drawings on the board (see Figure 1).

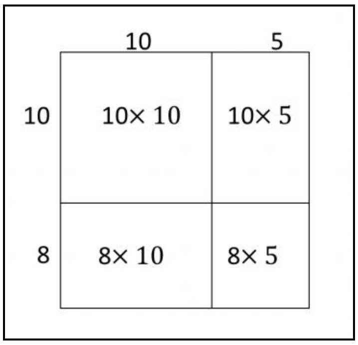


Figure 1. An Area Model of 18x15

Students can come up with several representations and teachers should challenge students to see if there are additional representations they can consider. For example, while one student can apply it by considering 18×15 as $(10+8)\times(10+5)=10\times10+10\times5+8\times10+8\times5$ (see Figure 1), another one can consider it in terms of factors and write it as $18\times15=9\times2\times5\times3=9\times10\times3=27\times10=270$ (see Figure 2).

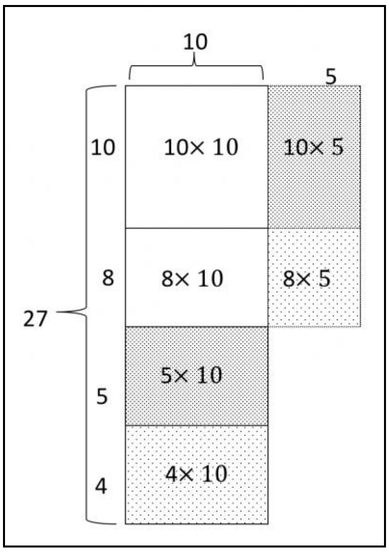


Figure 2. Another Area Model of 18x15

After this number talk activity, start a quick fraction number talk activity and say how we write 1/5 of 1/2 symbolically. Students can write this as $(1/5) \times (1/2)$. Then, ask students to represent both 1/5 and 1/2 by using rectangular areas. Then, tell them to shade 1/2 of 1/5. They can either use different colors or patterns to show the double shaded area as their answer (see Figure 3).

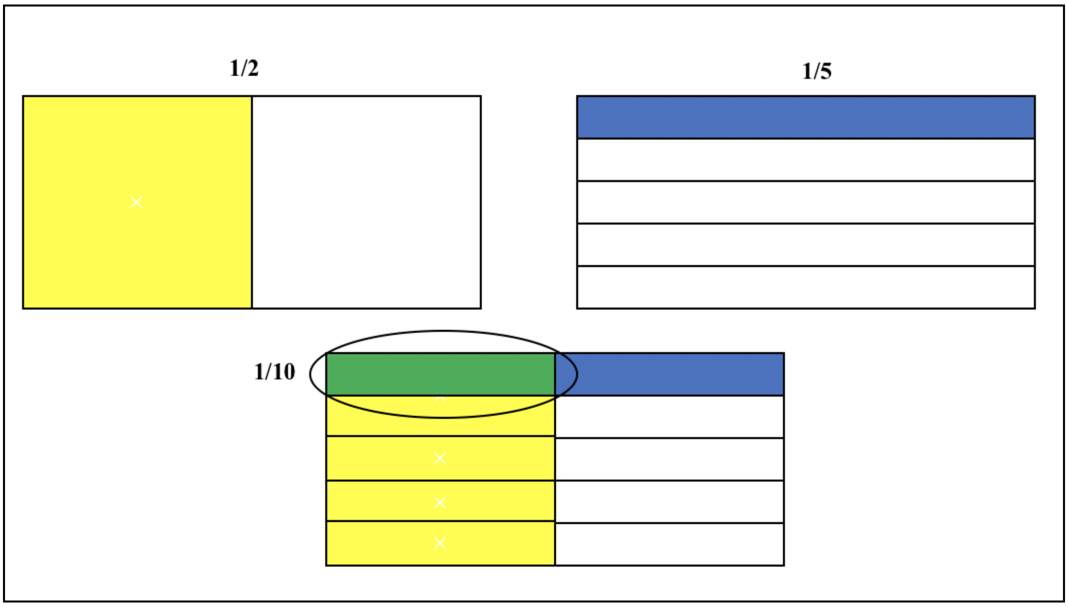


Figure 3. An Area Model of (1/2)×(1/5).

Ask students if we could write the fraction multiplication as $(2/10) \times (5/10)$. Students will recognize the equivalent fraction of 1/5 as 2/10 and the equivalent fraction of 1/2 as 5/10. Once students see the equivalent fractions, ask them to write the equations by using the decimal format 0.2×0.5. Then, let students conclude how this multiplication is the same as Figure 3.

Students can draw pictures and we expect them to see dividing each 1/5 horizontally will give 2/10 and dividing each 1/2 vertically give 5/10 and the answer will be $\left(\frac{2}{10}\right) \times \left(\frac{5}{10}\right) = 10/100$. Then, we will ask students to simplify this and see the answer is 1/10 and this will deepen students' understanding of the connection of decimals and fractions as well as let students create meaning of decimal multiplication by using their previous knowledge of fraction multiplication. Provide pictures for students by using base-ten materials (see Figure 3). Now, students are ready to apply decimal multiplication to model the costs of their map of towns.

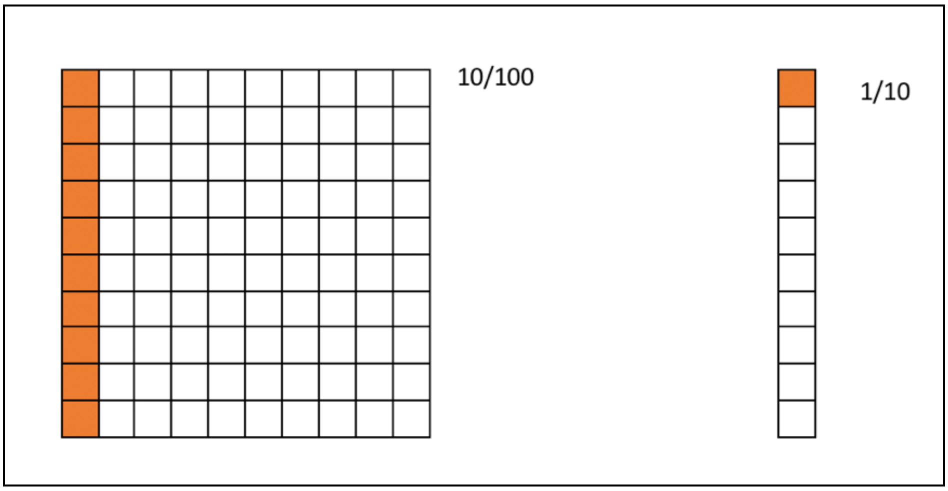


Figure 4. Representing 1/100 and 1/10 with Base Ten Materials

Explore-Part B

Students now are ready to apply the area model of decimal multiplications. Teachers should challenge students to use several representations by applying tools (e.g., grid papers, base-ten materials, drawings) strategically. For example, if students come up with an area of 80 in the first three days, they will divide this by 100 and have 0.8. Then, they will multiply this by the first-day price as 1.5 by applying an area model. They will then see that this is 1 whole and 0.5 or 5/10 or 1/2 of the whole. Students can use different colors on their models. For example, one student may use red to represent 1 and 0.5 and then color the 8/10 or 0.8 of these with yellow.

Teachers need to check how students read the equation as we expect them to say the question is asking what is 8/10 of 1.5. Then, the double-colored area (orange color) is their answer, which is 80/100 and 40/100. Students need to see several different ways of writing these fractions or decimals such as 0.80 and .40 or 0.8 and 0.4 and the teacher should ask students why 0.80 and 0.8 are the same.

If students have difficulty, ask them to write the expanded form of it and they will see that adding zero to the right side after decimal does not change the decimal. Then, let students work on new hundred charts to show the addition of 0.8 and 0.4 by coloring squares with orange color. Here, teachers should be careful to observe if students can flexibly see if they need more than one hundred charts as the addition of 0.8 and 0.4 is bigger than 1 or whole.

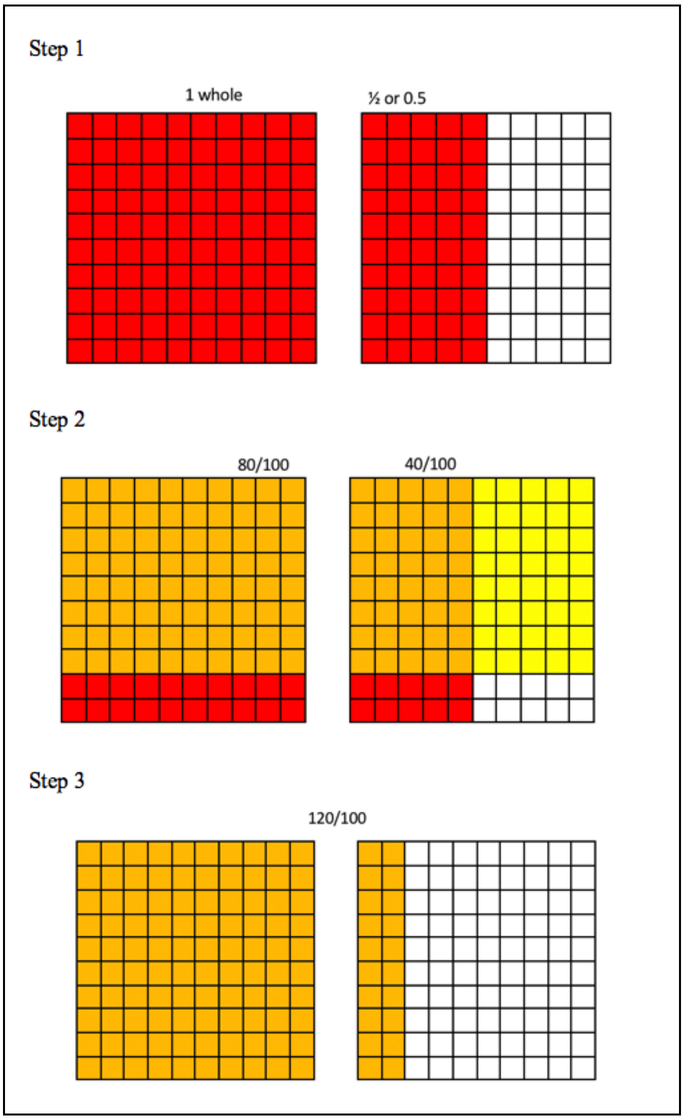


Figure 5. Base Ten Model for Multiplying Decimals

Let's see another example when students work on the fourth day. Remember we divide the area of our feature by 10 on the fourth day. Suppose a student finds an area of his/her feature as 48 and dividing it by 10 will result in 4.8. Since the price on the fourth day is \$0.25. We have to find what is 4.8 $\times$ 0.25. In this case, a student may think 4.8 as 4 and 0.8 and then write the multiplication as $(4 + 0.8) \times 0.25 = 4 \times 0.25 + 0.8 \times 0.25$. Then, they can compute this multiplication by applying base ten materials or grid papers. Students can think of this as 4 groups of 0.25 and then think what is 1/4 of 0.8. They will say four groups of 0.25 makes a whole (see Figure 6). They will then look 1/4 of 8/10 and count that there are 16 little squares and 8 half squares that make 20 little squares. Then, they will say the answer is adding 1 and 20/100 or 1 and 2/10 or 1 and 0.2 (see Figure 6). Teachers must let students discover additional drawings or materials when they multiply decimals rather than following exactly what teachers or their peers introduced. Teachers always challenge students how else you can represent it? Are there different ways you can think about showing this multiplication of decimal? Do you think you can apply different materials/pictures/drawings to show your decimal multiplication? To give an example, enabling students to think 0.25 as 1/2 of 1/2 is a creative way that a student may come up with.

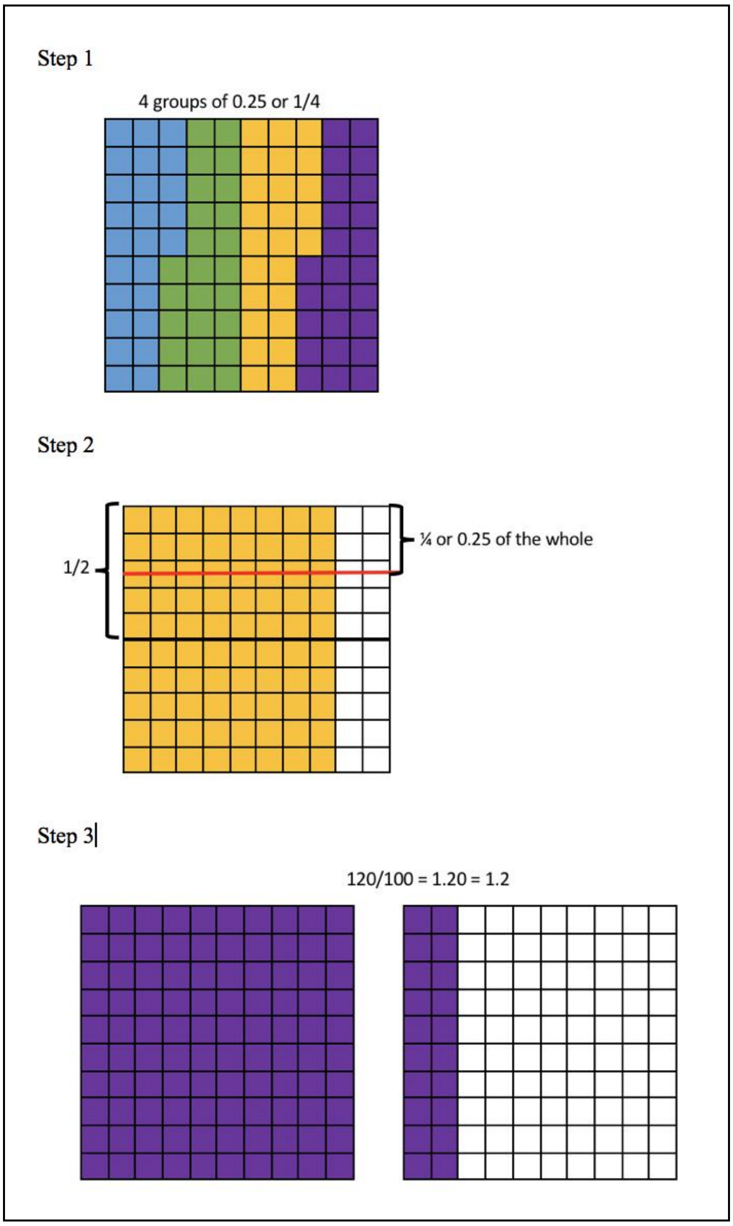


Figure 6. Showing Alternative Ways of Interpreting Area Model of Decimal Multiplications

Extend

Before students paste their features to their poster, teachers will ask them to think about the roads that connect their features. The unit they will use for their measurement is a length of a square on a grid of paper. They can either draw lines or use strings and glue to connect the features. They can also cut and paste the squares as many of them as they need to connect the features. Since they are now about to construct the roads, teachers provide Table 3 to students and ask them to use the predetermined decimals as daily rates. Inform students that the length of a road needs to be between 10 and 100 sides of a square on a grid paper and they will round the length to the nearest 10. Also, inform students that they will be charged only 1/10 of the length of their rods. Even they use a part of a square's side, it does cost as "1 side of a square". So, suppose they found or determined the length of a road as 83. They will then round it to 80 and divide it by 10. Then, they will multiply 8 with a daily rate (e.g., 1.5 on the first day) to find a cost on a corresponding day. For example, a student wrote the equation 8×1.5 and read it as eight groups of 1.5. Another student read it as 1.5 times eight and think it as 1×8 and 0.5×8 (or half an eight) (see Figure 7). Teachers should challenge students to read and represent them both ways. If students need more than 10 roads, they can continue using 0.01 as their daily rate. Students will draw lines (roads) or use strings to glue them on their board once they show the length model of corresponding multiplications. There are many user-friendly online websites that teachers and students can easily use to draw their number lines (e.g., https://apps.mathlearningcenter.org/number-line/).

Table 3. Total Cost for Each Day for the Roads

Days	Cost of "a side of square unit"	(The number of sides of a square)/10	Total Cost
1st road	1.5		
2nd road	1.25		
3rd road	0.5		
4th road	0.25		
5th road	0.2		
6th road	0.1		
7th road	0.05		
8th road	0.04		
9th road	0.02		
10th road	0.01		
Total cost to design a map of roads for refugees' town			

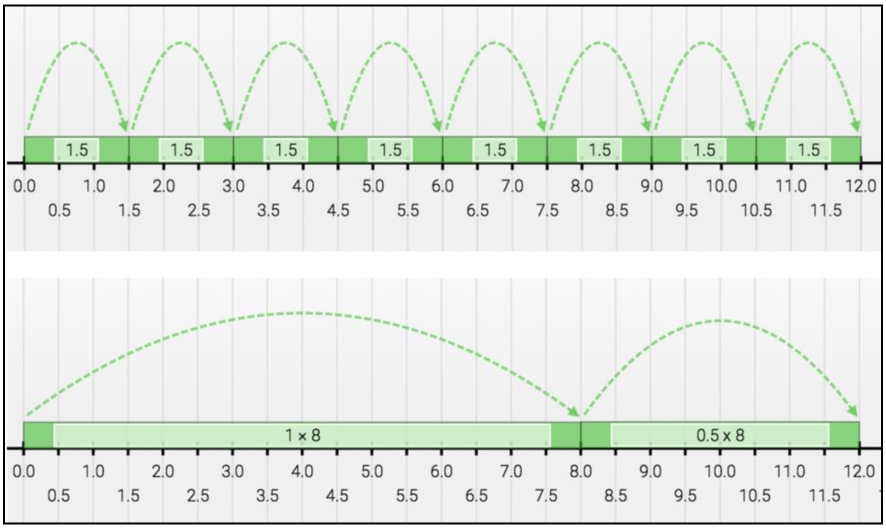


Figure 7. Two Ways of Reading and Representing Decimal Multiplication through Length Model

Once students complete their roads, ask them to find the total cost that they can either apply area or length model to add costs from features and roads. Since students already applied the area and length model, teachers should ask them to apply set models. For example, a student may have the costs as 19.95 dollars from the features and 24.46 dollars from the roads. In this case, a student first adds the whole parts as 19+24 and decimal parts as 0.95+0.46. Let's students apply a set model by using the website (https://apps.mathlearningcenter.org/money-pieces/). They will make the addition by converting decimals to a higher unit (see Figure 8). Then, they will have 44.41 as an addition of 19+24+1.41.

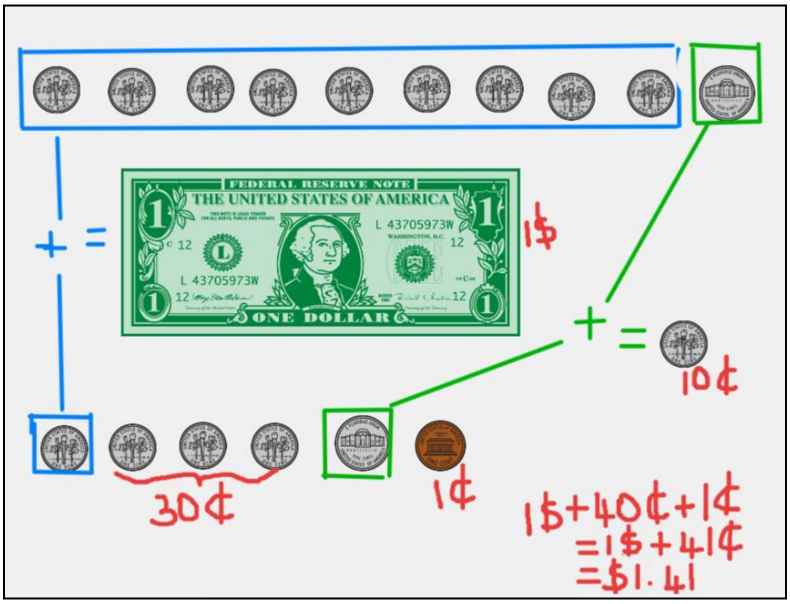


Figure 8. Set Model of Decimal Addition

This lesson can also easily be extended to teach decimal division through models. For example, teachers can ask students to represent $0.6 \div 3$ by using the area and length model. If students have a strong understanding of the decimal sense and decimal operations, they can easily apply their knowledge and represent it by using area, length, and set models (see Figure 9).

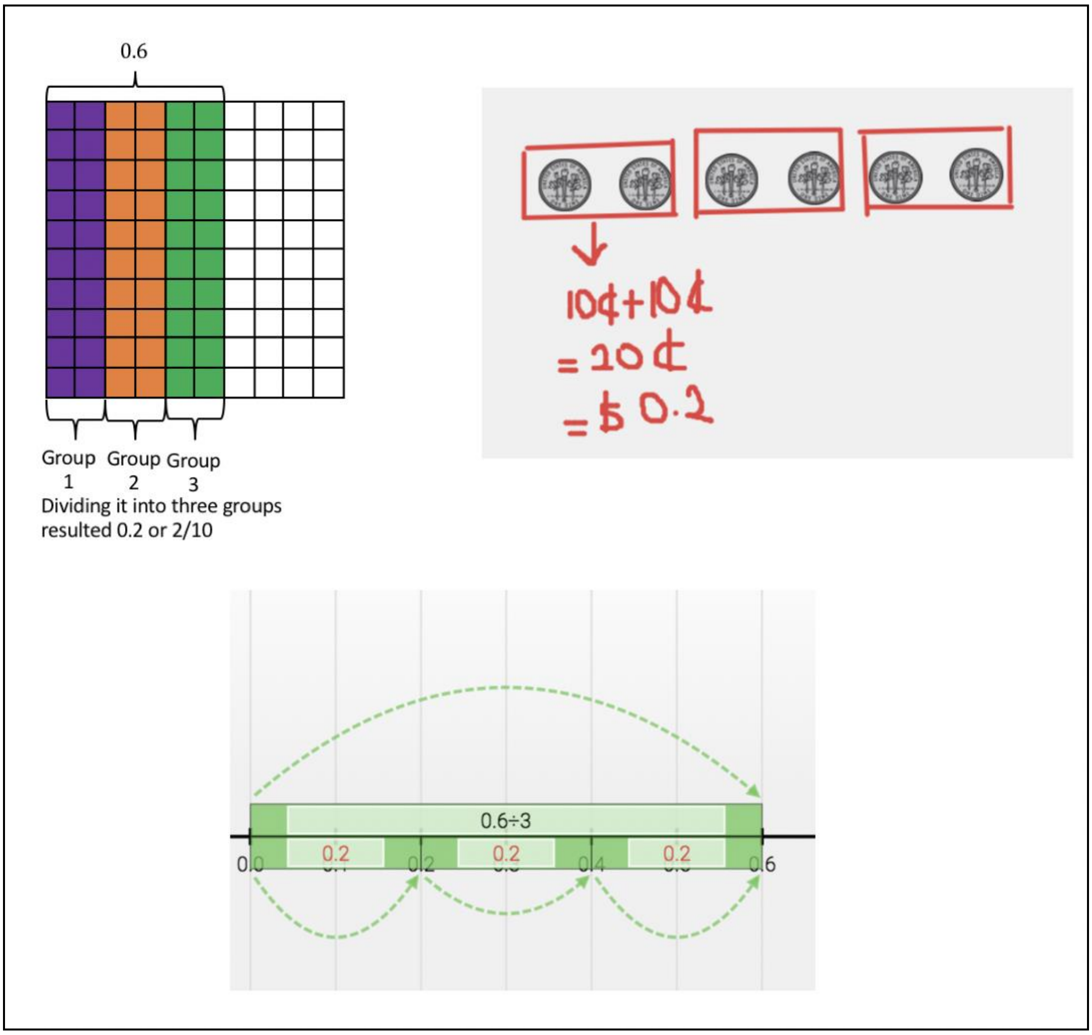


Figure 9. Three Models for Decimal Division

Evaluate

Teachers should first look for if students have a strong understanding of the connection between decimals and fractions and if they can interchangeably use them to add or multiply fractions. If students have difficulties with converting fractions to decimals or decimals to fractions, teachers should provide more activities. For example, give students a commonly used faction (e.g., 1/5) to convert a decimal by using based ten materials. While teachers can use base ten materials, they can ask students to show the same conversion on the grid line. Another suggestion might be to provide students the following fractions: 1/2, 1/4, 1/8, 1/16, 1/32… Provide a hundred charts to students and tell them first shade 1/2. Then, let them say how many little squares they shaded. They will then say 50. Then, ask how many totals we have on this chart. They will say than 100. At this point, students will see it as 50/100 or 0.50 is equivalent to 1/2. Then, do the same with 1/4 and let them see that it can be written as 25/100 or 0.25. Let students use different colors to make the decimals visible to them.

Extension part: The extension of this activity can be asking students what the sum of 0.50+0.25+0.125+…they think it is. Since students shade the squares, they can see a pattern and see that this sum is getting close and close to being a whole or 1.

Once we see that students are flexible to convert decimals to fractions or fractions to decimals, we can also look for if they have a decimal sense by asking them an estimation of some questions. While students work with their group members to find the models for adding or multiplying their fractions or decimals, ask them if they can estimate the results. For example, teachers can ask students to estimate the followings: (1) 4.95+123.01+56.12, (2) 5.91 x 6.1, (3) 0.51 x 9.91, (4) 149.9 x 0.11. For example, for #4 we expect our students to estimate 149.9 is almost 150 and 0.11 is about 1/10, and dividing 150 by 10 is about 15. These are essential skills as students can check if their answer makes sense after they apply different models (e.g., area, length, set). If students have difficulty with making estimations, then engage them in estimating activities by letting them use hands-on materials.

Another important dimension of this lesson is being flexible to use any models (e.g., area, length, and set) to represent decimal addition and multiplication. Teachers should look for if students can explain their reasoning of decimal multiplication and/or addition by using number lines, monetary concepts, diagrams, grid papers, and base 10 blocks. Other than shifting one model to another model to represent a decimal multiplication or addition, teachers must challenge students to generate additional representations or ways within a model. For example, after a teacher or student apply area model for 2.4 x 3.5 by using grid paper, we can ask how we could use it by using place value concepts (see Figure 10) similar to what we did in the 18x15 number talk activity (see Figure 1 & 2).

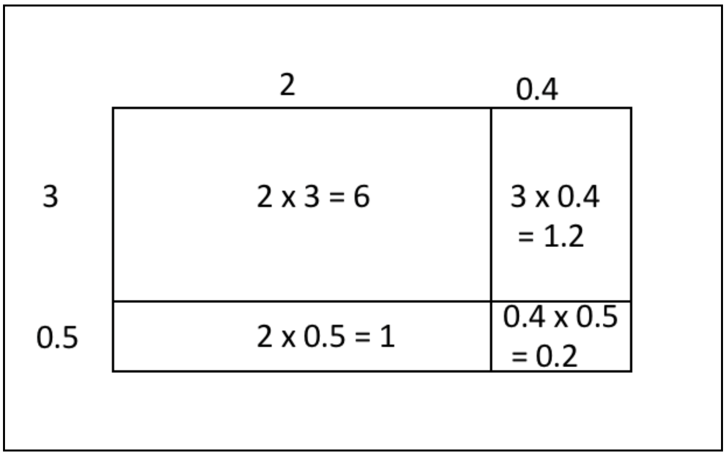


Figure 10. Applying Place Value to Decimal Multiplication

Task 5 - Use Equivalent Fractions as a Strategy to Add and Subtract Fractions: Infinite Fraction Patterns

Mathematical Content Standards

CCSS.MATH.CONTENT.5.NF.A.1

Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. *For example, 2/3 + 5/4 = 8/12 + 15/12 = 23/12. (In general, a/b + c/d = (ad + bc)/bd.)*

CCSS.MATH.CONTENT.5.NF.A.2

Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. *For example, recognize an incorrect result 2/5 + 1/2 = 3/7, by observing that 3/7 < 1/2.*

Mathematical Practice Standards

CCSS.MATH.PRACTICE.MP2

Reason abstractly and quantitatively.

CCSS.MATH.PRACTICE.MP4

Model with mathematics.

CCSS.MATH.PRACTICE.MP5

Use appropriate tools strategically.

CCSS.MATH.PRACTICE.MP7

Look for and make use of structure.

Vocabulary

Equivalent fractions, fraction addition, fraction subtraction, and fraction benchmark

Materials

Hundred chart, square paper, fraction bar or tower, grid papers, colored pencils, and counters.

Lesson Objective

Students will use their understanding of equivalent fractions to add and subtract fractions. One of the essential aims of this lesson is to promote students' mathematical creative thinking by enabling them to see how we can represent fractions differently even when we represent commonly applied fractions (e.g., 1/2, 1/4). Students will have a chance to develop the meaning for standardized algorithms of fraction additions and subtractions through using area, length, and set models. They will also have a chance to develop their estimation skills of fractions additions and subtractions using fraction benchmarks (e.g., 1/2, 1). This lesson will be covered within 4 class times.

Engagement

(15 minutes) Start the lesson with a YouTube video titled "infinite patterns" (https://www.youtube.com/watch?v=ZF3CgNpkSTQ). Then, ask students to share their observations. After hearing their observations, tell students that we all can see mathematical patterns differently and then solve mathematical problems by using different approaches. Then, tell students that in this lesson we will create different patterns and identify given

infinite patterns' rules while learning how to add and subtract fractions using our understanding of fraction equivalence.

Explain

(30 minutes) Before adding and/or subtracting fractions, it is suggested to refresh students' minds about fraction equivalence. Fraction bars (see Figure 1) or fraction towers can be very helpful manipulatives: https://mathigon.org/polypad#fraction-bars to refresh students' understanding of equivalent fractions.

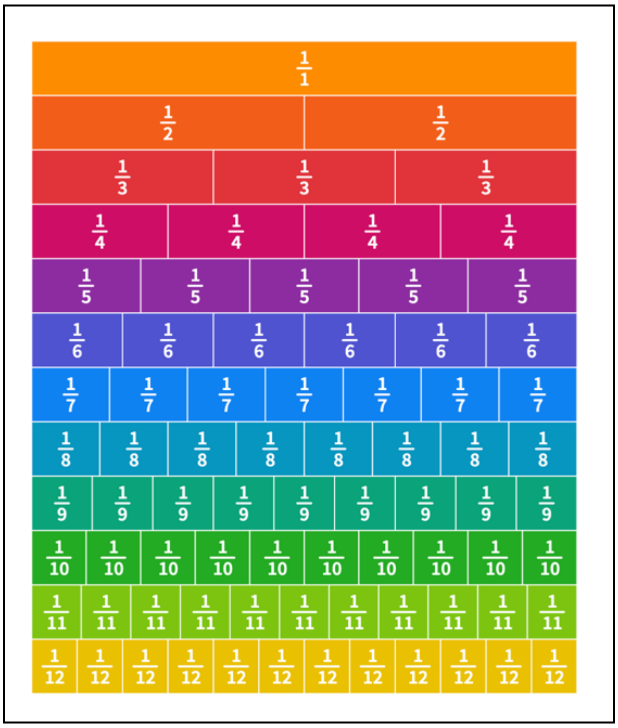


Figure 1. Fraction Bars

Ask students first to show all unit fractions from 1/2 to 1/12 listed under 1 or a whole. Then, ask students how many halves they need to make 1 or how many fourths they need to make 1. Students will see that they need 2 halves to make 1 and 4 fourths to make 1, etc. After these observations, ask students to find equivalent fractions for 1/2. Students will see that they need 2 fourths, 3 sixths, 4 eights, 5 tenths, and 6 twelfths to make 1/2. Ask them to write these as:

$$\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{4}{8} = \frac{5}{10} = \frac{6}{12}$$

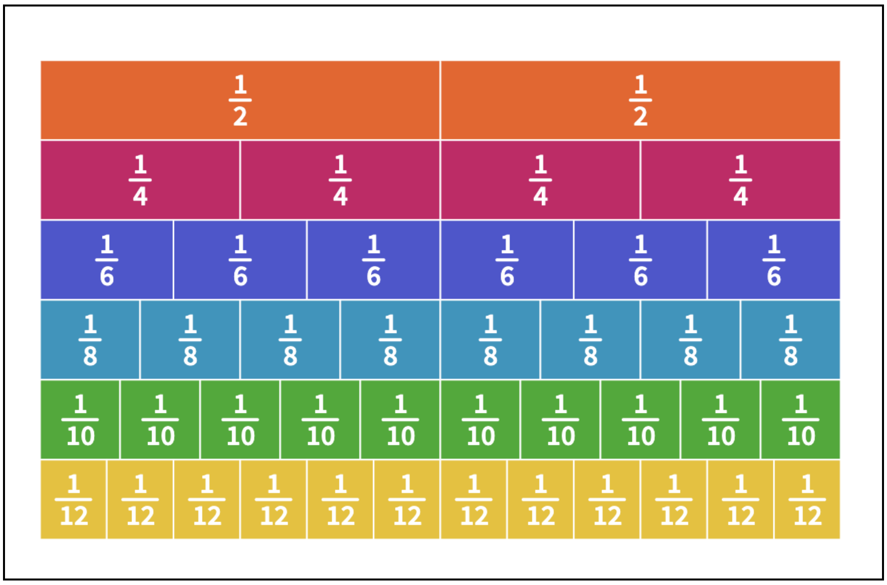


Figure 2. Equivalent Fractions of 1/2

Then, ask them to look if they see any patterns. Students might say, "the numerator increases by 1, and the denominator increases by 2s." Then, ask what about the next equivalent fraction, students will then say 7/14. If students cannot identify this pattern, encourage them to see by asking questions (e.g., what is happening in the numerator or the denominator from one to another equivalent fractions?). Teachers can also ask them to identify the relationship between the numerator and the denominator within the fractions. Students will say that the denominators are twice the numerators in each fraction. Then, tell students to write the same equations for 1/3 and ask them to generate three more equivalent fractions. Once students are fluent in generating equivalent fractions for a given unit fraction, it is time for teachers to ask students to generate equivalent fractions for 3/4 or 5/6. For 3/4, students will pick the bars or towers (see Figure 1) that exactly match with the 3/4 and they will write the following:

$$\frac{3}{4}=\frac{6}{8}=\frac{9}{12}$$

Then, teachers will ask students to write the next three equivalent fractions that are not available through manipulatives (e.g., fraction bars, fraction towers). Students are expected to see that the numerators increase by 3 and the denominators increase by 4 from one to another equivalent fraction. Then, they will write 9+3=12 and 12+4=16 and the next equivalent fraction is 12/16. Once students write three more equivalent fractions (i.e., 12/16, 15/18, 18/21), teachers should ask students to tell the relationship among these fractions. Students should note that the simplest form of these fractions is the same as 3/4 and be able to show their reasoning by writing one of these fractions (e.g., 12/16) as:

$$\frac{12}{16} = \frac{3 \times 4}{4 \times 4} = \frac{3}{4}$$

Once students are ready to generate more equivalent fractions for a given fraction and simplify a fraction to write its simplest form, they are ready to explore how to use equivalent fractions to add and subtract fractions.

Explore-Part A

(30 minutes) Ask students to draw a square and then let them shade one half of it. Let them share how differently they divide a square into half. Encourage your students to show 1/2 more than one way by using the squares (see Appendix A). This might be considered as either a fraction talk or a geometry talk based on the overarching goal of the class. Ask students to shade half of their squares blue and encourage them to show half of a square in as many different ways as possible by using different squares. Then, tell your students to show 1/4 of their squares without touching the shaded part (1/2). After students show 1/4 of their squares, ask them to shade the area green. Again, encourage students to show their 1/4 in as many different ways as possible by using different squares. After they shade 1/4 of their squares, tell them to show 1/8 and shade it by using orange or purple without touching the previously shaded areas. Ensure that students use two different colors if they have two 1/8 within a square so that they do not confuse it with 1/4 by combining them.

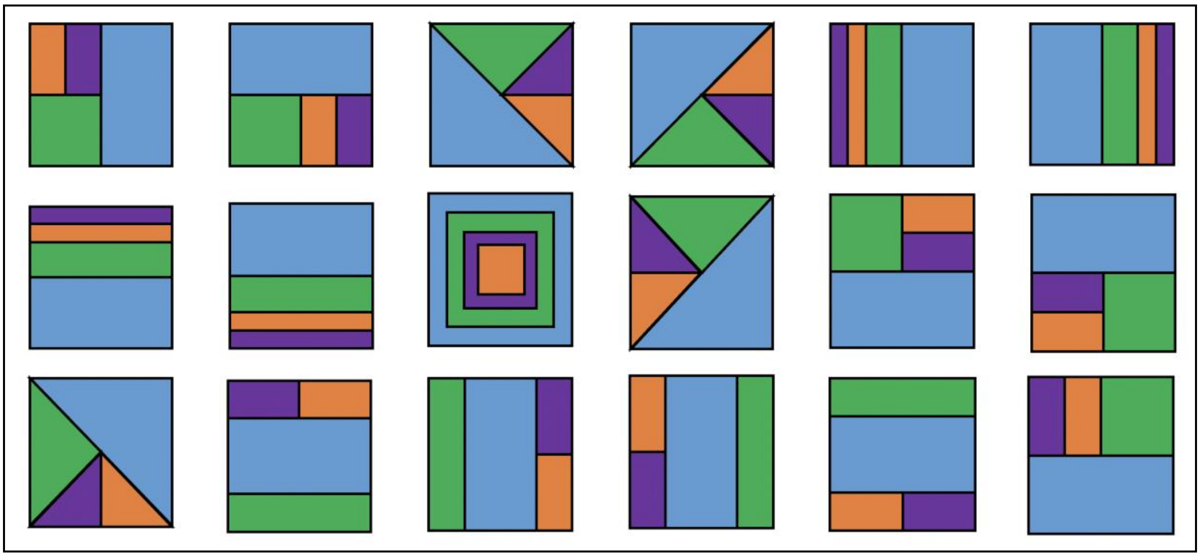


Figure 3. Many Ways of Representing 1/2, 1/4, and 1/8 within a Square

Explore-Part B

(40 minutes) Once students shared their works during a whole-class discussion, let them code the fractions with the colors. Students will then write blue = 1/2, green = 1/4, orange =1/8, and purple = 1/8. Observing if students can come up with correct coding is essential for teachers to see if students have a strong sense of fractions. Then, let students write many fraction addition and subtraction problems by using colors first and then fractions.

For example, students can write orange + purple = green and then translate this into fractions as 1/8 + 1/8 = 1/4. Encourage students to read it as 2/8 = 1/4. Since they are expected to use equivalent fractions to add or subtract fractions, asking students to interpret their fraction equations is a convenient exercise for students.

Another equation a student write might be purple + orange + green = blue along with its translated fraction form as 1/8 + 1/8 + 1/4 = 1/2. In this equation, we want students to first add 1/8 (purple) and 1/8 (orange) and say it is green (or 1/4). Then, write green + green = blue as well as 1/4 + 1/4 = 1/2.

Here, again let students read it like 2 times 1/4 is 1/2 or say 2 fourths is a half. Also, ask students to write subtractions (e.g., blue - green = purple + orange and 1/2 – 1/4 = 1/8 + 1/8) and tell them to explain the fraction equation to their group members or you. Since students already observed that 1/8 and 1/8 make 1/4 or two eights is a fourth based on the square they draw or the fraction bar model, they can write this as 1/2 – 1/4 = 1/4. At this part, let students write the equivalent fraction of 1/2 as 2/4 by looking at their fraction bar models (see Figure 4). Then, write the equation as 2/4 – 1/4 = 1/4. Again, use the fraction bars (see Figure 4), get two fourths, and then subtract one fourth. This results in 1/4.

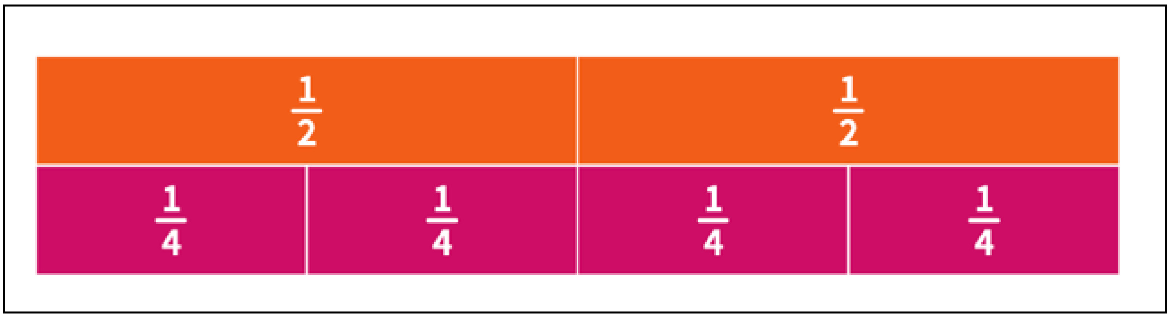


Figure 4. 2/4 is Equivalent to 1/2

Color Equations	Fraction Equations
orange + purple = green	1/8 + 1/8 = 1/4
purple + orange + green = blue	1/8 + 1/8 + 1/4 = 1/2
blue – (orange + purple) = green	1/2 – (1/8 + 1/8) = 1/4
green – purple = orange	1/4 – 1/8 = 1/8
blue – green = purple + orange	1/2 – 1/4 = 1/8 + 1/8
blue + green + purple + orange = 1	1/2 + 1/4 + 1/8 + 1/8 = 1
blue – green – orange = purple	1/2 - 1/4 - 1/8 = 1/8
blue – green = green	1/2 – 1/4 = 1/4

Once students see how they are using the equivalent fraction concept to add or subtract fractions with the same denominators, we can start working on fractions with different denominators. Ask students to find 1/3 + 1/4. In this case, students can be asked to add 1/3 and 1/4 together by using the fraction bars. They can see if its length matches any of the bars' lengths. They will see that the matching one is 7/12. At this point, teachers can ask students to think about how we can get 7/12 by adding 1/3 and 1/4. By looking at the length model, students can say that 1/3 corresponds to 4/12 and 1/4 corresponds to 3/12. We expect our students to see that they are using equivalent fraction concepts to add or subtract fractions. Then, add 4 twelfths and 3 twelfths pieces together to get 7 twelfths (see Figure 5).

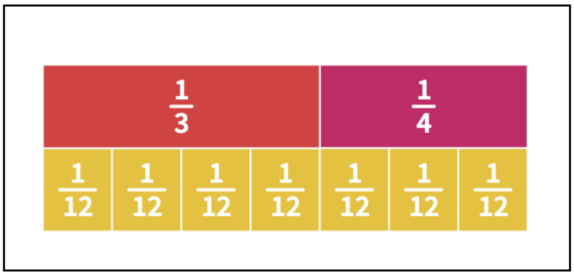


Figure 5. Adding 1/3 and 1/4 using Equivalent Fraction Concept

Explain & Explore-Part C

(45 minutes) After using the length model to add fractions with different denominators, we can show an area model to students to add fractions by using equivalent fractions. Let's show them how we can find the answer 1/5+1/2 by using an area model. Since there are two fractions, model each by using rectangular areas as:

1) Draw two rectangles
2) Break into parts. Partition the first rectangle vertically into fifths and the second one horizontally into half (see Figure 6). Or the first one horizontally into fifth and the second one vertically into half.

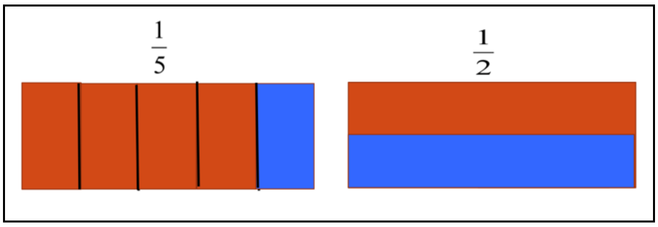


Figure 6. Partition the Rectangles Vertically and Horizontally

3) Divide the first box into the same units as the second rectangle (1/2 horizontally). Then, divide the second box into the same units as the first rectangle (1/5 vertically).

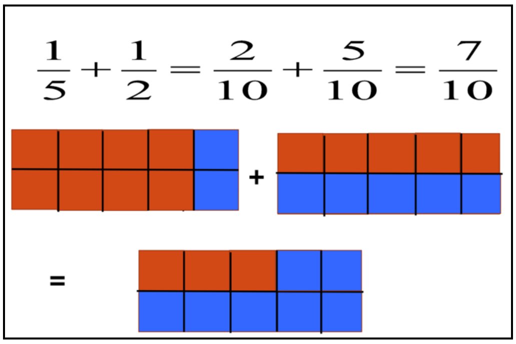


Figure 7. Finding Equivalent Fractions to Add Fractions

Teachers should ask questions (e.g., what did we do in this model) to enable students to see that we again use equivalent fraction concepts as we wrote 2/10 for 1/5 and 5/10 for 1/2 as their equivalent fractions. They then see that coloring two-tenths and five-tenths makes seven-tenths. After this example, teachers can ask another question (e.g., 3/2+1/4) for students and let them apply both the length model (e.g., fraction tower or bar) and area model to solve it. In this example, we expect our students to think of it as 1+1/2+1/4 when drawing or building their models.

When students are comfortable with the area and length model, it is time for them to apply the set model. Let them add fractions by giving them two fractions like 1/3 + 1/2 (see Figure 8). Represent each fraction with counters as follow:

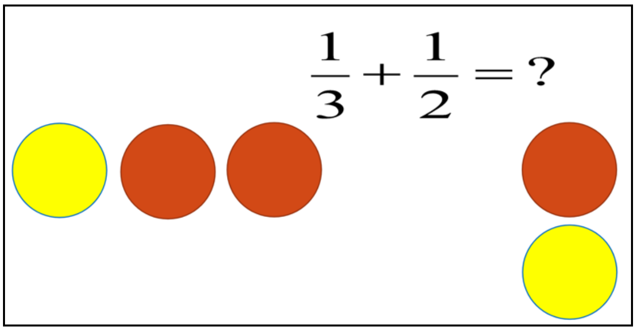


Figure 8. Applying the Set Model to Add Fractions

We expect our students to write the equivalent fractions of 1/3 and 1/2. It is possible that some can write 2/6 for 1/3 and 2/4 for 1/2 as equivalent fractions. It is important for teachers to create questions (e.g., why do think it did not work when you wrote 2/4?). Teachers should encourage students to see that they do have sixths in one set and the fourths in the other set. Then, teachers should ask: What can we do to make both the same (i.e., sixths)? It is expected for students to add two more counters to the second set as:

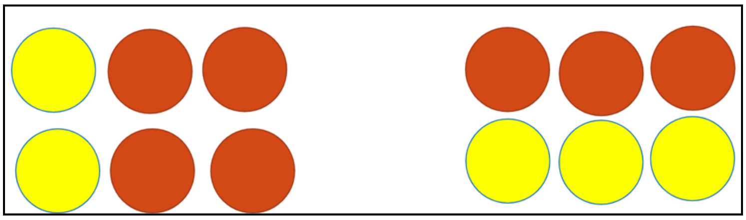


Figure 9. Finding Common Denominators to Add Fractions by using the Set Model

Once students have sixths in both sets, they can add the yellow ones to write the numerator and add all counters to write the denominator, and write 5/6 as their answer. After these, teachers can start introducing non-unit fraction additions. Teachers can ask students first to estimate the following fraction additions by using fraction benchmarks: a) 9/10 + 12/ 13, and b) 3/7 + 4/9. Create a number talk activity for students to discuss their reasoning with others. We expect students to say that 9/10 is almost one as it is missing only 1/10 to be a whole and 12/13 is also almost a whole as it is missing only 12/13 to be a whole. Then, adding two fractions that both are almost 1 makes the total almost 2. Then, they answer that 9/10 + 12/13 is a little less than 2. If some cannot make these estimations, provide them with manipulatives and let them make their estimation by using these manipulatives before mental estimations. We want students to do a similar estimation for 3/7+4/9 as they will say that both are less than 1/2 since half of 7 is 3.5 and half of 9 is 4.5. Adding two fractions that are both less than

1/2 makes the sum less than a whole or 1. Since they are both a little less than 1/2, the sum is a little less than 1.

After hearing students' estimations and letting them add or subtract fractions by using equivalent fraction concepts as area, length, and set models, students now are ready to generalize their understanding about adding or subtraction fractions. Ask them what they do when they add two fractions (e.g., 3/7 + 4/9) by using any models. For this, create a discussion environment for students to see their observations by using any models. For example, students who applied area models can think that they have to divide a rectangular area into sevenths horizontally and then later into ninths vertically and create 63 units that will be the denominators of their fraction. This is a nice observation that we want our students to do to develop reasons for standardized algorithms of fraction additions and subtractions. Then, let students multiply both numerator and denominator of 3/7 by 9 and have 27/63. Let them find equivalent fractions of 4/9 by multiplying both numerator and denominator by 7 and writing 28/63. Since they have now 63 in their denominator, they can add the numerators as the shaded areas and have (27+28)/63 = 55/63. The purpose is not to teach standardized algorithms to students, but to develop students understanding of the meaning of standard algorithms of faction addition and subtraction. This will help them later quickly grasp the concept and reason why certain procedures work in fraction addition and subtraction.

Extend

(40 minutes) It is time for teachers to let students observe different ways of finding equivalent fractions to add or subtract fractions. Teachers can use a hundred charts (See Appendix B) and ask students to divide them into half first. It does not matter how they make their division. Then, let them shade the areas. Then, let students draw lines to create 1/4 and ask them to shade it. Then, ask students to add 1/2 and 1/4 by counting the number of squares in the shaded areas. Students then will write the following: 1/2 + 1/4 = 50/100 + 25/100 = 75/100. Similarly, ask students to shade 1/8 and ask them to add 1/2 + 1/4 +1/8 by writing the equivalent fractions. They will then write 1/2 + 1/4 +1/8 = 50/100+25/100+12.5/100 = 87.5/100. You can then ask students to shade 1/16 and add it to the sum. They will then have 1/2 + 1/4 +1/8 + 1/16 = 50/100+25/100+12.5/100 +6.25/100. Then let students draw the visuals they have (see Figure 10). A student may have the following:

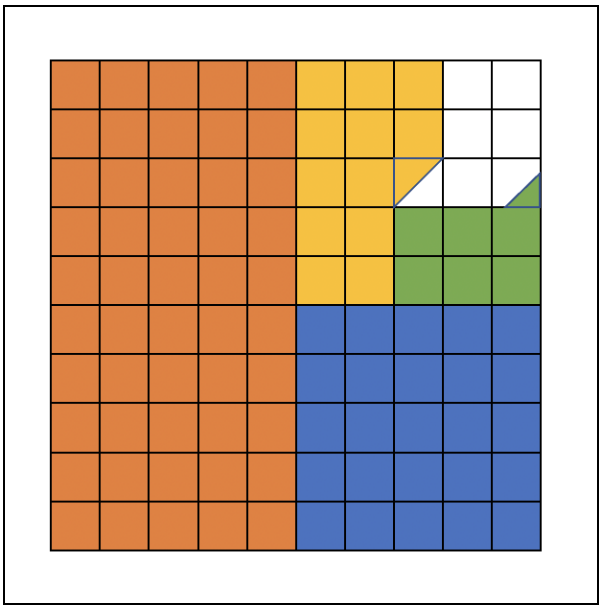


Figure 10. Adding 1/2, 1/4, 1/8, 1/16 by using Equivalent Fractions in a Hundred Charts

Since students may create their visuals differently, ask them to draw on the board so that others can see how their approaches are similar or different from their visuals (see Figure 11).

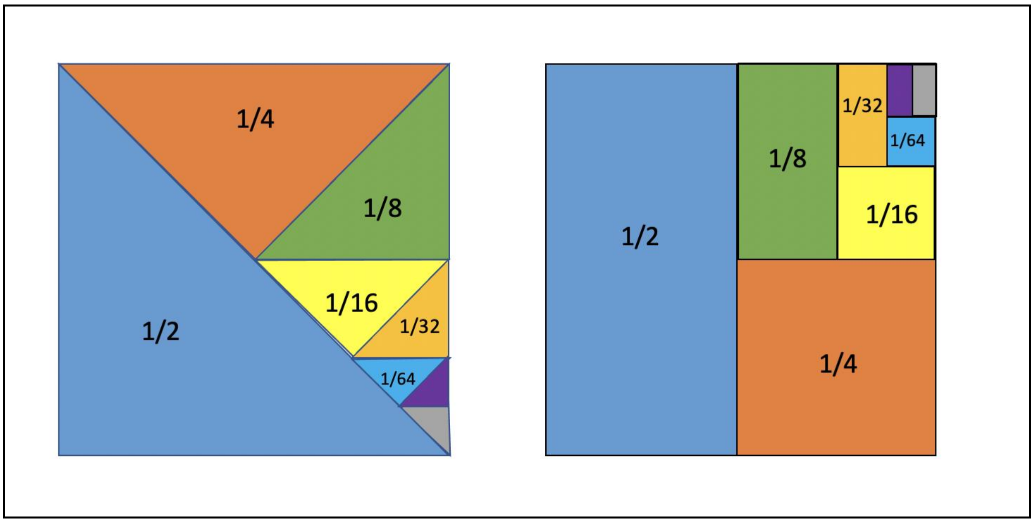


Figure 11. Two Different Ways of Adding 1/2, 1/4, 1/8, 1/16 by using the Area Model

Let students share what they notice when they see the pictures or visuals of 1/2 + 1/4 + 1/8 + 1/16 + 1/32 + 1/64+ … We expect our students will say that the area or fraction they continue to add is getting smaller and smaller. Then, ask them what the area will be when you infinitely add all of these fractions. Let students work in groups and hear their ideas. At this point, students will look at the squares and say “it will be the whole square or 1”. This is a high ceiling activity for students. It is not that we expect our students to understand the limit concept, but we want them to identify visual patterns while learning fraction addition and subtraction. This activity can also be extended to teach students percent as they were changing each fraction to percentages.

If you like to challenge your students more with fraction additions, you can do the following activity as well. Show students the following pattern and let them share what they think is happening in the patterns.

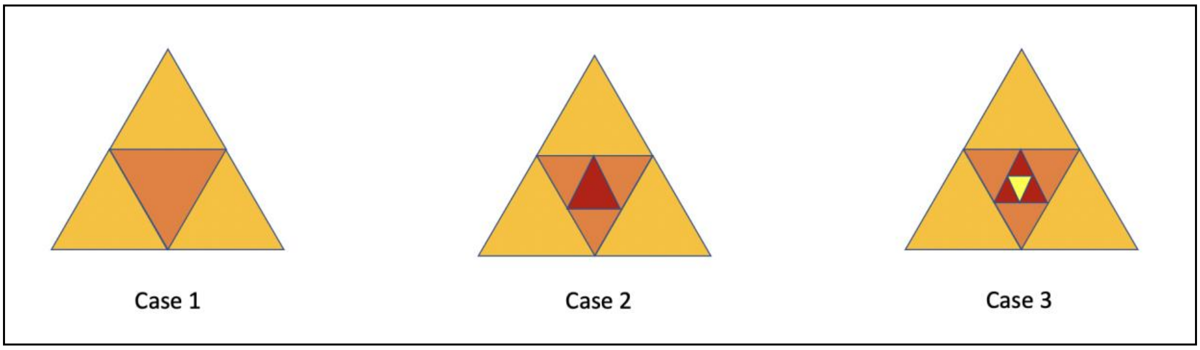


Figure 12. Investigating Fraction Addition with Infinitely Growing Patterns

Ask students to represent the area other than the middle triangle in each case with fractions. After students write the fractions as 3/4, 3/16, and 3/64, ask them what the sum of these fractions would be if you continue this pattern infinitely many times. Help students write the fractions as 3/4, 3/(4x4), 3/(4x4x4). Then ask them what is the next one. Students will see the next one is 3/(4x4x4x4) and realize that the added triangles areas are getting less and less each time when they continue adding triangles into the middle of the shape. Encourage students to think how many times the fractions get less from one to the next case. Students will see that they are dividing into 4 equal pieces or dividing by 1/4 each time, so the unit/area or fraction gets 4 times smaller in each case.

It is possible that some students visually see that sum is getting to be 1 and state “When you do this calculation infinitely many times, you will have no gaps in the triangle and the sum will be equal to 1 or the whole triangle”. If some cannot see this, let them create a table and observe what is happening after adding one more case. They can also practice adding fractions by using the equivalent fraction concept. Computing the first five cases can be enough for students to see that the sum of these fractions is getting close and close to being 1 and when you do this infinitely many times, the sum of these fractions equal 1.

Case Number	Adding fractions using equivalent fractions
1	3/4
2	3/4 + 3/16 = 12/16 + 3/16 = 15/16
3	15/16 + 3/64 = 60/64 + 3/64 = 63/64
4	63/64 + 3/256 = 252/256 + 3/256 = 255/256
5	255/256 + 3/1,024 = 1,020/1,024 + 3/1,024 = 1,023/1,024

If students cannot compare the fractions (e.g., 3/4, 15/16), provide them with grid papers and let them draw two number lines or two circular areas. Let students see that 15/16 is missing 1/16 to be a whole and 3/4 is missing 1/4 to be a whole. Then, ask students to compare 1/4 and 1/16 by using any models. They will see that 15/16 is missing little to be 1 compared to what 3/4 is missing. Then, they will conclude that the fractions are getting higher as the case number increases, but each fraction is less than 1. This is also an ability for students to compare fractions by using benchmarks.

Evaluate

Teachers should look for if students are flexible to represent any given fractions by using manipulatives or drawings. Without having pictures of commonly applied fractions in their minds, students cannot develop the meaning of fraction addition using equivalent fractions. If students have issues with representing commonly applied fractions (e.g., 1/2, 1/3, 1/4, 3/4, 5/10), teachers should provide more opportunities to students to play with manipulatives and ask them to represent commonly applied fractions.

Teachers also should look for if students can find different ways of representing the fractions (e.g, 1/2, 1/4) by using the same model or visuals. If students get stuck on a single representation, they may not see the relationship between two given fractions or they may not identify some mathematical details. This is because some mathematical details or patterns on a certain model or picture may not be as visible to students as to when they were on the other models and/or visuals. It is possible that they may not see that two fractions are equivalent on a certain drawing, but this might be straightforward on another drawing. Therefore, they may not develop the meaning of fraction addition and subtraction like 1/8 + 1/8 = 1/4 or 1/2 – 1/4 = 1/4 if they get stuck on a single representation. Teachers should let students share their drawings, ideas, and offer other ways so that they can capture different meanings of the fraction through various visuals.

Teachers need to observe if students can fluent in generating equivalent fractions and flexible to change unit fractions to each other. For example, they should be flexible to see that 2 fourths are halves, or 2 eights is one-fourth. If they have issues, teachers should let students engage in fraction bars, fraction towers, or fraction circles until they reach the equivalent fraction meaning with unit fractions.

It is also important for teachers to look for that their students see many patterns related to unit fractions. For example, when they hear one half, they should be able to generate an equivalent fraction and they should be able to tell the relationship between the numerator and the denominator by saying the denominator is twice the denominators. If students have issues with it, create a number talk activity for unit fractions so that students can describe their meanings and represent them by using manipulatives or drawings.

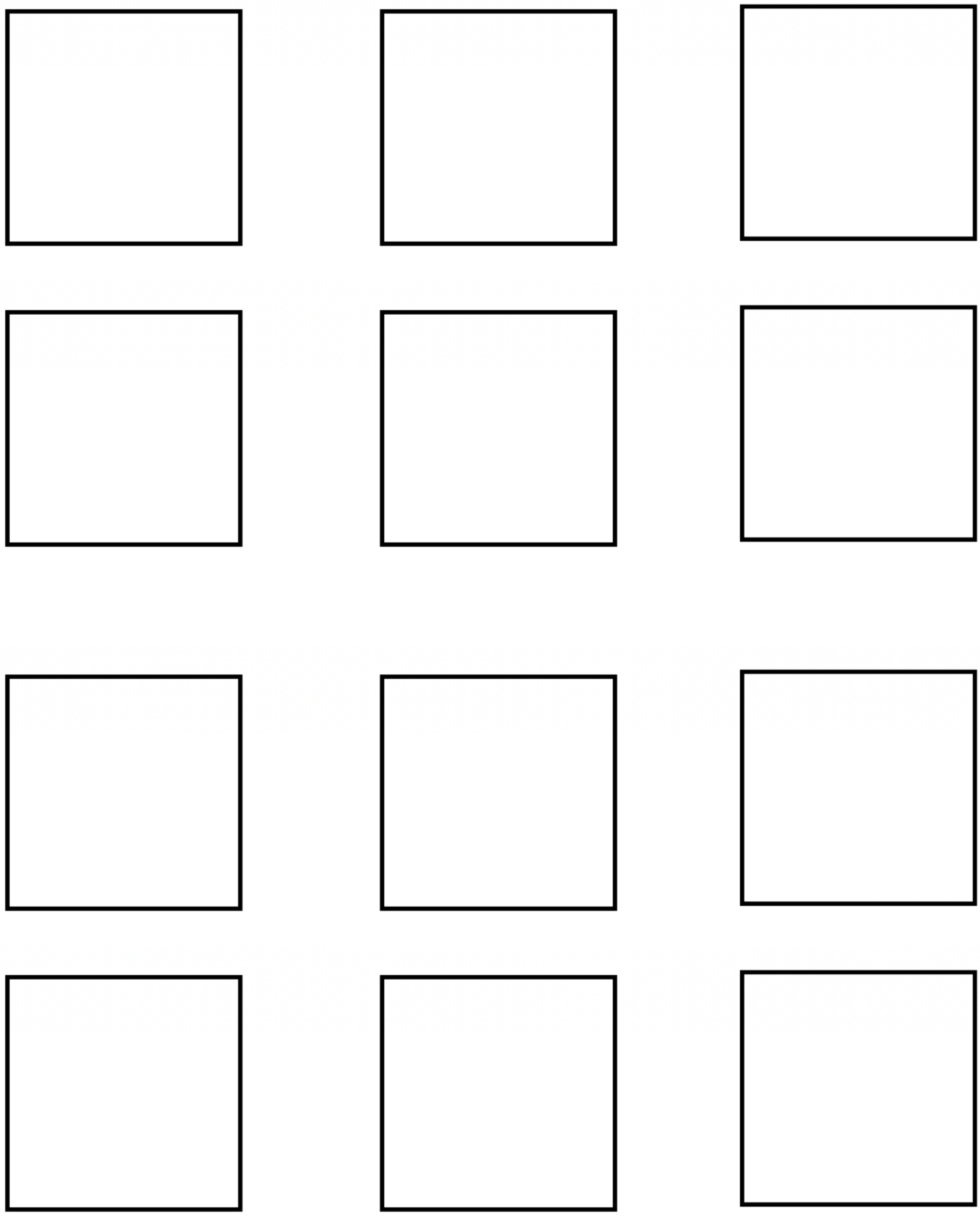
Teachers should look for if students are fluent in their multiplication facts as they are flexible to multiply fractions to find the equivalent fractions. If students have problems with their multiplication facts, ensure that they have time to practice multiplication facts by using manipulatives and drawings.

Teachers also should look for if students identify mathematical patterns by looking at the pictures or tables while adding the fractions. Teachers need to observe if students can associate the relationship between 1/2, 5/10, 50/100 when they use a hundred charts. Since 50/100 as a fraction include numbers that are high than what students are used to seeing at the beginning (e.g., 1/2, 4/8), using grid papers or a hundred charts is very helpful for students to develop their understanding of equivalent fractions further. If students have problems with it, ensure that students count the number of squares and write the fractions as well as show the corresponding fraction with visuals.

Teachers should look for if students partition two rectangular areas by using both horizontally and vertically. If students do not understand the meaning of horizontal and vertical partition, they may partition the areas either horizontally or vertically. This sometimes can make their solution process very complicated. Teachers should ask students to see if they understand the reason why they apply both horizontal and vertical partition. Let students do their method and communicate with them why they have difficulty with using only horizontal or vertical partitioning. Then, ask them to apply by using both horizontal and vertical partitioning.

Lastly, teachers should look for if students flexibly use benchmarks to compare fractions. If they have issues with determining the size of fractions, provide them with sets of fractions that they can compare. Teachers first should let students compare fractions by using manipulatives or drawings and gradually move to compare fractions mentally.

Appendix A. Square Paper



Appendix B. Hundred Chart

Task 6 - Apply and Extend Previous Understandings of Multiplication and Division to Multiply Fractions: Unfinished Projects

Mathematical Content Standards

CCSS.MATH.CONTENT.5.NF.B.3

Interpret a fraction as division of the numerator by the denominator ($a/b = a \div b$). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem.

For example, interpret 3/4 as the result of dividing 3 by 4, noting that 3/4 multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people each person has a share of size 3/4. If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?

CCSS.MATH.CONTENT.5.NF.B.4

Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.

CCSS.MATH.CONTENT.5.NF.B.4.B

- Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths.
- Multiply fractional side lengths to find areas of rectangles, and represent fraction

products as rectangular areas.

CCSS.MATH.CONTENT.5.NF.B.5

Interpret multiplication as scaling (resizing), by:

CCSS.MATH.CONTENT.5.NF.B.5.A

Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication.

CCSS.MATH.CONTENT.5.NF.B.6

Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem.

Mathematical Practice Standards

CCSS.MATH.PRACTICE.MP1

Make sense of problems and persevere in solving them.

CCSS.MATH.PRACTICE.MP4

Model with mathematics.

CCSS.MATH.PRACTICE.MP5

Use appropriate tools strategically.

CCSS.MATH.PRACTICE.MP8

Look for and express regularity in repeated reasoning.

Vocabulary

Fraction multiplication, modeling fraction multiplications, and fraction multiplication as a scaling.

Materials

Pattern blocks, hundred charts, snap cubes, fraction bars/towers, circular fractions, colored pencils.

Lesson Objective

Students will investigate fraction multiplication as they will apply contextual examples and models to solve problems involving: 1) multiply a fraction by a whole number, 2) multiply a whole number by a fraction, 3) multiply a fraction by a fraction-no subdividing, and 4) multiply a fraction by a fraction-with subdividing. Students' mathematical creativity will be promoted by enabling them to illustrate representations of fraction multiplications in their own way. This lesson will be covered within about 4 class time.

Engagement

(20 minutes) Start the lesson by telling students that a construction company needs interns to design interior areas of an apartment complex project. The project they will be working on includes flooring, decorating, and designing. They will help the company finish some projects by applying their previous knowledge understanding of fractions and multiplication while illustrating fraction multiplication in their ways. They will work as a group of 3 to 4 to complete a given project. As soon as they complete a given project, they will be asked to convince the whole class about why their illustrations make sense. The other groups will act as the company and be skeptical to ask as many questions as they want until they are completely satisfied with their reasonings. Once everyone is satisfied with their explanations, they will move to another project.

To refresh students' minds about whole number fractions, start the class by applying a number talk. Ask students to find 12 x15 by applying area models and discuss with them their reasoning. It is important to tell students that they are not allowed to apply standardized algorithms and encourage them to find as many different ways as possible. Students can come with several ways as: a) (6+6) x15, b) (10+2) x15, c) 6x2x5x3=18x10, d) 12 x (10+5), e) 3x4x5x3=3x20x3, f) 12 x (12+3), and etc. Tell students to explain their reasoning and highlight their models of multiplication as equal sets, area or array, and number line models.

Explain for Project 1

(Multiplying a fraction by whole number) (30 minutes) After students fluently come up with several explanations by applying several models, let them discuss how they can interpret multiplication of a fraction by a whole number such as 5 x (1/4), 6 x (1/8), 10 x (3/4), and 3 x (7/3). Although this was introduced in grade-4 by CCSS-M, students can benefit from this discussion to build a conceptual understanding of a fraction multiplication by a whole number. It is suggested to provide students with scenarios. For example, for 5 x 1/4, we can say you have 5 pans each with 1/4 of a pizza. How much pizza do you have? We expect our students to see this as five groups of 1/4 and write 1/4 + 1/4 + 1/4 + 1/4 + 1/4. Then, let students apply different models (i.e., area, set, length) to show their answers (see Figures 1, 2, & 3).

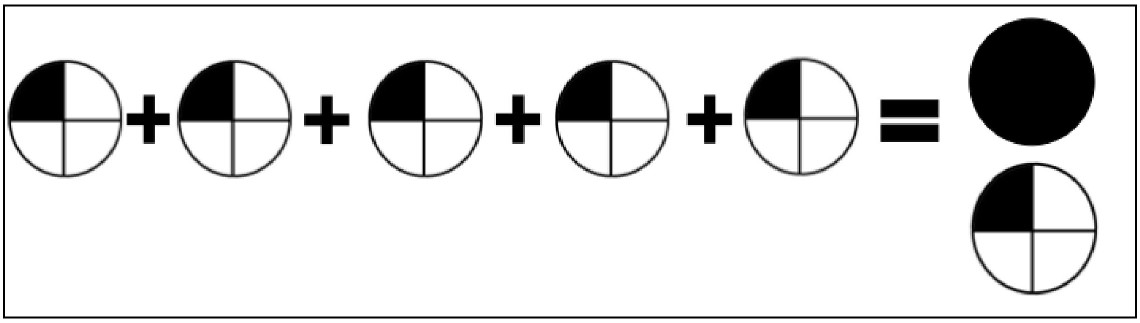


Figure 1. 5 x (1/4) with Area Model

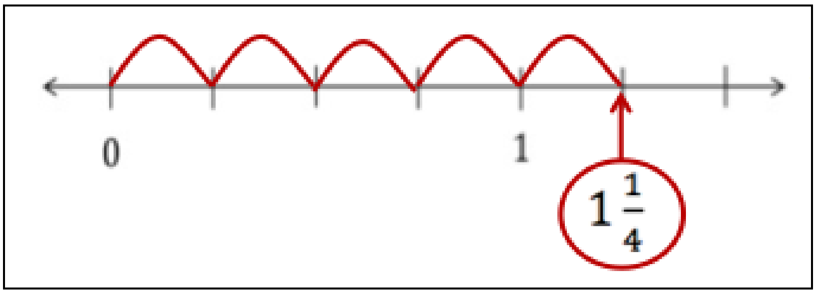


Figure 2. 5 x 1/4 with Length Model

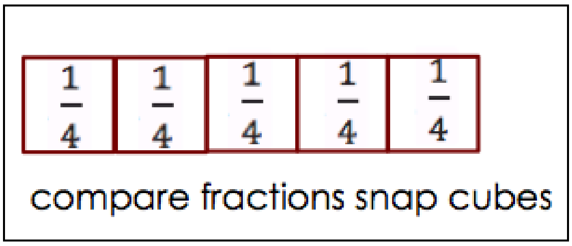


Figure 3. 5 x 1/4 by using Snap Cubes that can be Considered as Either Length or Set Model

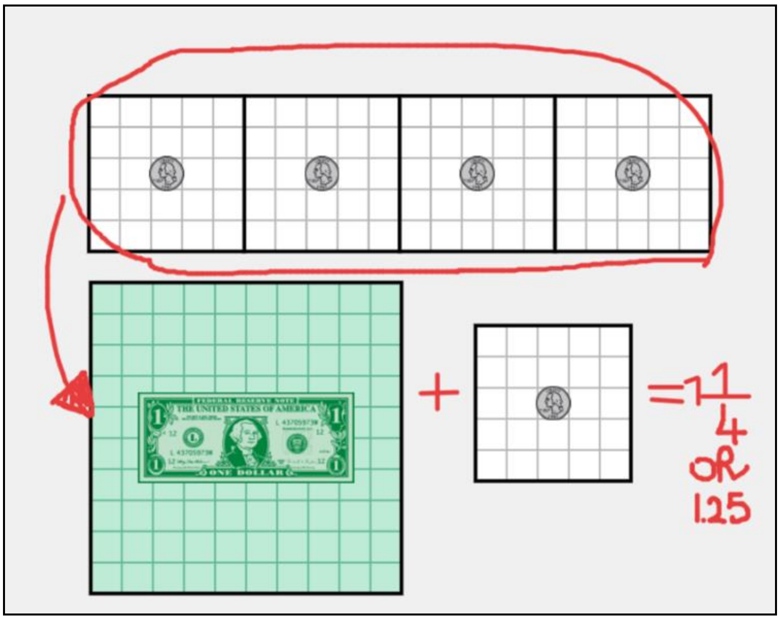


Figure 4. 5 x 1/4 by Applying Set Model

Explore-Project 1

(Multiplying a fraction by whole number) (20 minutes) Once students understand the meaning of a fraction multiplication by a whole number, it is time to engage them in creative work. For project 1, let students know that they are expected to create a pattern that will be used for tiling a bathroom's walls. Provide students with pattern blocks (https://mathigon.org/polypad/qVSgBQIpBsPJ9Q) and ask them to create a pattern on paper. Ensure that their patterns do not exceed a regular page. Tell students that you want them to create their patterns in a way that their multiplications of a fraction (i.e., representing the corresponding pattern blocks in their pattern) by a whole number (i.e., representing the number of times they used the corresponding pattern blocks) results in a whole number. For example, students can come up with the following pattern (see Figure 5). Then, teachers should ask them to write down the corresponding expression as: $6 \times 1 + 12 \times \frac{1}{2} + 6 \times \frac{1}{3} + 12 \times \frac{1}{6}$. Then, ask students to look at their patterns and tell the answer to their expressions. Students are expected to see that 12 times 1/2 (red or trapezoid) makes 6 wholes or hexagons, 6 times 1/3 makes 2 wholes, and 12 times 1/6 makes 2 wholes (see Figure 6). Let students share their patterns and the results of their expressions with others and create a discussion that enables students to hear various strategies and analyze the reason why some expressions in their patterns work, but some do not result in whole numbers. Let them explain their strategies. Ask many questions to challenge them to think critically. For example, can I add as many hexagons as possible? What about trapezoids?

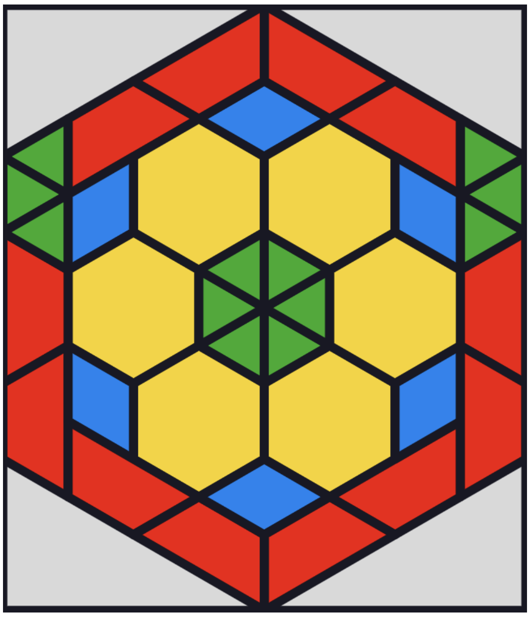


Figure 5. A Pattern Example that results a Whole Number

This activity can be extended by asking students how many 1/2, 1/3, or 1/6 pieces would cover their patterns. For example, teachers look for if students can think twice of their answer (16) when their unit is 1/2. If they cannot see this pattern, ask them to cover the shapes in Figure 6 and let them see the relationship between their answer when their unit was a hexagon and their answer when their unit was a triangle, a trapezoid, or a diamond. When students are comfortable with the meaning of multiplication of a fraction by a whole number that results in a whole number, ask them to generate examples that do not result in a whole number by using pattern blocks. Once students are comfortable with generating examples (i.e., 10 x (1/6) is equal to 1 and 4/6 or 1 and 2/3), let them move to the next project. Project 1 enables students to see a fraction (a/b) can be written as $a \times \frac{1}{b} = \frac{a}{b}$.

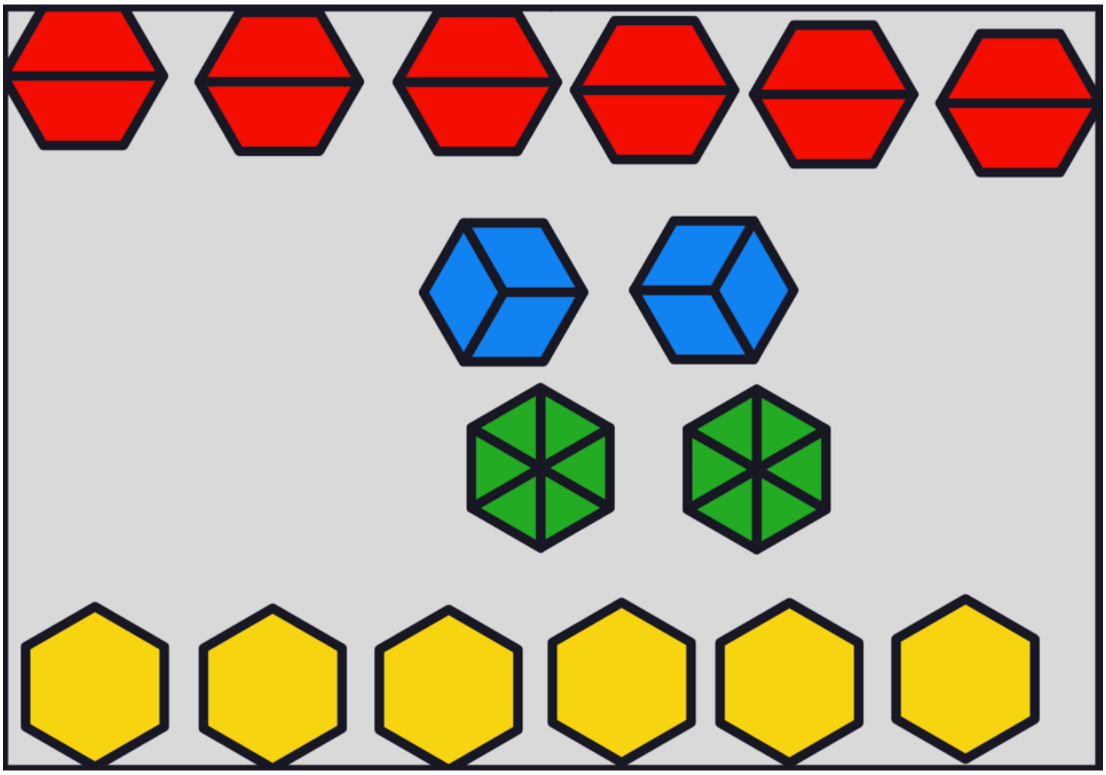


Figure 6. Reasoning of Multiplication of a Fraction by a Whole Number by using Pattern Blocks

Explain and Explore for Project 2

(Multiplying a whole number by a fraction) (30 minutes) Before students start their project 2, ask them multiplication problems involving whole numbers by using area models (i.e., 2 x 4). Once they completed their work, let them think about how they would change their models if they were asked to multiply 1 by 4. Ensure that students work on their existing models. Students would divide their areas in half (see Figure 7).

Once students are fluent and flexible with their modeling of multiplications by seeing the area as 1x4 and 1x4, ask them how the original area changed. Let students see that they ended up having half of the original area as they change a multiplier from 2 to 1. Introduce the word "scaling" to them and let them know that you scale the original area in half by changing a side of their rectangle from 2 to 1. Then, ask students to show (1/2) x 4. Students then need to divide 1 into half and see that the area is half the previous drawing, which is 2. It is important to ask students to read it as 1/2 of 4 is 2. Then, let students see that the multiplication of fractions is really about scaling.

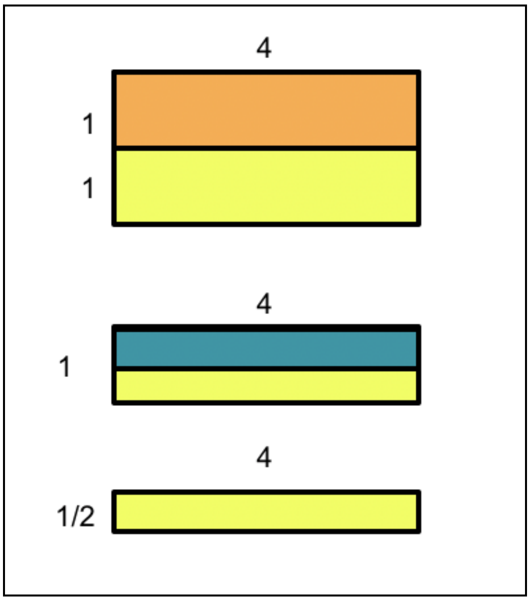


Figure 7. Seeing Multiplying 1/2 as a Scaling into Half

Support students' understanding of multiplying a whole number by a fraction by providing contextual problems or projects. Ensure that students can apply area, set, and length models for each type of multiplication (e.g., multiplying a whole number by a fraction). As an example, for a set model, tell them the company finished 45 living rooms in an apartment complex and 2/3 of these rooms were yellow painted. Then, let them think of the rooms as a set model by using counters, snap cubes, or other manipulatives and partition 45 into three groups and then see how many are in two parts. Or, this can be considered as a rectangular area model by drawing 45 equal size rooms connecting and asking students to divide the whole into three equal pieces and see how many are in two parts.

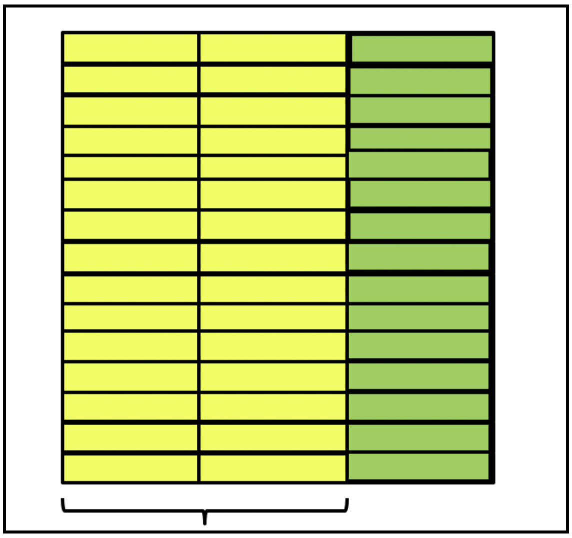


Figure 8. Visualization of (2/3) x 45 with an Area Model

This example can be supported with paper folding. Start with one paper to prepare students for the rest of the activity. Provide them with a paper and let make them two folds (one horizontal and one vertical) to have fourths. Expect students to see that they scaled their papers 4 times shorter. Then, let students unfold and shade a piece that represents 1/4 of a paper and write it as (1/4) x 1 = 1/4. Encourage students to represent 1/4 of a paper in various ways. In this case, encourage students to see that 1/2 of 1/2 is 1/4 by either looking at fraction bars or drawing area models.

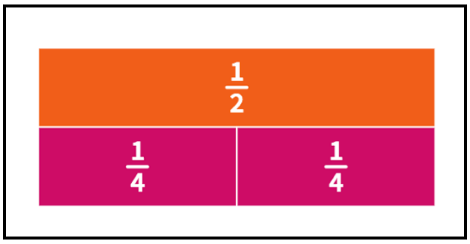


Figure 9. Representing 1/2 of 1/2 is 1/4

Then, let them stack 45 papers at top of each other and then ask them to make 2 either horizontal or vertical folds. Since 45 papers might be too many to make folds ask each member to folds 15 papers and then let them bring the papers together. Students can see the thirds by either cutting or shading the two parts to show or count how many are in two parts.

Explore-Explain- Project 3

(Multiplying a fraction by a fraction-no subdividing) (35 minutes) After students have experiences with wholes of fractions and fractions of wholes, it is time to introduce finding a fraction of a fraction. Teachers should select their tasks carefully so that students do not have to make additional partitioning at first. For this, teachers again should use contextual problems as follow: The construction company wants you to design a space that includes parking, pool area, children park, and grass area. The manager asks you to leave 1/2 of the lot for the parking area. Then, arrange 1/5 of the remaining lot for the children's park. What is the area allotted for the children's park? For this question, you can use a paper folding activity as students can make one horizontal fold to have halves and let them color the half (the outside/cover part). Then, ask them to make four vertical folds to have fifths and color 1/5 with another color. Tell them to unfold it and look at the double-colored area and write it as 1/10. Teachers should also explain this paper folding activity by using rectangular areas (see

Figure 10). Students should be encouraged to write this contextual problem as $(1/5) \times (1/2)$ read it as what is 1/5 of 1/2. Or, they can read it as how big is a 1/5 of 1/2 piece or area? For a rectangular model, ask students to represent both 1/5 and 1/2 by using rectangular areas. Ask them what the question was asking and let them say that what is 1/5 of 1/2. Then, let your students partition the rectangular area showing 1/2 to fifths. Then, shade 1/5 of the area. They will see that the double shaded area is their answer as it represents 1/5 of 1/2.

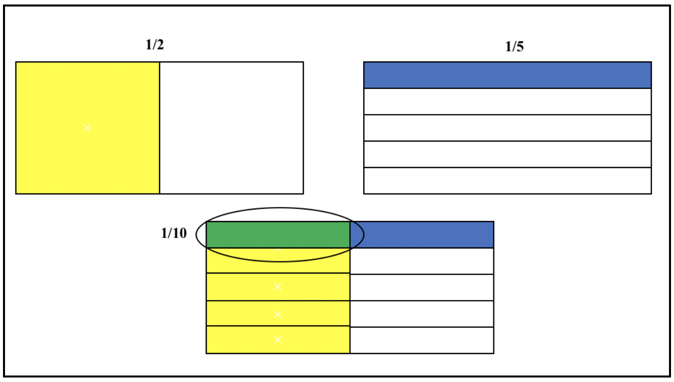


Figure 10. An Area Model of (1/2)×(1/5)

Another contextual problem can be as follow: The construction company wants you to paint one of the outside walls of the apartment complex. You started this project earlier and blue painted 1/10 of the walls. Then, the manager asked you to paint 2/3 of the remaining area with the color of orange and the rest of the wall be unpainted as the manager want to put his portray on that part. How much of the wall will be painted orange? Students can apply area models by using rectangular or circular models (see Figures 11 & 12). Encourage students to apply more than one way to promote their creativity. As you can see in these examples, students should see that they can model their solutions based on what the problems are asking rather than imitating previously applied models.

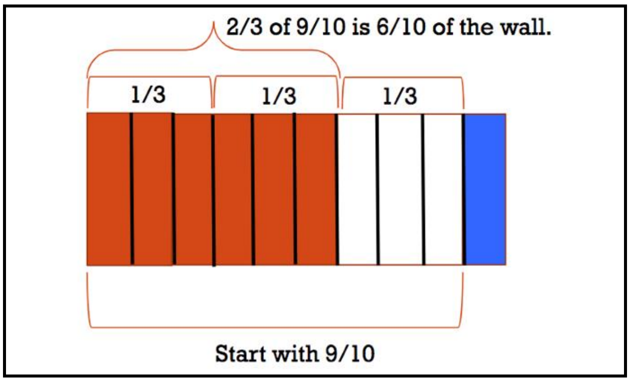


Figure 11. Representing a Fraction of a Fraction with a Rectangular Area

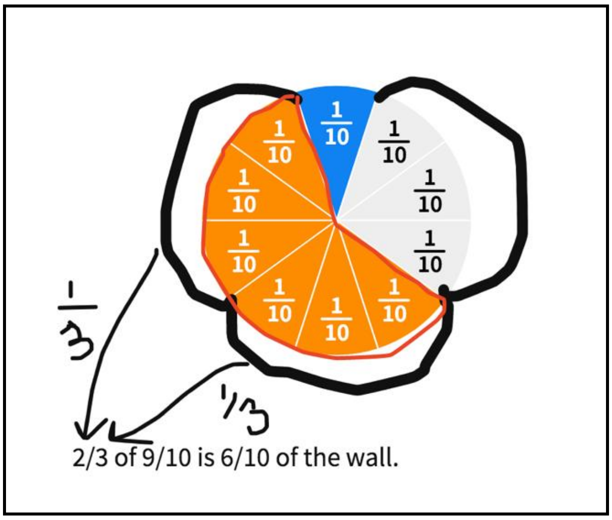


Figure 12. Representing a Fraction of a Fraction with a Circular Area

Extend

(Multiplying a fraction by a fraction-with subdividing) (35 minutes) It is time for teachers to introduce a new contextual problem. The company asks you to design 2/3 of the long hall's base by using roppe. Although you worked very hard on the first day, you were able to finish 3/4 of it. How much of the hall's base were you able to complete? Teachers should encourage students to restate the problem and expect students to state that the problem is asking how much 3/4 of 2/3 of the hall is. Students can apply a length model by using snap cubes, fraction bars, fraction towers, or drawing lines. For example, a student may use represent 2/3 at first. Then, partition each piece into fourth to find three fourth of two-thirds (see Figure 13).

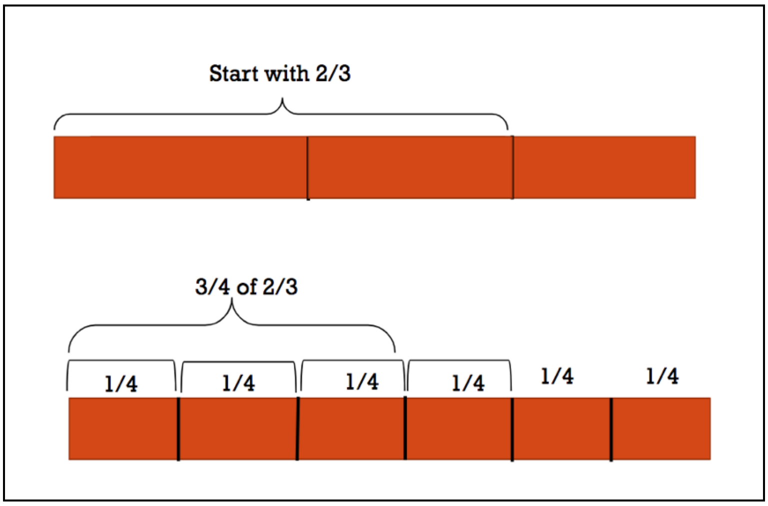


Figure 13. A Model of a Fraction of a Fraction with Subdivision

Students may have difficulty with the length model when the fractions both are close to 1. They may have difficulty partitioning small length intervals even into smaller ones. Therefore, teachers should encourage students to apply any models to represent a fraction of a fraction.

For example, teachers can ask a similar problem to the previous problem that requires students to find 2/3 of 3/5. Although this problem might be more appropriate to apply a length mode, students can apply an area model (see Figure 14).

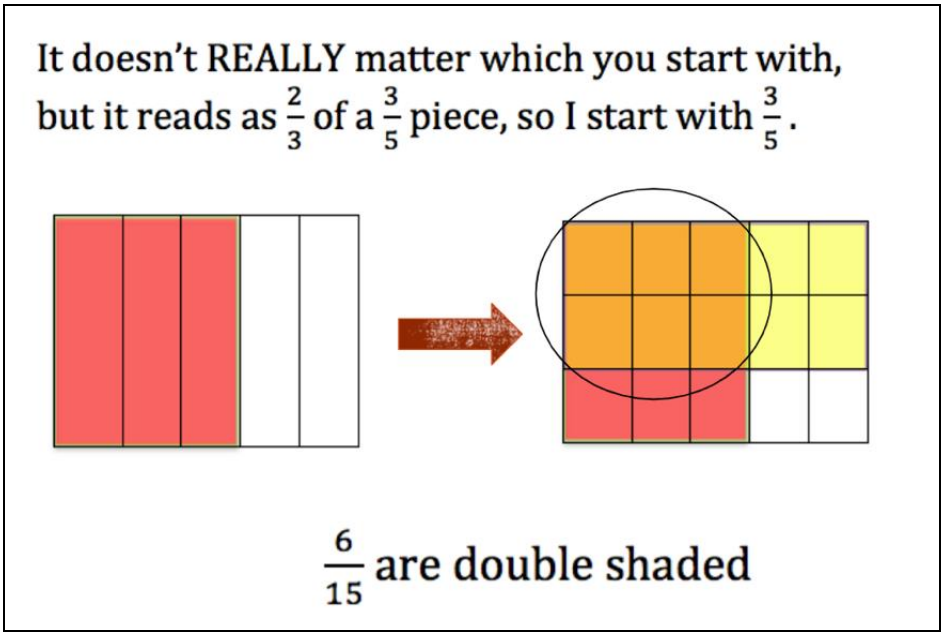


Figure 14. Representing a Fraction of a Fraction with a Rectangular Area

Teachers can also extend this lesson by introducing multiplying fractions in which at least one of the factors is a mixed number.

For example, teachers can introduce the following project: The project manager bought a new carpet to cover the bedrooms' floor. The dimensions of each bedroom are 6 and 2/3 feet by 4 and 1/4 feet. How many square feet of carpet do you have to use for each bedroom?

In this project, let students state the problem with their own words and enable them to draw (see Figure 14) and write it as (6 + 2/3) x (4 + 1/4) = (6 x 4) + (6 x 1/4) + (2/3 x 4) + (2/3 x 1/4). Since they learned how to multiply each term on this expression, they can apply any models they like to find the answer.

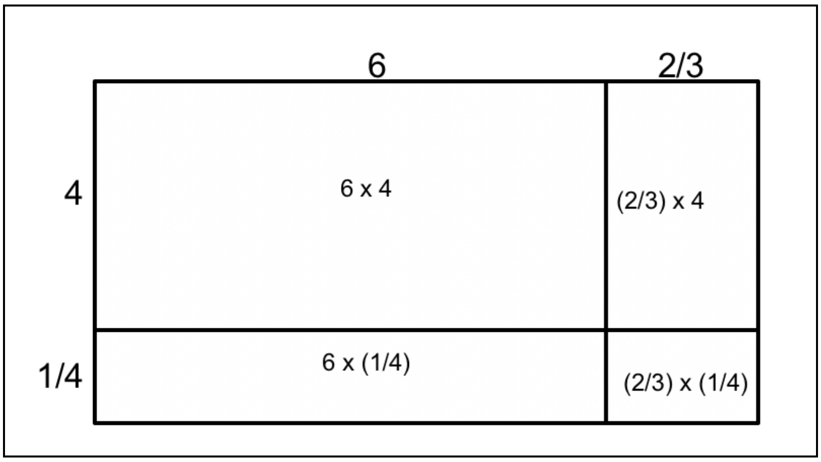


Figure 15. Applying Area Model for a Fraction of Fraction with Mixed Numbers

Evaluate

Teachers should look for if students have pictures or representations of each fraction in their minds. If they have problems with representing fractions, ensure that students work with various manipulatives to emphasize different models of fractions. For example, use fraction towers/bars for the length model and circular fractions for the area model. Second, a teacher should look for if students strategically design their patterns in a way that multiplying a fraction by a whole number results in a whole number. If students have problems with this, let students play with manipulatives (e.g., fraction bars) to see patterns such as 4 fourths make 1 or a whole, 6 sixths make 1 or a whole and then let students see that 8 fourths make 2 or two-wholes, etc. Third, teachers should look for if students can interpret fraction multiplication as scaling. Ensure that students can see they are finding 1/2 of 1/4 when they are asked to find 1/2 x 1/4. If they cannot see a fraction as a scaling, encourage them to see what happens when they multiply a whole number (5) with a fraction (1/2) by using paper folding, drawing rectangular areas, or applying a set model. Encourage them to see that the result is smaller than 5 and it is exactly 1/2 of 5. Then, encourage them to see that they scaled their sets or areas to half of the original. Teachers should also look if students are flexible to find a fraction of a fraction when they need to have additional partitioning. It might be challenging for some students to have additional partitions or they may forget to apply partition the remaining parts. Encourage them to see that they have to apply additional partitioning to the wholes so that they can find their denominators. Lastly, teachers should look for if students are comfortable applying several models and are flexible to use one model in more than one way.

Task 7 - Apply and Extend Previous Understandings of Multiplication and Division to Divide Fractions: Time to Spread Love!

Mathematical Content Standards

CCSS.MATH.CONTENT.5.NF.B.7

Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions.[1]

CCSS.MATH.CONTENT.5.NF.B.7.A

Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. For example, create a story context for (1/3) ÷ 4, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that (1/3) ÷ 4 = 1/12 because (1/12) × 4 = 1/3.

CCSS.MATH.CONTENT.5.NF.B.7.B

Interpret division of a whole number by a unit fraction, and compute such quotients. For example, create a story context for 4 ÷ (1/5), and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that 4 ÷ (1/5) = 20 because 20 × (1/5) = 4.

CCSS.MATH.CONTENT.5.NF.B.7.C

Solve real world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem. For example, how much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 1/3-cup servings are in 2 cups of raisins?

Mathematical Practice Standards

CCSS.MATH.PRACTICE.MP2

Reason abstractly and quantitatively.

CCSS.MATH.PRACTICE.MP3

Construct viable arguments and critique the reasoning of others.

CCSS.MATH.PRACTICE.MP4

Model with mathematics.

CCSS.MATH.PRACTICE.MP8

Look for and express regularity in repeated reasoning.

Vocabulary

Dividing a fraction by a whole number, dividing a whole number by a fraction, connecting multiplication and division in fractions, mathematical modeling, generalizing.

Materials

Super Sticky Easel Pad, 25 inches by 30 inches, bright yellow, scissor, colored pencils, 12-unit (e.g., inch) ribbon, a bag of bows in different length as their lengths are unit fractions (1/2, 1/3, 1/4, 1/6, 1/8) of 12-unit ribbon. Note: Teachers can ask students to cut and fill their bags with unit fractions.

Objective

Students will explore dividing a whole number by a unit fraction and they will extend this knowledge to dividing a whole number by any fractions. They will also explore dividing a unit fraction by a whole number and they will extend this knowledge to dividing any fractions by a whole number. Students' creativity will be fostered as they will have opportunities to generalize fraction division by investigating the relationship between fraction division and fraction multiplication (e.g., 4/ (1/3) = 12 and 12 x (1/3) = 4). This lesson will be covered within about 3 class time.

Engagement

(10 minutes) Start your class by telling your students Valentine's Day is for celebrating the love of all kinds, not just your significant other who should get a special Valentine's Day note from you on February 14. Watch a YouTube video (https://www.youtube.com/watch?v=VXgNa-jy56E) so that students can see that they can prepare very lovely Valentine's Day cards only by using paper and colored pencils or markers. Then, tell your students that today they will be learning how to divide a whole number by a unit fraction and how to divide a unit fraction by a whole number through crafting Valentine's Day cards for our classmates. There are many children-friendly Valentine's Day crafts (https://www.countryliving.com/diy-crafts/how-to/g1584/valentines-day-crafts-for-kids/) that teachers can easily integrate into their teaching to cover any mathematics concepts. However, in this class, we will investigate fraction division by crafting simple Valentine's Day cards.

Explore-Part A

(30 minutes) Let students work as a group of three and provide each group with a Super Sticky Easel Pad, 25 inches by 30 inches, bright yellow. Tell groups that each group will pick one of the following fractions to decide their cards' length. The group can pick the following sets of fractions: Set 1: 1/2, 1/3, 1/6, and Set 2: 1/2, 1/4, 1/8. After each group selects their set of fractions, tell them that they will cut their poster paper in a way that each group will have representations of each fraction in their set. For example, if they pick the set of 1/2, 1/3, and

1/6, they need to cut their poster in a way that they will have 1/2, 1/3, and 1/6 of the poster paper. Encourage students to come up with creative ways of showing or representing their fractions. After students cut their poster papers, they are ready to design their Valentine's Card notes. Then, tell students that each of them needs to make 4 cards for their friends by using the size of a paper. Ensure that students understand that the sizes of their cards depend on what fraction of a poster paper they are working and each of their four cards needs to be identical in terms of their sizes. Let students apply different strategies to estimate the sizes of a card they will give to their friends. For example, if they have 1/2 of a poster paper, they can partition it into four pieces, cut a piece, and see what fraction or size of his/her card is by comparing it with the original poster. If they have 1/2 of a poster paper, they can also see that their partition of their fraction of a poster paper into four groups equal to the groups or individuals that they have 1/8 of an original poster paper. Let students have original poster paper and draw/cut/play with it until each group members know what sizes of their cards will be. Since some of the sizes might be more challenging than the other, encourage students to work collectively. Ask students to fill out the table (see Table 1 & Appendix A) to see what sizes of Valentine's cards they will give to their friends.

Table 1. Length Model of a Fraction divided by a Whole Number

A poster paper	The fraction they picked to have their cards	The number of people they will give the cards	Equation
1 unit	1/2 of a poster paper	4 people	$\frac{1}{2} \div 4 = \frac{1}{8}$
1unit	1/3 a poster paper	4 people	$\frac{1}{3} \div 4 = \frac{1}{12}$
1unit	1/4 a poster paper	4 people	$\frac{1}{4} \div 4 = \frac{1}{16}$

Explain-Part A

(30 minutes) Once students fill out the table, encourage them to generalize by asking what pattern they see on their table. Let them share their ideas and encourage them to use additional models to justify their ideas. For example, if they have (1/2)/4 = 8, let them show this by using area (see Figure 1), length, or set model. Encourage students to explain their

reasoning by asking questions (e.g., why does the denominator become 8?) so that others can hear their friends' mathematical ideas and reasoning.

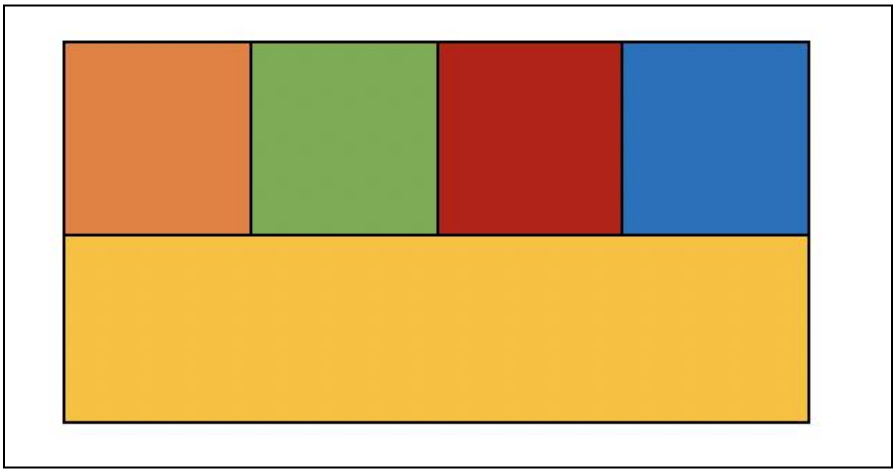


Figure 1. Area Model of (1/2)/4

Explore-Part B

(30 minutes) After creating your Valentine's Day notecard, it is time to decorate it with bows and they will explore how many bows of given lengths of unit fractions can be made from a given 12-unit piece of ribbon. Provide your students with a 12-unit piece of ribbon along with boxes or bags that involve unit fractions made from the 12-unit piece of ribbon. Ask students to align the 12-unit piece of ribbon with each given unit fraction to see how many given unit fractions they need to match with the length of the 12-unit piece of ribbon.

For example, how many 1/2-unit pieces of ribbon match with the length of the 12-unit piece of ribbon (see Figure 2).

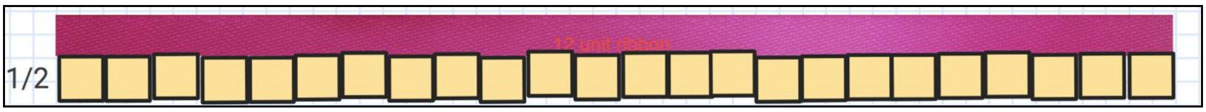


Figure 2. How Many 1/2 Units needed to Match with the Length of the 12-unit Piece of Ribbon?

Provide students table (see Table 2 & Appendix B) to fill out the information: the length of the 12-unit piece of ribbon, the length of ribbon needed per bow, the number of unit fractions to match with the length of the 12-unit piece of ribbon, and, equations.

Table 2. Length Model of a Whole Number divided by Fractions

12-unit piece of ribbon	The length of ribbon needed per bow	The number of bows from	Equation
12 unit	1/2 unit	24	$12 \div \frac{1}{2} = 24$
12 unit	1/3 unit	36	$12 \div \frac{1}{3} = 36$
12 unit	1/4	48	$12 \div \frac{1}{4} = 48$

Explain

(20 minutes) Once students completed filling out Table 2, encourage them to generalize by asking what pattern they see on their table. Let them share their ideas and encourage them to use additional models to justify their ideas. Also, encourage students to see fraction division in terms of multiplication. Have the following discussion with your students:

Teachers: What is 6/2 =?

Students: It is 3.

Teachers: 6 is a number, 2 is a number, and 6/2 results in a number.

Teachers: What about 6 / (1/2)?

Students: 6 is a number, 1/2 is a number, so 6/ (1/2) should result in a number.

Note: If students do not make this statement, teachers can scaffold students to see that dividing two numbers or fractions results in a number.

Teachers: Teachers then should write on the board the following: if 6/2 =3, then 6 = 2 x 3.

Teachers: What about 6/ (1/2) =? We know that an answer is a number, but we do not know what number is yet.

Teachers: If 6 / (1/2) = a number, then 6 = 1/2 x (a number). How do we read this equation?

Students: 6 is 1/2 of a number. Or, 1/2 of what number is 6.

Teachers: Show by using an area model.

Students: We can use the area model (see Figure 3). Since 1/2 is that number is 6, then the other half is also 6. It makes that number is 12.

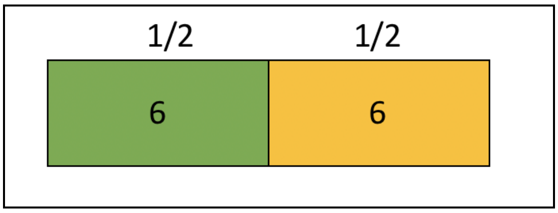


Figure 3. 1/2 of What Number is 6

Teachers: What about with a length model?

Students: Since 1/2 of it is 6, I can divide 1 on a number line into half (see Figure 4). Then, the other half is also 6 and makes the number is 12.

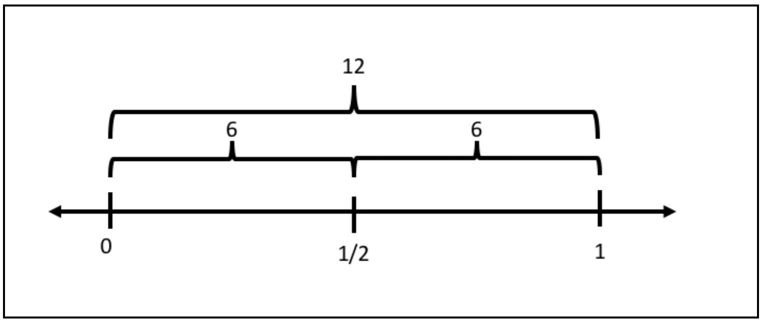


Figure 4. 1/2 of What Number is 6 with a Length Model

Teachers: What about the set model?

Students: I can show it by using counters (see Figure 5). If I have half of my counters that is 6, I can add another half to make it a whole which is the number I am looking for.

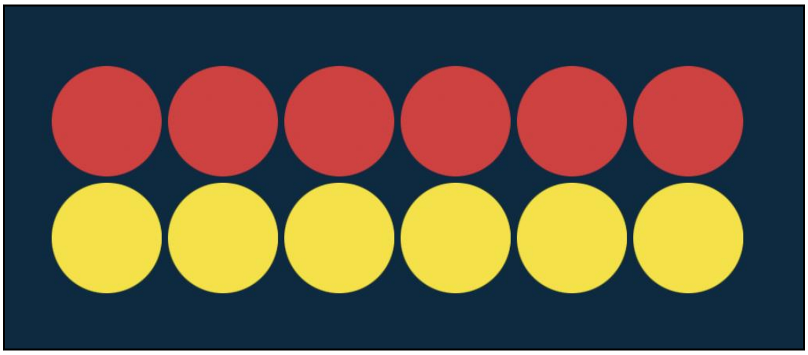


Figure 5. 1/2 of What Number is 6 with a Set Model

Extend

(30 minutes) After using three models that depict the relationship between division and

multiplication when a whole number is divided by a fraction, students can extend this knowledge to dividing a whole number by non-unit fractions. For example, teachers can ask students to compute 6/ (2/3) by using the relationship between division and multiplication. Then, students can see that they can write it as 6 = 2/3 x (what number) and read it as 6 is 2/3 of what number. Then, they will see that if 2/3 of that number is 6, 1/3 of it is 3. Then, 3 x 3 = 9 (see Figure 6).

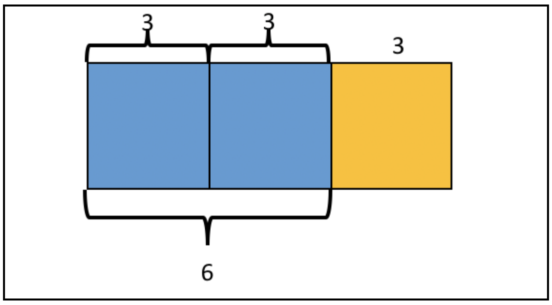


Figure 6. 6 / (2/3) by using Area Model

Another extension is dividing a fraction by a fraction. For example, a teacher can ask students to apply what they learned to compute (1/3) / (2/5). Students then write it as 1/3 = 2/5 x (what number) and read it as 1/3 is 2/5 of what number. They can use the area model to represent their computation (see Figure 7).

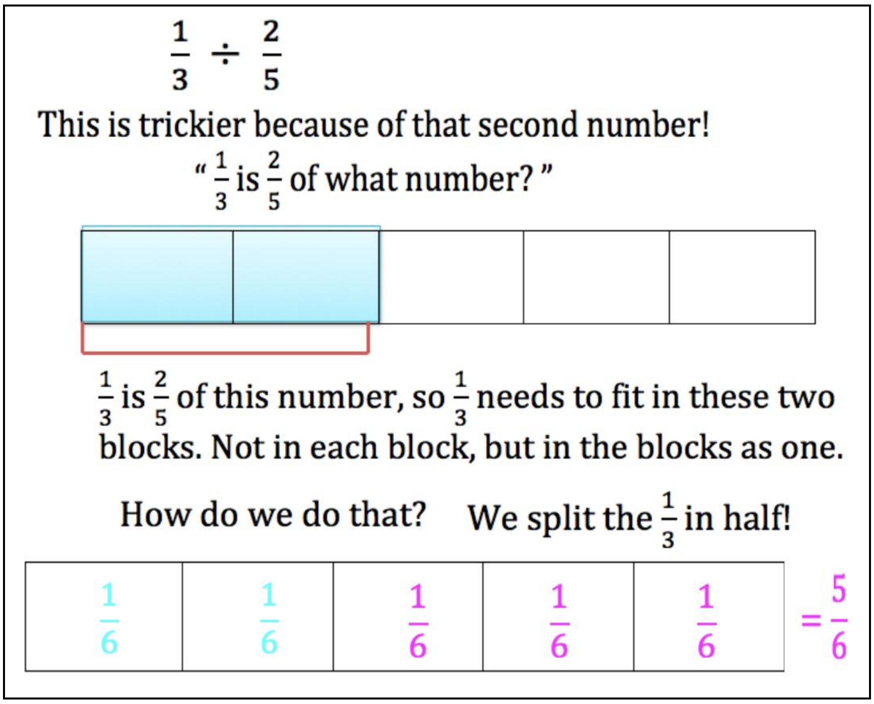


Figure 7. Dividing a Fraction by a Fraction by using Multiplication Area Model

Evaluate

Teachers should look for if students see the pattern when they divide a whole number by a fraction and when they divide a fraction by a whole number. It is possible that some students may overgeneralize these to any fractions (e.g., since 3 / (1/2) as equal to 3 x 2, 3 / (2/3) is 9). To avoid this, let students be aware that they are only working with unit fractions until you introduce non-unit fractions in the extension part. If you do not want intent to cover the extension part, then provide them with a couple of examples showing that they cannot generalize this to every fraction. Teachers should also look for if their students understand that they can have smaller cards when they picked smaller fractions. If they have difficulties with seeing this before starting on cutting their poster paper, ensure that they have a strong fraction sense. Let these students compare and contrast several fractions by using drawing, manipulatives, etc. Also ensure that students see equivalent fractions and can identify the size of 1/8 or 1/4 by looking at their whole (i.e., poster paper). If they have problems with it, again let them apply different models through drawing and using manipulatives. An important dimension of this lesson is to understand the connection between fraction division and fraction multiplication. The notion that 2 / (1/4) can be thought of as 2 x 4 is essential for students to grasp the meaning of fraction division. In this case, we want students to see that there are four fourths in each whole, so 2 x 4 is 8, which is connected to the idea that 8/2 is 4. Enable students to apply models to show this kind of relationship when they divide a whole number by a fraction or when they divide a fraction by a whole number.

Appendix A

A poster paper	The fraction they picked to have their cards	The number of people they will give the cards	Equation

Appendix A

Appendix B

12-unit piece of ribbon	The length of ribbon needed per bow	The number of bows from	Equation

Task 8 - Convert like Measurement Units within a Given Measurement System: Designing a Basketball Court

Content Standards

CCSS.MATH.CONTENT.5.MD.A.1

Convert among different-sized standard measurement units within a given measurement system (e.g., convert 5 cm to 0.05 m), and use these conversions in solving multi-step, real world problems.

Practice Standards

CCSS.MATH.PRACTICE.MP1

Make sense of problems and persevere in solving them.

CCSS.MATH.PRACTICE.MP3

Construct viable arguments and critique the reasoning of others.

CCSS.MATH.PRACTICE.MP6

Attend to precision.

CCSS.MATH.PRACTICE.MP8

Look for and express regularity in repeated reasoning.

Vocabulary

Unit conversion, metric system, customary units, metric units.

Materials

Measuring tapes, grid papers, pencils, A4 regular papers, scissors.

Objectives

In this lesson, students will be learning how to convert commonly used metric units and customary units to each other. Students' creative thinking will be developed as they will be challenged to estimate how many regular A4 papers can cover their school's basketball court. Students' creative thinking will be also developed by enabling them to deduce possible relationships for any units (e.g., inch, meter) by using given relationships (e.g., 1 inch = 2.54 cm and 1 meter = 100 cm).

Engagement

(20 minutes) Let students work as a group of three to four to design a basketball court (see Figure 1). Give each group 1-cm grid paper and tell them to use the paper to design a basketball court for their school however they like to make their design. After students complete their design, ask them to find the area of their basketball court. They can either count the number of squares inside of the region of their basketball court. Or, they can measure the length and width of their basketball court and multiply them to find the area. Teachers should observe if students can apply row and column models to find the area of the basketball court. Once students find the area, bring them to your school's basketball court and ask students how many of their drawn models they need to cover the basketball court. Let students make their estimations and tell their final estimation as a group. Write down their estimation on the board so that everyone can see others' estimations as well. Ensure that students are aware that their estimations are different since the area of their basketball courts is not identical. So, we also expect them to see that their estimation should be somewhat close since each group is working on a regular A4 grid paper (21 cm by 29.7 cm).

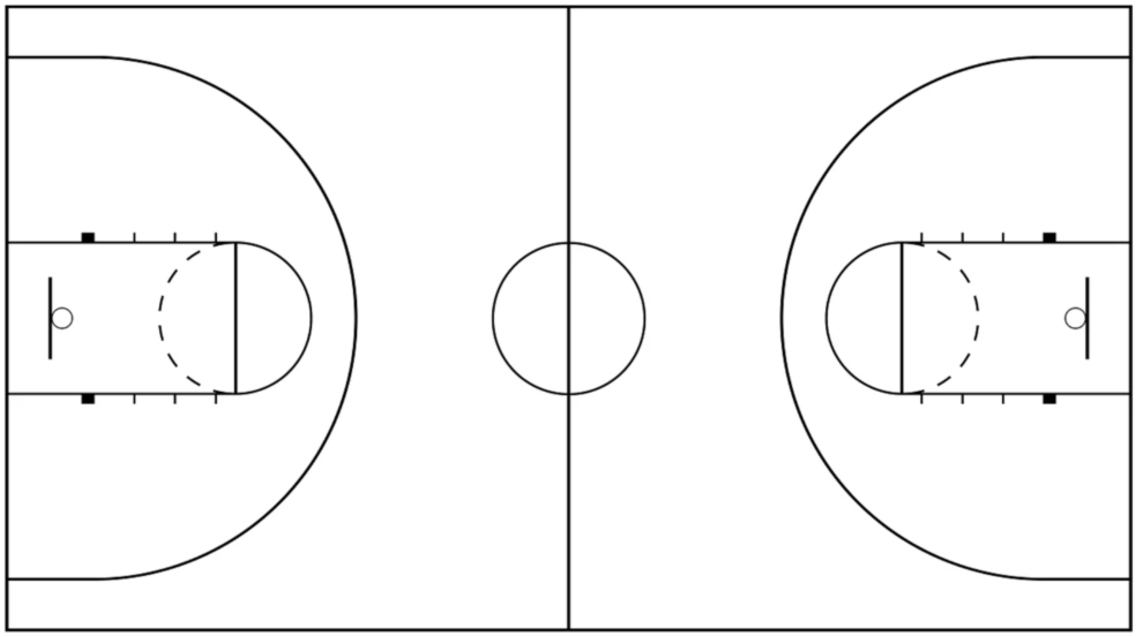


Figure 4.A Basketball Court

Explain

(20 minutes) After students make their estimations, ask them if they know the length of the squares on their grid paper. If they do not know, introduce them to meter and centimeter with examples that they are familiar with. For example, let students know that the height of a door is approximately 1 meter from the floor. Then, ask students what about the height of the door and expect to hear that it is about 2 meters. Once students are familiar with what 1 meter is, then provide them with the conversion chart (see Table 1).

Table 1. Conversion Chart

Kilometer	Hectometer	Dekameter	**Meter**	Decimeter	Centimeter	Millimeter
1×1000	1×100	1×10	1	$1 \div 10$	$1 \div 100$	$1 \div 1000$

Since students previously learned the power of tens, they are ready to grasp this metric unit conversion. To ensure students understand it, ask them what they understand from the table. You can also tell students to create their chart by changing meters to 10, 100, and 1000 to see if students can comfortably convert them to each other. Then, ask students the height of the door knob from the floor in terms of centimeters. Since students learned that it is about 1m, it is equivalent to 100 centimeters.

Explore

(30 minutes) Once students see the relationship between metric units, ask students to find the area of their basketball court in terms of meters. Suppose a group used the entire A4 paper, then they know that their area is $21cm \times 29.7cm = 623.7\ cm^2$. Then, we expect students either to covert each dimension to meter and multiply them ($0.21m \times 0.297m = 0.06237m^2$ or they may divide 623.7 by 100 by looking the table 1. Bring the groups to the basketball court again and provide them with a measuring tape to measure the width and length of the basketball court. Students will also need paper and pencil to record their measurements since the measuring tape will not reach all of the ways. Once groups find out the actual dimensions (e.g., 28 by 15 meters) they are ready to find the actual area. Let students make sense of the problem as they will figure out what they need to do next to see how much their estimation is close. Teachers should observe that students apply divisions and round the decimal numbers to attend precision in their answers. Once students compute to find how many of their models (e.g., $(28 \times 15) \div (0.21 \times 0.297) = 6734.0067$) can cover the entire basketball area, ask them to round their decimals while they are comparing it with their initial estimates. So, a group can tell the following "we estimated 5,000, but the actual area is about 7,000 meters squared." It is possible that another group state, "we estimated 4,000 but the actual area is about 6750.". Or, another group of students can say, "we estimated 6,000 but the actual area is about 6734." Discuss students what rounding is more precise and how much precisions they need. This is essential discussion since we want students to make sense of what unit they are using, what numbers they are strategically rounding by comparing the estimates and the actual area, and how they are evaluating their estimates.

Extend

(30 minutes) When students understand the power of 10 relationships among the metric units, ask students to be more specific on their basketball court models in terms of the length and/or width of the elements in their basketball court model. For example, how much free throw circle will be away from the end line/baseline. Ensure that students write their estimations in terms of centimeters and let them convert those later to meters. Once they finalized their models, ask them to visit other groups and see if their estimations are similar or different across groups. After they observe other groups' estimations and models, either bring them to

the basketball court and measure the actual length and/or the width of the features on their designs or show them the picture (see Figure 2) so that they can compare the actual measures with their estimates on their designs.

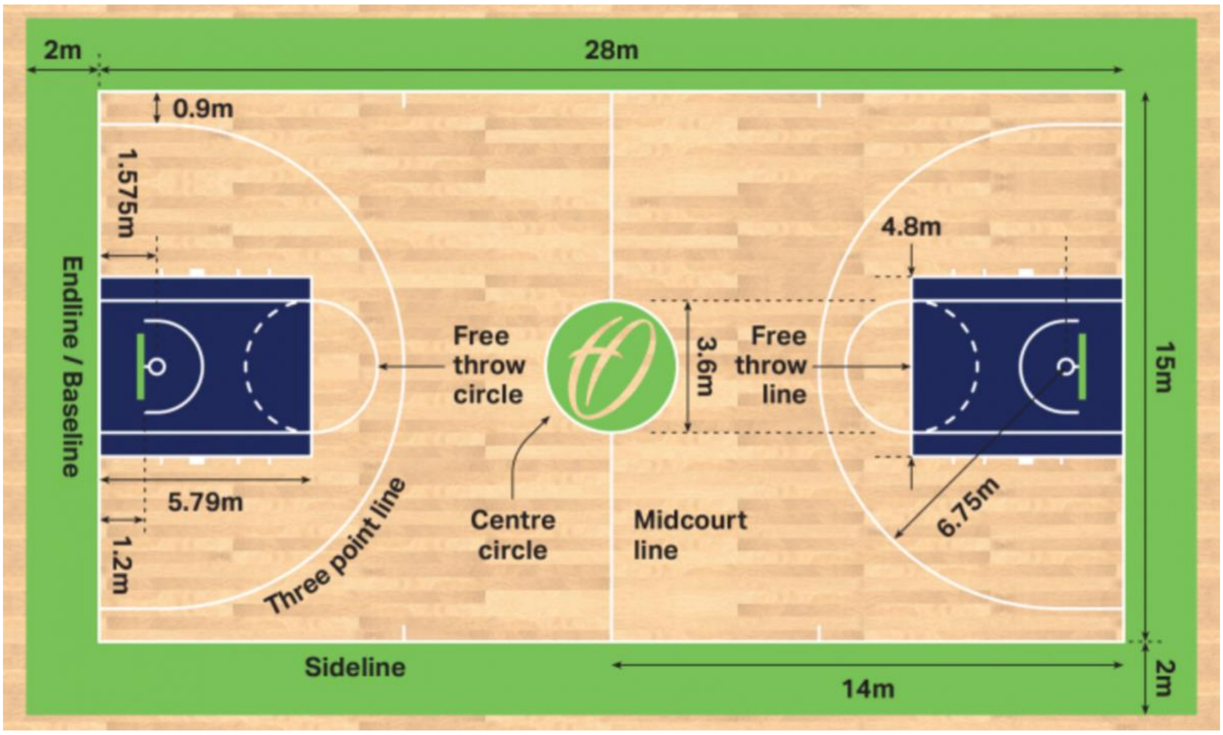


Figure 5. A Basketball Court Dimensions with Meters

Once students are comfortable with the metric unit, it is time for them to be introduced to customary units. Provide students with the conversion table (see table 2), and ask them to design their models of basketball court by using appropriate customary units. Students can pick yards or feet since they used meters and centimeters previously and the corresponding units were given on the table. Again, create a discussion environment why they did not pick inch, but they prefer yard and/or feet (See Figure 3 & 4). It is also possible that students can pick both yard and feet within their models.

Table 2. Conversion from Customary to Metric Units

Customary units	Metric units
1 inch	2.54 cm
1 foot	30.48 cm
1 yard	0.914 m
1 mile	1.609 km

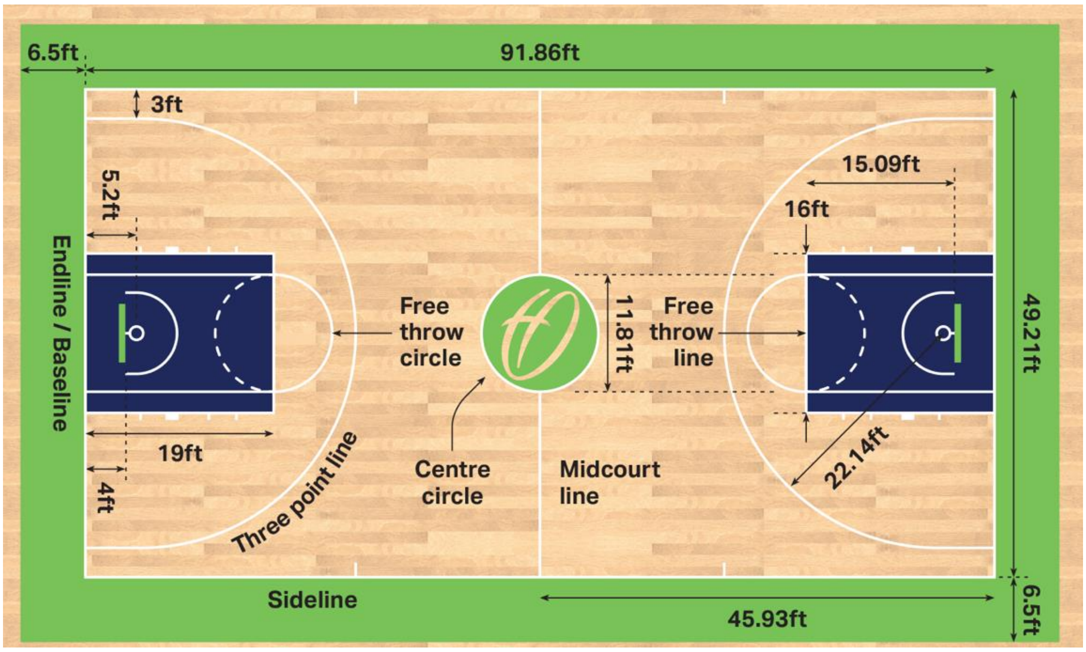


Figure 6. A Basketball Court Dimensions with Feet

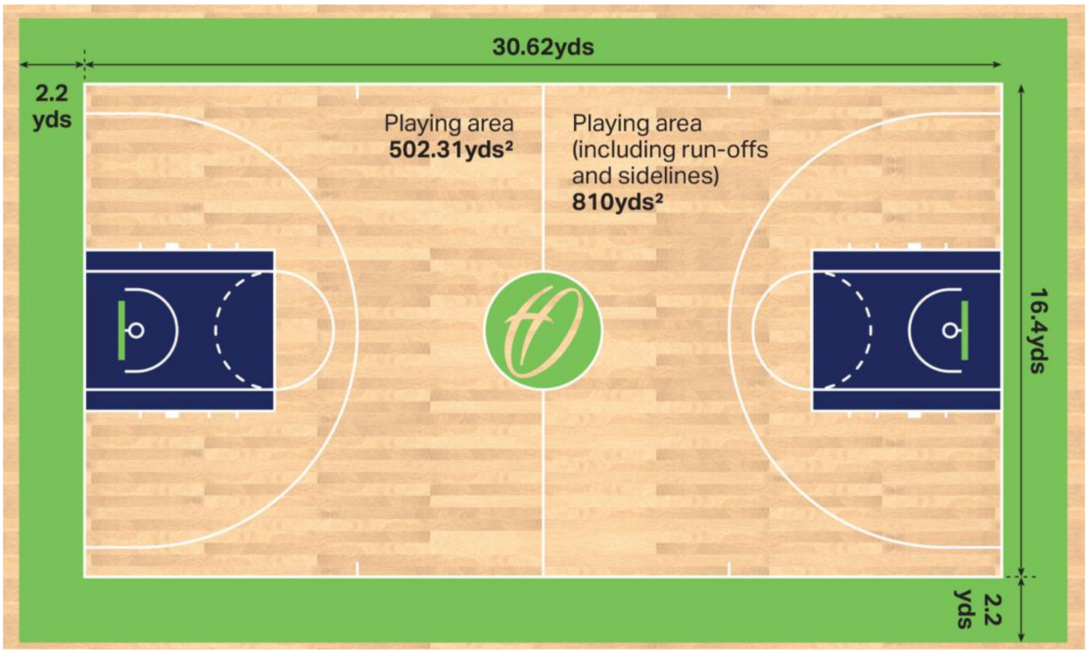


Figure 7. A Basketball Dimensions with Yards

Another extension might be considering the height of some of the features on their designs. It is relevant since they are designing a basketball court. Similar to what they did above, you can ask them to estimate the height of the features (e.g., hoop). Then, ask students to first make the conversion from cm to m (see Figure 5) and then to customary units. It is better if students use several units within their designs that can be indicators of their comprehension of the metric and customary units.

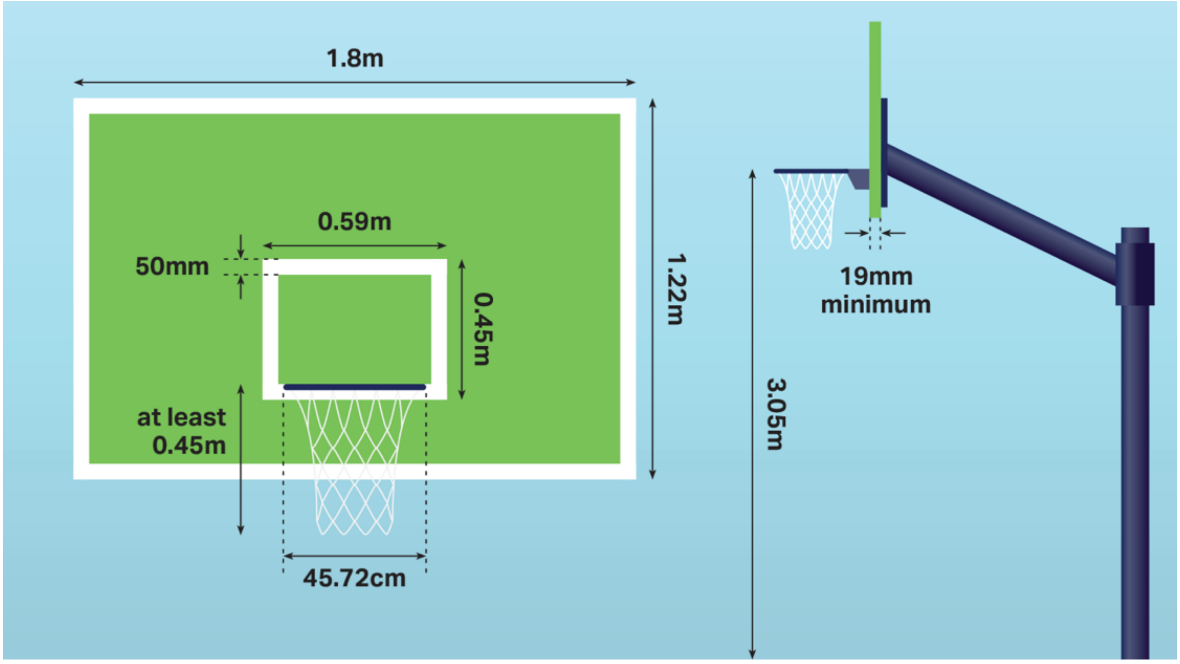


Figure 8. The Height of the Features with Metric Units

Evaluate

Teachers should look for if students identify the relationship among the metric units as the power of 10s that they need to either multiply or divide by power of 10s depending on the situation. If they cannot identify the relationship (e.g., 1 meter is equal to 100 cm), it is better that they engage in activities that emphasize the relationship between the units of power 10s with length models. Teachers also should look for if students are flexible to find the area of the basketball court by multiplying the width by the length of their basketball court models. If not, let students engage in rows and columns multiplication and see how it is related to the area concept. Teachers also should look for if students make reasonable estimations when they were asked how many A4 regular papers can cover the basketball court. Teachers can set expectations for students to tell a number that is more than 1000 or 2000. If their estimation is like 100 or 500, challenge students to cover a small part/area of the basketball court with their models and let them see that they need a lot more than what number of A4 paper they estimate. The same activity can solve if they estimate a lot more (e.g., 20,000) than what it is. Teachers should also look for if students can make reasonable conversions. For example, teachers can observe if students can make deductions to convert one unit to another one by using tables or charts. For example, students should be able to connect given information (e.g., 1 inch = 2.54 cm and 1 cm = 0.01 m) and deduce that 1 inch is equal to 0.0254 m.

Teachers should emphasize that we need to have measurement sense without memorizing as students can find the conversion charts or conversion apps/websites everywhere. The important idea teachers should focus on is to understand commonly used units (e.g., meter, miles, inch).

The pictures were retrieved from Harrod Sport website: https://www.harrodsport.com

References

Harrrod Sport (2022). *Basketball court: Size dimensions and markings*. Retrieved from https://www.harrodsport.com/

Task 9 - Represent and Interpret Data: How Much Sugar in Ali's Cake

Mathematical Content Standards

CCSS.MATH.CONTENT.5.MD.B.2

Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Use operations on fractions for this grade to solve problems involving information presented in line plots.

For example, given different measurements of liquid in identical beakers, find the amount of liquid each beaker would contain if the total amount in all the beakers were redistributed equally.

Mathematical Practice Standards

CCSS.MATH.PRACTICE.MP1

Make sense of problems and persevere in solving them.

CCSS.MATH.PRACTICE.MP3

Construct viable arguments and critique the reasoning of others.

CCSS.MATH.PRACTICE.MP5

Use appropriate tools strategically.

Vocabulary

Line plot, operation on fractions, column chart, row chart, line chart, area chart, pie chart, and donut chart.

Materials

Paper, pencil, sticky notes, snap cubes, and available computer or online graphic designers (e.g., https://mathigon.org/polypad#fraction-bars or Microsoft Excel).

Objective

Students will collect data and make a line plot to display a data set. They will be asked to generate data along with its representation on a line plot that includes fractions. They will pose problems based on their data sets and hand their problems and a line plot to other groups to solve these. Students' creative thinking will be fostered by enabling them to display their data sets by using line plots and then converting their line plots to other visuals (e.g., line charts, bar, and pie). Students' creativity will be developed by enabling them to see the relationship between district (e.g., set model) and continuous (e.g., length model). Students' creativity will be also developed by letting them generate their mathematical questions by using their charts.

Engagement and Explain

(20 minutes) Start the class by drawing a line and numbers under it from 1 to 10. Then, provide students with sticker notes and ask them to put their sticky notes above the number representing the number of members in their family. Ensure that students leave some distance between their sticky notes and others since we like to be able to read how many students place their sticky notes on a certain number (see Figure 1). Then, ask students many questions like the followings:

1) How many people did answer this question?
2) How many students do have several family members more than 5 or less than 4?
3) What is the highest/least family size?

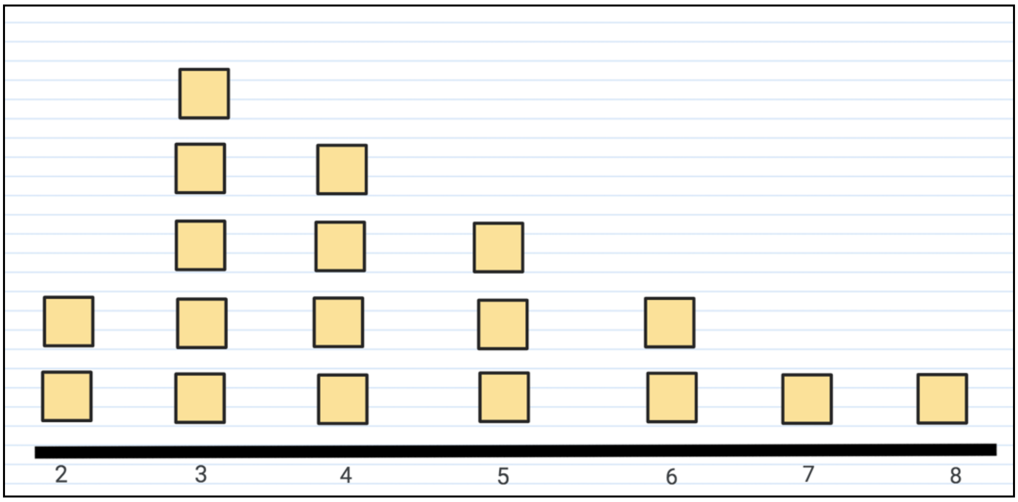


Figure 1. Line Plots with Sticky Notes

After students answered these questions, replace each sticky note with a star/circle/square/or a letter x (see Figure 2) to show them how various ways they can represent each data point. Then, tell students that this is named as a line plot that they will be investigating today by collecting what they like to learn from others and representing it with line plots to communicate their findings.

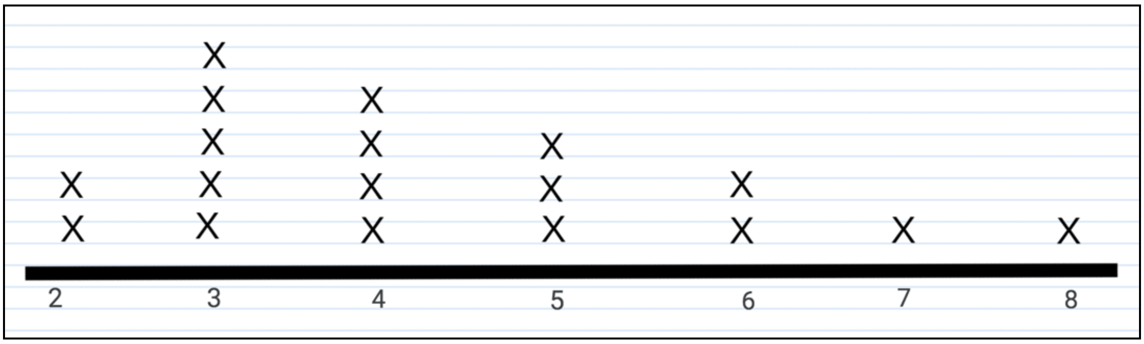


Figure 2. Line Plots with Letter X

Explore

(20 minutes) Students are now ready to collect data from their friends. Although students have the freedom to ask any questions to collect data, teachers should monitor if students ask for information that can be sensitive to others. If students have a hard time finding a question that they like to ask their friends to collect data, teachers can provide them some options such as: What is your favorite pizza (i.e., vegetarian, cheese, pepperoni, seafood)? What is your favorite season (i.e., winter, spring, summer, fall)? What is your favorite leisure activity (i.e.,

watching, reading, bowling, skating, swimming, etc.)? When students decide what they like to ask and determine the options as their answers, ask them to create tables (see Table 1) to organize their data sets. Table 1 represents data only from six samples and teachers should ensure that students collect data from all their classmates. Once they fill out their table, ask them to represent their data by using line plots. For example, if they have 20 friends who answered their questions, they may have their line plot like in Figure 3.

Table 1. Creating a Table to Organize their Data

Participants	Types of Pizza			
	Pepperoni	Vegetarian	Cheese	Seafood
Person A	X			
Person B		X		
Person C			X	
Person D	X			
Person E				X
Person F			X	

Figure 3. A Line Plot of Favorite Pizza with 20 Samples

Explain and Explore

(20 minutes) After students create their line plots, ask them to use snap cubes (https://www.didax.com/apps/unifix/) for each letter x and recreate their line plots. When students complete their line plots with snap cubes, let them connect the cubes and create columns (see Figure 4). Let students see that this is another representation of their data as

they can use an additional line (i.e., a vertical line) to represent the number of pizzas for each type. Let students know that this is a column chart that is widely used to represent data sets.

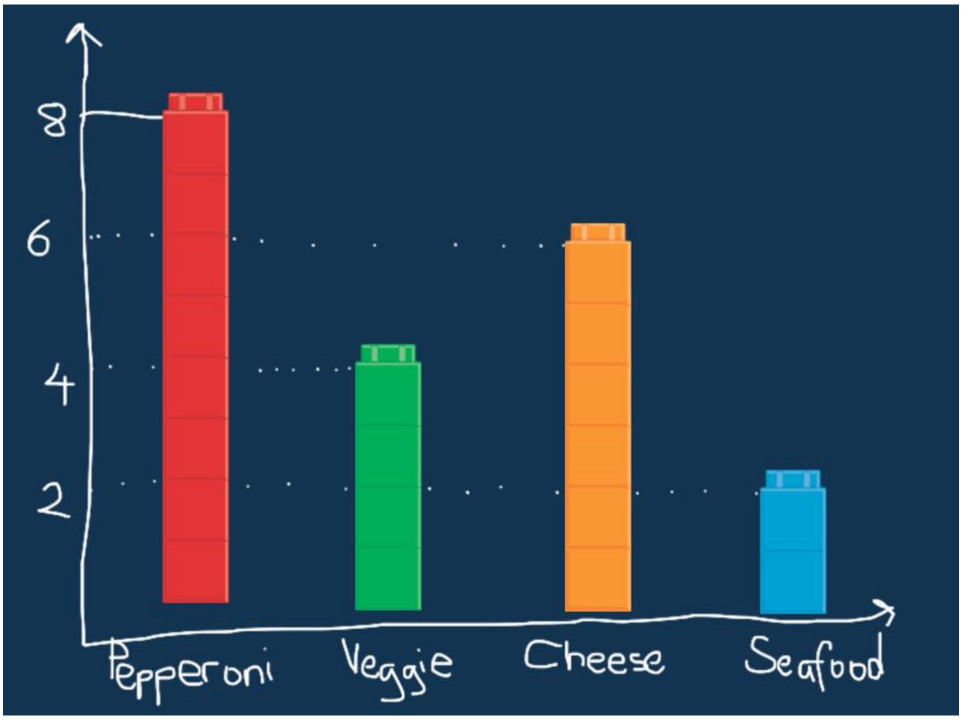


Figure 4. Representing Data with a Column Chart

After students understand and create their column charts, introduce them to the following link: https://mathigon.org/polypad where they can convert their tables to column charts. Show students x represents the horizontal line and y represents the vertical line on the table/graph. Once students fill out the table, let them use the arrow to connect it with the graph (column chart).

Horizontal line (x)	Vertical line (y)
Pepperoni	8
Veggie	4
Cheese	6
Seafood	2

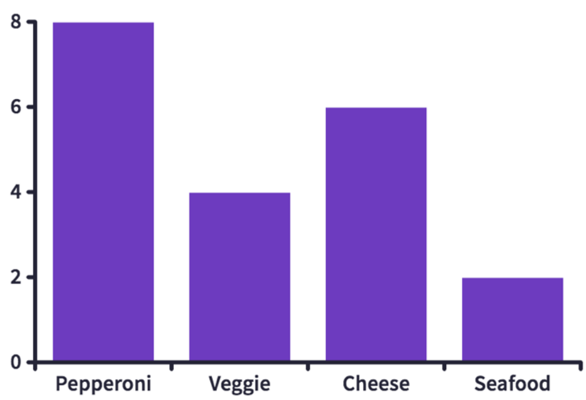


Figure 5. Using a Table to Represent Data with a Column Chart

When they create their column charts, ask them to connect their table with a row chart and let them see how it is similar to or different than their column charts. Ensure that students label x and y correctly as x represents horizontal and y represents vertical lines.

Students must make connections between discrete (e.g., set model) and continuous models (length model).

Vertical line (y)	Horizontal line (x)
Pepperoni	8
Veggie	4
Cheese	6
Seafood	2

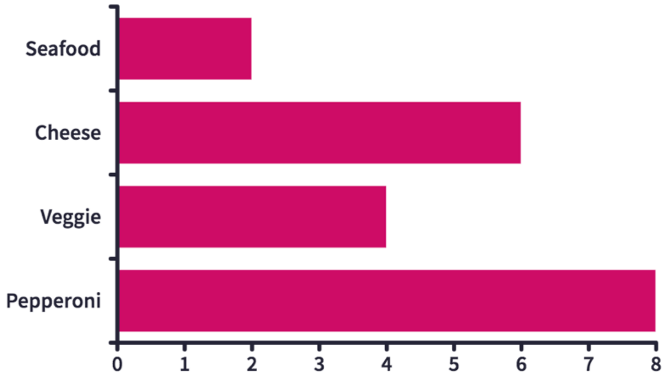


Figure 6. Using a Table to Represent Data with a Row Chart

After students complete their row charts, let them use line chart, area chart, pie chart, and donut chart to represent their data (see Figure 7 & 8).

Horizontal line (x)	Vertical line (y)
Pepperoni	8
Veggie	4
Cheesee	6
Seafood	2

Figure 7. Using a Table to Represent Data with a Pie and Donut Chart

Ask students to explain each visual to their friends. If you have time, you can now collect data about their favorite charts and represent the result with each chart/graph. Students must make connections between discrete (e.g., set model) and continuous models (area model).

Horizontal line (x)	Vertical line (y)
Pepperoni	8
Veggie	4
Cheesee	6
Seafood	2

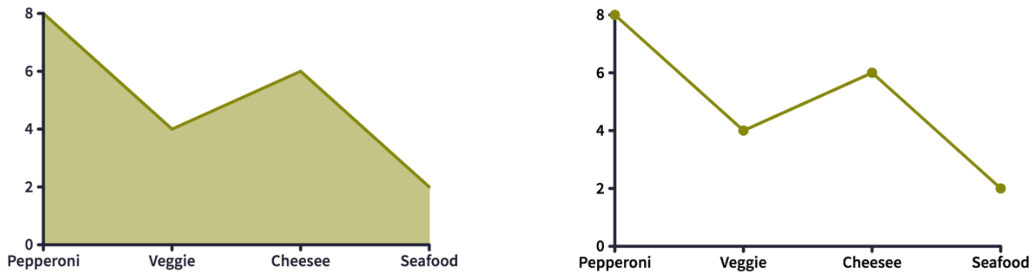


Figure 8. Using a Table to Represent Data with a Line and Area Chart

Extend

(30 minutes) Since students are familiar with a line plot and other charts to represent a data set that includes whole numbers, it is now time to introduce a data set that includes fractions (i.e., 1/2, 1/4, 1/8). Show students the following line plot (see Figure 9).

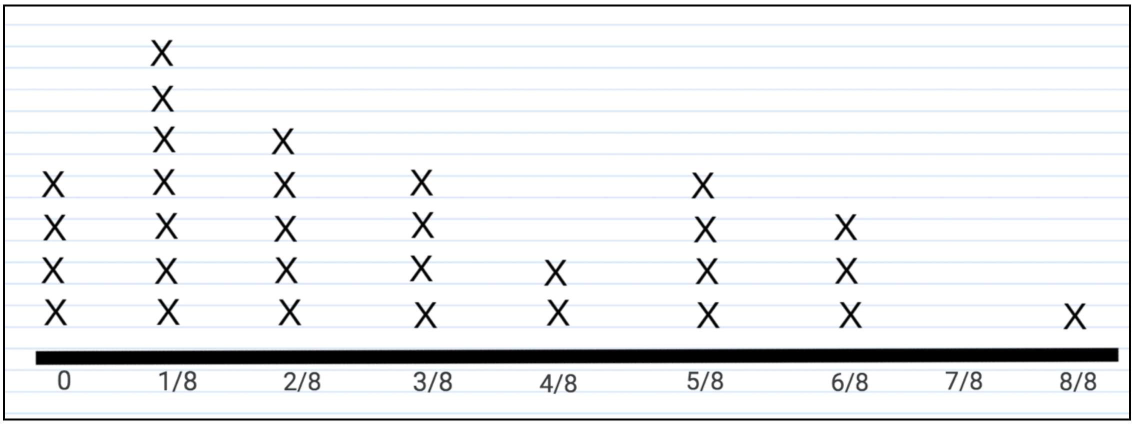


Figure 9. A Line Plot showing the Cups of Sugar Ali's Cakes' Recipes

After showing the data set with a line plot, ask students to answer the following questions:

1) What is the fraction of the most sugar contained in one of Ali's cakes?
2) How many of Ali's cakes are sugar-free?

3) Of the cakes that contained sugar, what is the least amount of sugar in a cake?
4) Of the cakes that contained sugar, how much more sugar did Ali use with a cake that contains the most amount of sugar than a cake that contains the least amount of sugar?
5) How many cakes recipes are presented in the line plot?
6) How much sugar did Ali use after cooking all of the cakes?
7) If Ali combined the sugar he used in his cakes and redistributed it evenly, how much sugar would be in each cake?

After students show that they can use operation on fractions represented in line plots, ask them to work as a group of three and randomly generate a data set that includes fractions and whole numbers. After representing their data with a line plot, ask them to write as many questions as possible for other groups. This activity will let students critically think about what they like to learn and carefully evaluate if their line plots include necessary information for other groups to answer the questions.

Evaluate

Teachers should look for if students read the data and understand that a line plot represents the frequency of answers in the data sets. Teachers also should look for if students can convert one chart to another and communicate the similarities and differences among the charts. Students should make connections between discrete (i.e., set model) and continuous models (i.e., length model, and area model). Teachers should also look for if students can comfortably compute fraction operations to answer questions when the problems include fractions. If students have problems with fractions, teachers should support their understanding of operations in fractions with manipulatives and draw (i.e., mathematical modeling) activities.

Task 10 - Geometric Measurement - Understand Concepts of Volume and Relate Volume to Multiplication and to Addition: How Many Krispy Kreme Donuts?

Mathematical Content Standards

CCSS.MATH.CONTENT.5.MD.C.5

Relate volume to the operations of multiplication and addition and solve real-world and mathematical problems involving volume.

CCSS.MATH.CONTENT.5.MD.C.5.A

Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication.

CCSS.MATH.CONTENT.5.MD.C.5.B

Apply the formulas $V = l \times w \times h$ and $V = b \times h$ for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths in the context of solving real-world and mathematical problems.

Mathematical Practice Standards

CCSS.MATH.PRACTICE.MP3

Construct viable arguments and critique the reasoning of others.

CCSS.MATH.PRACTICE.MP4

Model with mathematics.

CCSS.MATH.PRACTICE.MP5

Use appropriate tools strategically.

CCSS.MATH.PRACTICE.MP8

Look for and express regularity in repeated reasoning.

Vocabulary

Volume of a right rectangular prism, area model, array model, height by the area of the base.

Materials

Geoboard, base-ten materials, snap cubes, paper, pencil.

Objective

Students will build connections between two-and three-dimensional representations of a solid box by using views of the Krispy Kreme Donut box (a rectangular prism). Students will estimate how many donuts are in the box and they will investigate how the side view of the box they create may change the corresponding top view of their box. Students' creativity will be fostered as they will be asked to generalize the formula of a rectangular prism by seeing its meaning as multiplying the height by the area of the base of the Krispy Kreme Donut box.

Engagement & Explore

Start the class with the video (https://www.youtube.com/watch?v=Rn0XsW2l4d4&t=6s) about Krispy Kreme Donut. Then, show students the picture (see Figure 1 from https://www.thrillist.com/eat/nation/krispy-kreme-2-400-donut-box) and ask them to predict how many doughnuts are in the box?

Figure 1. How Many Donuts are in the Box?

After hearing students' predictions, ask them to explain how they did their estimations through using several multiplication models such as equal group model, area model, area model with base ten blocks, array model, and partial product algorithm. Students can pick any two numbers to visualize their multiplication, but encourage them to pick at least one factor as a two-digit number. For example, a student can apply an equal group model for 5x38 by using base-ten materials (see Figure 2).

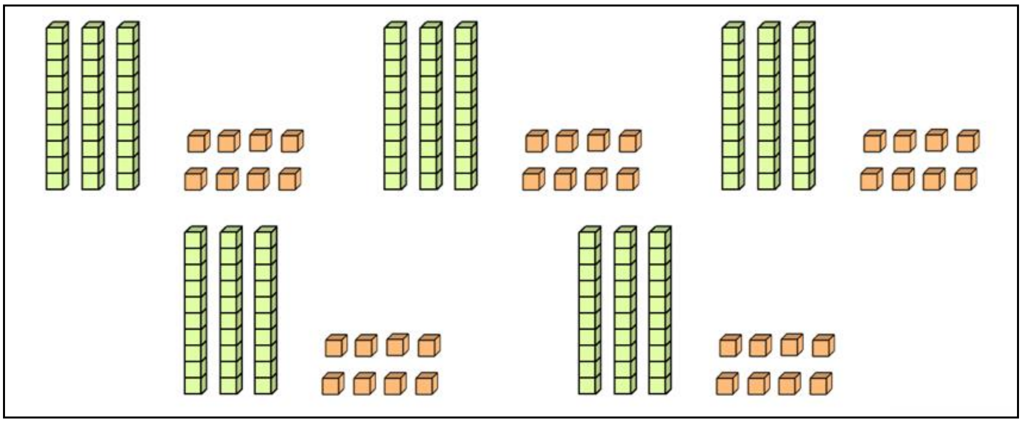


Figure 2. Applying Equal Group Model for 5x38

Another student can apply the array model by using a geoboard for 3x19 and counting the intersection points of rows and columns (see Figure 3).

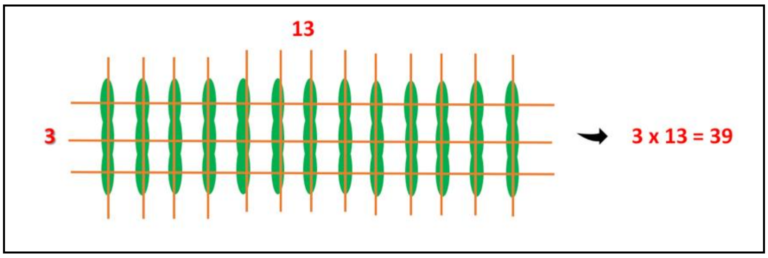


Figure 3. Applying Array Model for 3x13

Teachers should students to use base ten materials to apply area models involving two-digit factors (e.g., 36x54). (see Figure 4).

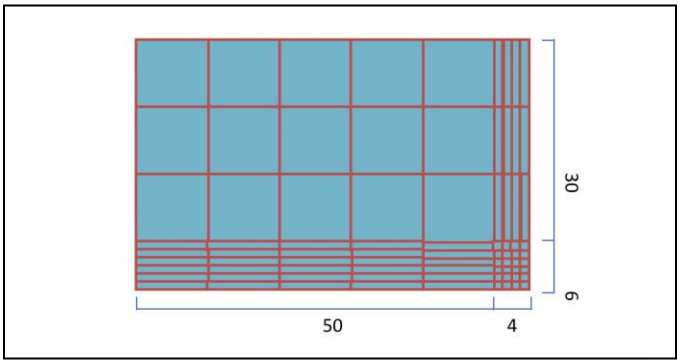


Figure 4. Applying Area Model by using Base-ten Materials

Let students identify hundreds, tens, and ones by drawing (see Figure 5). Applying area or array models without using manipulatives is an essential step for students as they will be introduced to volume in this lesson that requires them to consider one more factor.

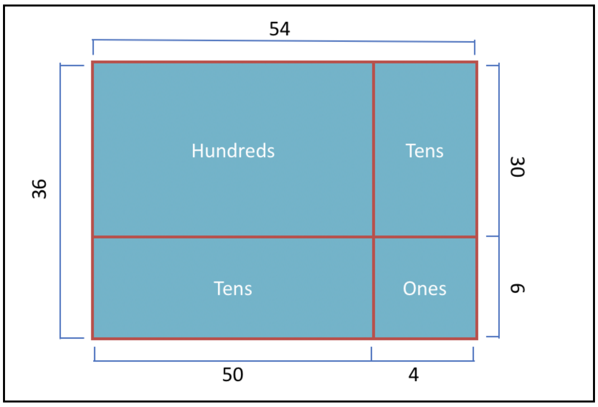


Figure 5. Applying Area Model without Manipulatives for 36x54

Once students fluently and flexibly applied several multiplication models, it is time to check if their predictions of how many donuts are in the box are close to the actual number of donuts are in the box. Teachers should enable students to see the relationship between array and area model since students will be asked to use the area model based on the number of donuts (array model) in this lesson.

Explore

It is time to provide students with the number of donuts in each row and column (see Figure 6).

Figure 6. The Number of Donuts in Each Row and Column

Then provide students with base-ten blocks to cover the image or construct the given area to determine the number of donuts in the box. Once students create the area that each unit cub represents a donut (see Figure 7), ask them to draw a model area model showing hundreds, tens, and ones (see Figure 8).

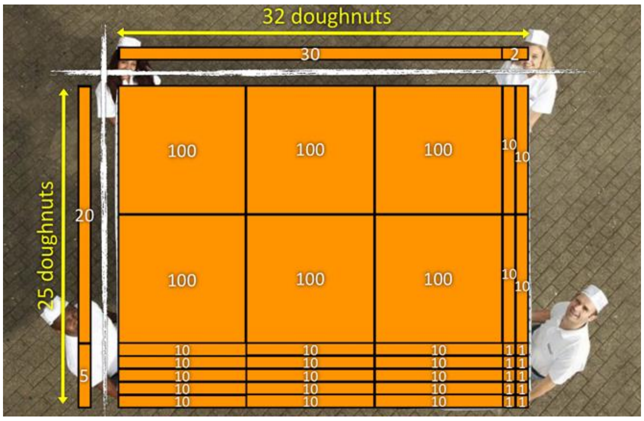


Figure 7. Find the Number of Donuts in the Box by representing Each Donut with a Unit Cube

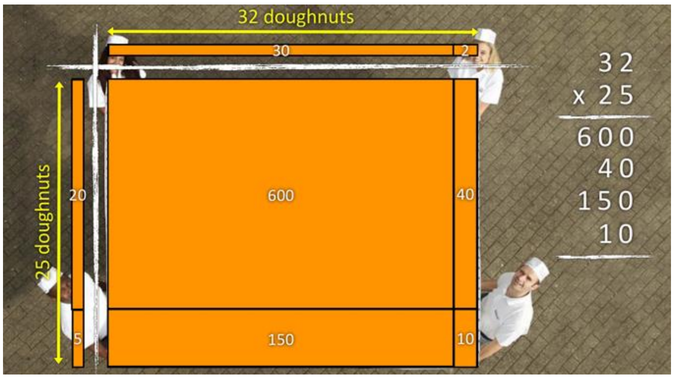


Figure 8. The Number of Donuts in the Box through an Area Model

Once students add 600, 40, 150, and 10, they will find 800 and compare this with their predictions. Then, the teacher will say she knows that the number of donuts is more than 800 (see Figure 9) as the top of the box states “DOUBLE HUNDRED DOZEN” (see Figure 10).

Figure 9. The Number of Donuts in the Box is more than 800

Figure 10. The Number of Donuts is Double Hundred Dozen

Then, ask students to calculate how many donuts are in the box. They will then compute 2x100x12 and see that there are 2,400 donuts in the box (see Figure 11).

Figure 11. There are 2,400 Donuts in the Box

Explain

Let students discuss how it is possible that there are 2,400 donuts when they know the dimensions as 800 (32x25). It is possible that some students can come up with the idea that there might be possible additional layers. Then, show students one more picture showing the side view (see Figure 12).

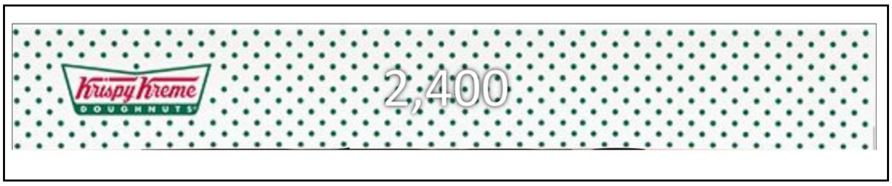


Figure 12. Side View of the Box including 2,400 Donuts

Seeing the side view will enable students to see that there are more layers since one layer includes 800 donuts. Then, ask students to think about how many layers are the box has. It is possible that they can divide 2,400 by 800 or they can add 800s until reach 2,400. They, they will find that there are 3 layers as 800x3=2,400 donuts (see Figure 13).

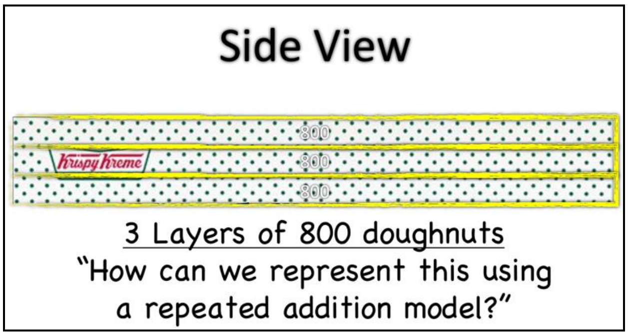


Figure 13. There are Three Layers of 800 Donuts

At this point, teachers should ask students if they knew that there were three layers along with the dimensions or the number of donuts in each row and column, what they would do to find the number of donuts in the box. Let students come up with that they would multiple the numbers as 3x25x32. Then, tell them if they could apply the same procedure if they had different dimensions (e.g., 5x16x24) and let them generalize that they can multiply to height, length, and width to find the number of donuts. Tell students that this is known as a volume of a right rectangular prism (or box).

Extend

Tell students that there are four types of donuts (i.e., glazed, chocolate, marble, and cinnamon) in the box. Tell students that 1/2 of the number of donuts are glazed type, 1/4 of the number of donuts are chocolate, 1/6 of the number of donuts are marble type, and 1/12 of the number of donuts are cinnamon). Tell students that they will arrange the side view in a way that people can tell how many of each type of donuts are in the box. Tell students that you like to see at least three different ways of side view arrangement. Once they have three different ways that each represent that 1/2, 1/4, 1/6, and 1/12 of the number of donuts are glazed, chocolate, marble, ad glazed types. Then, ask them to represent top views for each side views they made. Once students complete the views, ask them to engage in the following problem: if one donut contains 190 calories, how many calories are there in the box? Expect students to multiply 2,400 by 190 to get the total calories available in the box. Encourage students to apply different techniques such as 2,400x(200-10), 2,400x(100+90), and 24x(20-1)x1000. A number talk activity can be helpful for students to see different ways of computing this multiplication.

Evaluate

Teachers should look for if students can use several models when multiplying multiple digit numbers. If students have problems with any models (e.g., area, array, equal group), ensure that students engage in appropriate manipulatives such as geoboard for an array model, base ten materials for an area model, and snap cubes for an equal group model. Another thing teachers should look for is whether students can connect two-dimensional representations of three-dimensional representations. Because there are several ways students can arrange their side views and their top views of the box may or may not change correspondingly, they may struggle with the top view. Ensure that students discuss with their group and as a whole class about why or why not their top view of the box is right or wrong based on its side views. If students have a problem, engage them in representing a rectangular prism by using snap cubes. You can replicate the problem by using smaller numbers (e.g., 36 snap cubes represent donuts). Teachers should also look for if each student generalizes the formula of a rectangular prism's volume by asking them similar problems including different numbers. If students have problems with generalization, ensure that students see that you have height as repeated addition of layers. This can be accomplished by asking students to create a rectangular area by using snap cubes and asking them to replicate it more than one more time. After they see that they are repeatedly adding the number of models they build, they will see that they are multiplying the area by the height or the number of models they create to find the total number of snap cubes or volume. A teacher should provide students with snap cubes or similar manipulatives when students create their layers and see how they should multiply the height to find the volume of a rectangular prism. This is because when they actively involve building or construing their models with manipulatives (e.g., using snap cubes or similar manipulatives) that are not directly ready for them to use, they have a stronger connection with mathematical concepts to generalize compared to when they use manipulatives that they are ready to for them to use (e.g., using flats of base ten materials by placing them top of each other to see the height).

Task 11 - Graph Points on the Coordinate Plane to Solve Real-world and Mathematical Problems: Yummy-Yucky & Healthy-Unhealthy

Mathematical Content Standards

CCSS.MATH.CONTENT.5.G.A.1

Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates. Understand that the first number indicates how far to travel from the origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond (e.g., *x*-axis and *x*-coordinate, *y*-axis and *y*-coordinate).

CCSS.MATH.CONTENT.5.G.A.2

Represent real-world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation.

Mathematical Practice Standards

CCSS.MATH.PRACTICE.MP3

Construct viable arguments and critique the reasoning of others.

CCSS.MATH.PRACTICE.MP4

Model with mathematics.

CCSS.MATH.PRACTICE.MP6

Attend to precision.

Vocabulary

Coordinate plane, first quadrant, vertex, origin, and axes.

Materials

Post-it Super sticky Easel Pad, 25x30 inches, paper, pencil, coordinate plane templates (see Appendix).

Objectives

Students will develop the meaning of the coordinate plane by using their qualitative comparison of "things" that can be objects, activities, foods, humans, places, transportations, animals, etc. Students should select their items in a way that the selected items have at least two distinguishing characteristics. For example, if a group of students selects food, they can consider two characteristics of food (e.g., yummy vs. yucky and healthy vs. unhealthy). Students' creativity will be developed as they use their previous knowledge to construct the meaning of the coordinate plane. Students will also develop the convention of (x, y) and be able to locate numbers in the first quadrant. Their creativity will be developed as they will be generalizing the fact that (a,b) is not equal to (b, a) other than when a = b. Students' creative thinking skills also be developed as they will be encouraged to think strategically to find the location of vertices of the shapes after hearing feedback from their partners. This lesson will take two class time.

Engagement

(10 minutes) Start the class by asking students to work as a group of three and provide them with Post-it Super sticky Easel Pad, 25x30 inches to draw their coordinate planes. Let students pick their own "thing" based on their interests. Ensure what they select has at least two distinguishing characteristics. For example, if a group of students selects "food", the two

characteristics can be "taste" and "healthy". Also, ensure that the opposite of each characteristic is available (e.g., tasty-yucky, healthy-unhealthy) (see Figure 1). Another group of students can pick animals as their "thing" and they can think about two characteristics of animals such as animals' sizes (big vs small) and their speed (fast vs slow). The important thing is to ensure that each distinguishing characteristic can be represented in two opposite spectrums. Based on what they select, a teacher can ask students to collect more qualitative data from others to see if they miss important and interesting knowledge. For example, they may not list a sloth or a turtle as an animal before talking to others. However, since a sloth or a turtle moves very slowly, including this kind of example (e.g., sloth) can help them interpret coordinate planes easily (i.e., what happens moving from left to right or down to up on the coordinate plans).

Exploration

(25 minutes) Teachers can show a coordinate plane (see Figure 1) example and ask students to interpret it. It is essential for teachers to carefully listen to students' observations to see if they correctly interpret the foods represented on the coordinate plane (see Figure 1) in terms of their taste and health.

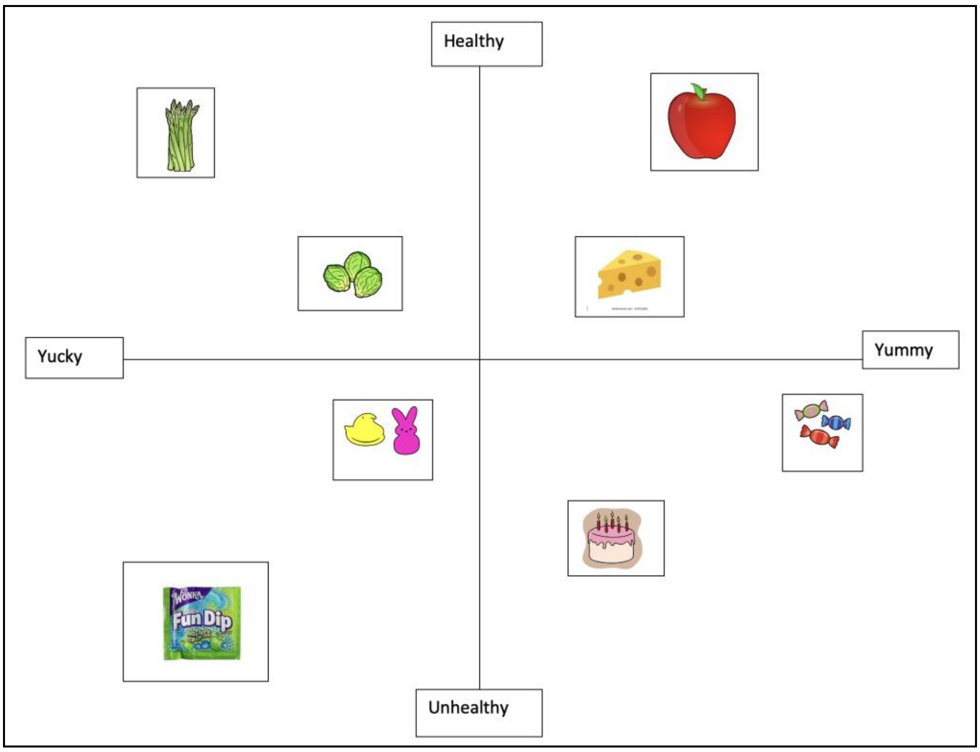


Figure 1. Two Characteristics of Foods as Yucky-yummy and Unhealthy-healthy

It is possible that some students only pay attention to one characteristic (e.g., asparagus tastes worse than brussels sprouts). Teachers should emphasize the other dimension by asking which one is healthier or unhealthier. Then, students can state "asparagus tastes worse than brussels sprouts, but it is healthier than brussels sprout." Teachers can ask several other questions to students as follow:

1) What food are both healthy and yummy?
2) What foods are both yucky and unhealthy?
3) Is there a food that you don't agree with the placement of? If so, which one?
4) Can you think of some more food items to add to the graph? If so, which ones and where would you place them?

Once every group of students complete their coordinate planes, ask them to hang them on the wall so that others can make their observations and share what ideas are interesting or new to them during a short whole-class discussion.

Explain

(25 minutes) After students understand how to interpret the coordinate planes qualitatively, it is time for them to understand how they can interpret the location of "things" on the coordinate plane quantitatively. A teacher should ask questions that students extend their number line understanding to the coordinate planes. For example, a question might be "what if a food is placed right in the middle of the coordinate plane? Is it yummy or yucky? Is it healthy or unhealthy? At this moment, students can say it is neither yucky nor yummy.

Similarly, it is neither healthy nor unhealthy. Then, a teacher asks if we represent that location (i.e., the intersection of the lines) with the numbers, what it would be? Students can think what number is neither negative nor positive and tell the answer is 0. Then, tell students that since they have two characteristics to represent, they can use the following format as (taste, health) = (0,0).

After locating (0,0) on the coordinate plane, tell students that this location is named as the origin. After they understand the meaning of origin, ask them what number can represent cheese (see Figure 2) on the coordinate plane. To eliminate ambiguity, you can say the highest number for taste and health can be 5.

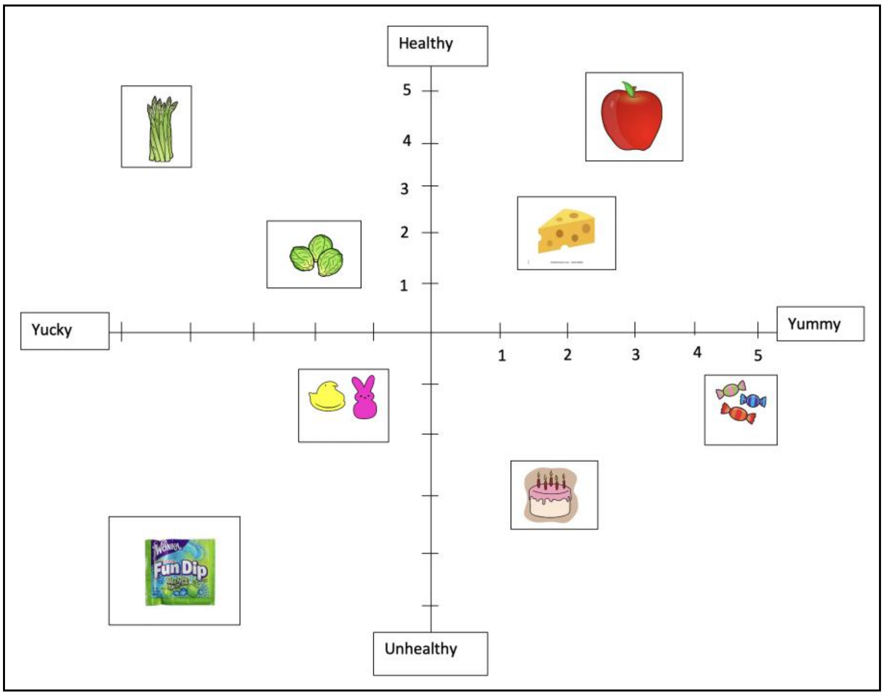


Figure 2. Locating Numbers on the Coordinate Plane

Because other food items' locations may require students to think fractions (see Figure 2), it is better for students to first engage in activities that only require whole numbers. Teachers can provide students with another example showing the locations of animals in the zoo (see Figure 3).

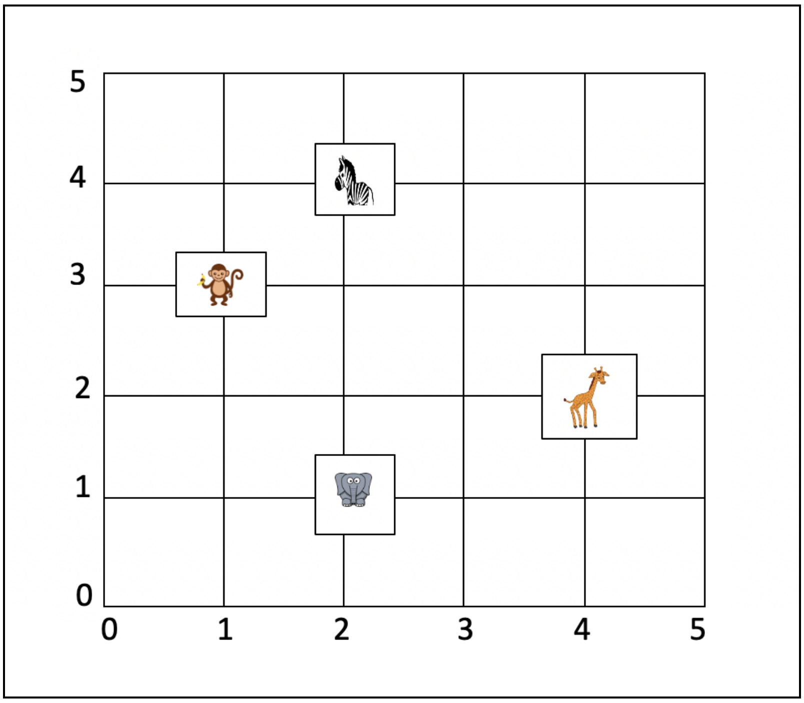


Figure 3. Location of Animals living in the Zoo

This activity takes place in the first quadrant according to the 5h grade measurement Common Core State Standards. Teachers can ask students to play games with similar coordinate planes that the things are at the exact points on the grid. For example, one student can guess the locations of the animals living in the zoo (see Figure 3) and the other two students can check if any animals are located on the point. When the student's guess states a correct point where there is an animal, the other two students with the grid say, “you got it” and tell the names of the found animal (e.g., elephant). When the student is guessing and does not state the correct coordinates of an animal, the student will say, “you did not get it”. They will continue to guess points until all of the animals are found.

Extent

(10 minutes) After students are comfortable with locating animals in the zoo, teachers can continue asking questions about the food items that their locations require students to think fractions. For example, a teacher can ask students to find the location of the apple (see Figure 2) on the coordinate plane. Students can see that on the taste dimension the apple is located right in the middle between 3 and 4 and they can state that it is on the point of 3 ½. For the health dimension, students can see that the apple is in the middle between 4 and 5 and state that it is on the point of 4 ½.

Explain and Explore

(30 minutes) It is time for teachers to tell students that we generally represent the horizontal line with the letter x and the vertical line with the letter y. Then, teachers can ask students to play more games by using the guessing and checking technique. Teachers can ask students to play the following game with partners. One partner can draw a 2D geometrical shape that has vertices (e.g., rectangle, square, triangle) in the first quadrant and the other partner can guess it (see Figure 4 & Appendix).

When the student who is guessing states a vertex of the shape, the partner says, “you are on it”. When the point is on a side or inside of the shape, the partner says, “you are near it”. When the point is inside of the shape, the partner says, “you are inside of it”. When the point is outside of the shape, the partner says, “you did not get it”. When all the vertices of a shape are found, the partner says, “here is my shape (e.g., triangle, rectangle)” and shows it to his/her partner. Students then change their roles and replay.

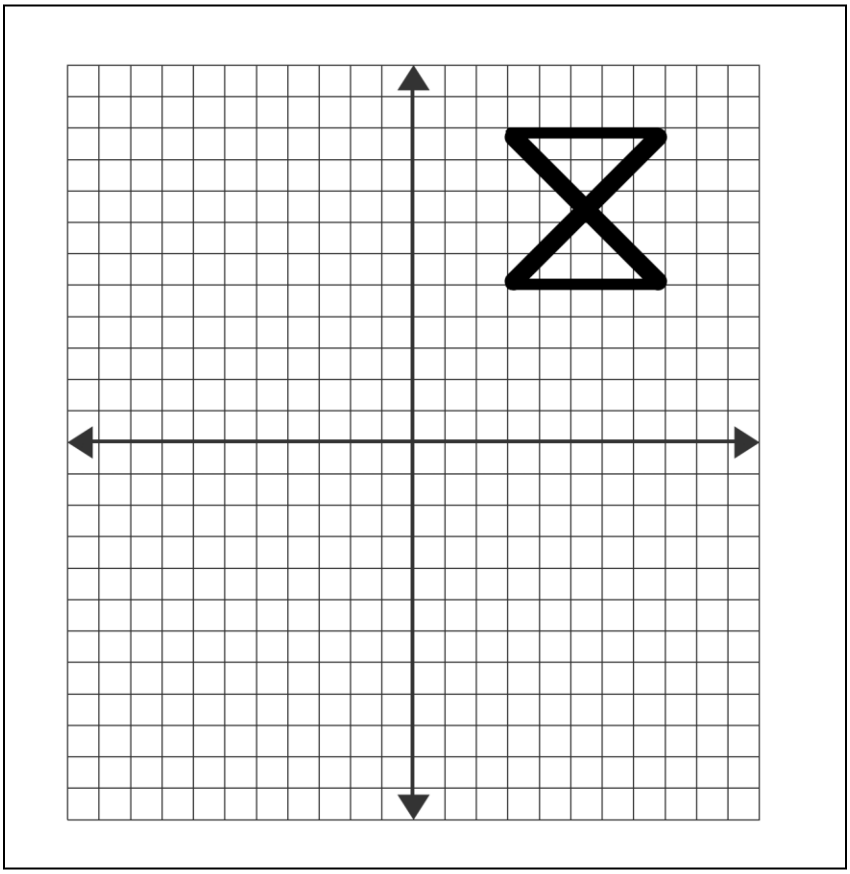
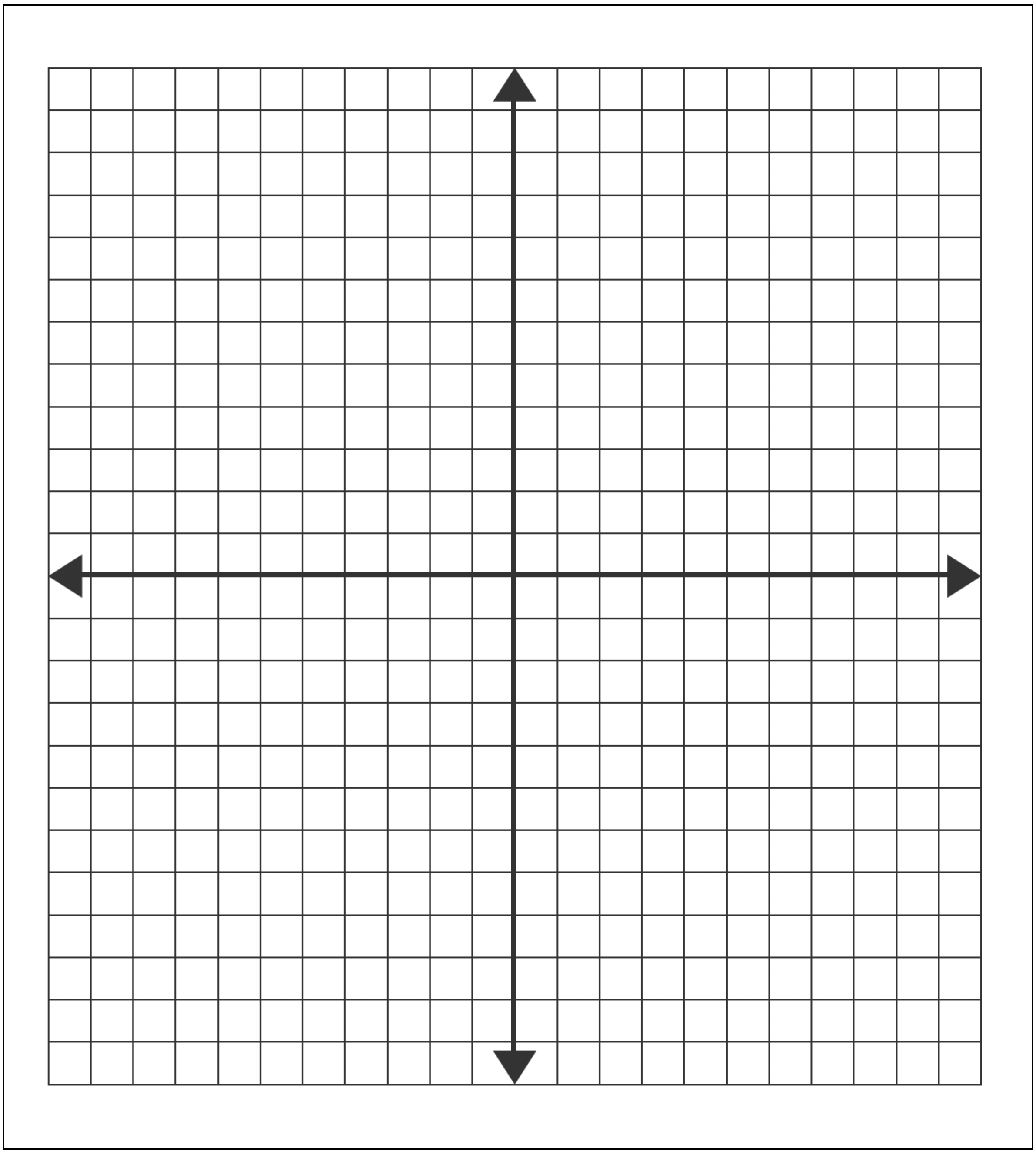


Figure 4. Find my Shape in the First Quadrant

Evaluate

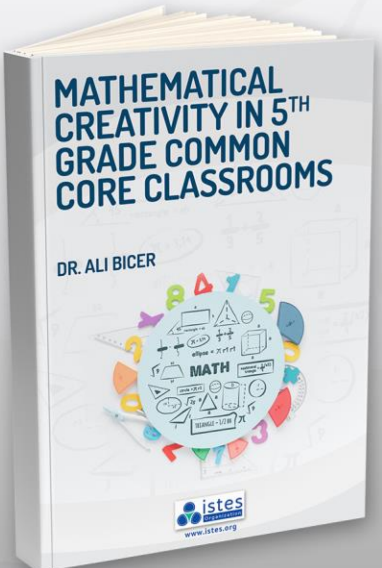
Teachers should look for if students simultaneously consider two characteristics of the selected “thing” represented on the coordinate planes. Teachers should emphasize the other distinguishing characteristic if students missed one by asking them questions about the other characteristics. Teachers should encourage students to construct a sentence that includes both characteristics of the selected “thing”. Teachers also should look for if students grasp the meaning of four quadrants when they are talking about the relationship between two or more different items in the coordinate plane. If students have difficulty with understanding the meaning of four quadrants, especially the first quadrant as it is the focus standard in the 5th grade, teachers can provide more examples showing what happens when they move from left to right, right to left, up to down, or down to up. Another look is if students use correct terminology by using the (x, y) convention. Students may confuse about which one was coming first horizontal or vertical dimensions. To eliminate this, teachers first should use what x and y represent (e.g., taste and health) and gradually move to represent them with (x, y). Teachers should carefully look for that if students locate points (a, b) and (b, a) interchangeably. If that happens, locate example points in the coordinate plane so that students can see that (1,2) is not the same as (2,1). Then, they can generalize that (a, b) is not equal to (b, a) in the coordination plane other than the case when a is equal to b. Lastly, a teacher should carefully observe if students think reasonably about the feedback they get from their partner while they are playing the games (i.e., find my shape). For example, when a partner says, “you are near it”, a student is expected to add or subtract from x or y or x and y together since the meaning “you are near it” is the point they said is on a side or inside of the shape. If they say a point that is very far away or random after they received the feedback “you are near it”, teachers should provide a couple of attempt examples showing how they strategically think to find the locations.

Appendix



ENGAGE YOUR STUDENTS IN RICH, VISUAL, CREATIVE, HIGH-COGNITIVE DEMAND, LOW FLOOR & HIGH CEILING, OPEN-ENDED MATHEMATICAL TASKS

Creativity-directed tasks aim to develop students' creative thinking skills while they construct new mathematical ideas, concepts, and knowledge. Although research suggests that teachers should implement creativity-directed tasks in their classrooms to foster their students' mathematical creative thinking skills, it is unclear how teachers can implement these tasks while trying to address Common Core Standards. This book includes 10 research-based, Common Core content and practice standards-aligned, visual, creative, low-floor & high ceiling, high-cognitive-demand, and rich open-ended tasks that will enable your students to think deeply and reason mathematically, problem-solve, problem-pose, discuss and convince, explore multiple solutions methods, connect multiple representations, justify their thought processes, and generalize their mathematical observations. The lesson plans encourage students to understand that mathematics is a creative field in which they can be creative! There are 11 chapters addressing 10 Common Core 5th-grade content practices along with Common Core practice standards. This book provides teachers with the lesson plans about the following content standards: 1) write and interpret numerical expressions, 2) analyze patterns and relationships, 3) understand the place value system, 4) perform operations with multi-digit whole numbers with decimals to hundredths, 5) use equivalent fractions as a strategy to add and subtract fractions, 6) apply and extend previous understanding of fraction multiplication, 7) apply and extend previous understanding of fraction division, 8) convert like measurement units within a given measurement system, 9) represent and interpret data, 10) geometric measurement: understand concepts of volume, and 11) graph points on the coordinate plane to solve real-world and mathematical problems.



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